1. Let $f : [0, 3] \to \mathbb{R}$ be given by
$$f(x) = \begin{cases} -1 & \text{if} \quad 0 \le x \le 1, \\ 0 & \text{if} \quad 1 < x \le 2, \\ 1 & \text{if} \quad 2 < x \le 3. \end{cases}$$
Using the definition of riemann integrability show that $f \in \mathcal{R}[0, 3]$.

2. Let $f : [0, 1] \to \mathbb{R}$ be defined as
$$f(x) = \begin{cases} x & \text{if} \quad x = 1/n \text{ for some } n \in \mathbb{N}, \\ 0 & \text{if} \quad x \ne 1/n \text{ for any } n \in \mathbb{N}. \end{cases}$$
Show that f is riemann integrable.

3. Let $f : [0, 1] \to \mathbb{R}$ be given by
$$f(x) = \begin{cases} x & \text{if} \quad x \text{ is rational}, \\ 0 & \text{if} \quad x \text{ is irrational}. \end{cases}$$
Show by using the cauchy criterion that f is not riemann integrable.

4. Consider the collection of functions $f, g : [0, 1] \to \mathbb{R}$ such that f and g are bounded, but, neither f nor g is riemann integrable. Find examples from such a collection satisfying each of the following
   (a) $f + g$ is riemann integrable
   (b) fg is riemann integrable
   (c) $|f|$ is riemann integrable.

5. Let $f \in \mathcal{R}[0, 1]$ and $(\dot{\mathcal{P}}_n)$ is a (particular) sequence of tagged partitions of $[0, 1]$ such that $\lim\limits_{n\to\infty} \|\dot{\mathcal{P}}_n\| = 0$. If $\lim\limits_{n\to\infty} S(f, \dot{\mathcal{P}}_n) = 0$, show that $\int_0^1 f = 0$.

6. Evaluate $\lim\limits_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\sqrt{in+n^2}}$.

7. If $f, g : [a, b] \to \mathbb{R}$ are continuous and if $\int_a^b f = \int_a^b g$, prove that there exists a $c \in [a, b]$ such that $f(c) = g(c)$.

8. {Mean value theorem of integral calculus} For reals $a \le b$, let $f : [a, b] \to \mathbb{R}$ be continuous. Show that there exists a $\xi \in [a, b]$ such that $\int_a^b f = f(\xi)(b - a)$.

9. Let $f : [0, 1] \to \mathbb{R}$ be continuous. Suppose $\int_0^x f = \int_x^1 f$ for all $x \in [0, 1]$. Then, show that $f(x) \equiv 0$.

10. Find an $f : [0, 1] \to \mathbb{R}$ such that f is differentiable 4 times but not the 5–th time.
{Hint: Fundamental Theorem of Calculus}