1. (a) Show that for every $x \in \mathbb{R}$, there exists a unique $z \in \mathbb{Z}$, such that $z \leq x < z + 1$
   (b) Define a functon $f : \mathbb{R} \to \mathbb{Z}$ via $f(x) = z$, the unique z in part (a). f is called "the greatest-integer-function" and the value $f(x)$ is usually denoted as $[x]$.
   (c) Show that f is continuous at every $c \in \mathbb{R} \setminus \mathbb{Z}$, i.e., for every $c \in \mathbb{R}$ but $c \notin \mathbb{Z}$. Further, show that f is discontinuous for every $c \in \mathbb{Z}$.
   (d) Define $g : \mathbb{R} \setminus \mathbb{Z} \to \mathbb{Z}$ via $g(x) = f(x)$. Draw a graph of g. Show that g is a continuous function.
2. Applying the Weierstrass' Criterion, prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by $x \mapsto x^3$ is continuous.
3. Define $g : \mathbb{R} \to \mathbb{R}$ by $x \mapsto 2x$ if x is rational, and $x \mapsto x + 3$ if x is irrational. Find all points of continuity of g.
4. Prove that the (six basic) trigonometric functions are continuous.
5. Show that the polynomial $p(x) := x^4 + 7x^3 - 9$ has at least two real roots. Use a calculator to locate these roots to within two decimal places.
6. Show that every real polynomial of odd degree has at least one real root.
7. Find examples for or prove non–existence of continuous functions whose domains and ranges are from the collection
$$\{[0, 1], [0, 100], [0, 1), [0, 100), (0, 1], (0, 100], (0, 1), (0, 100), \mathbb{R}\}.$$