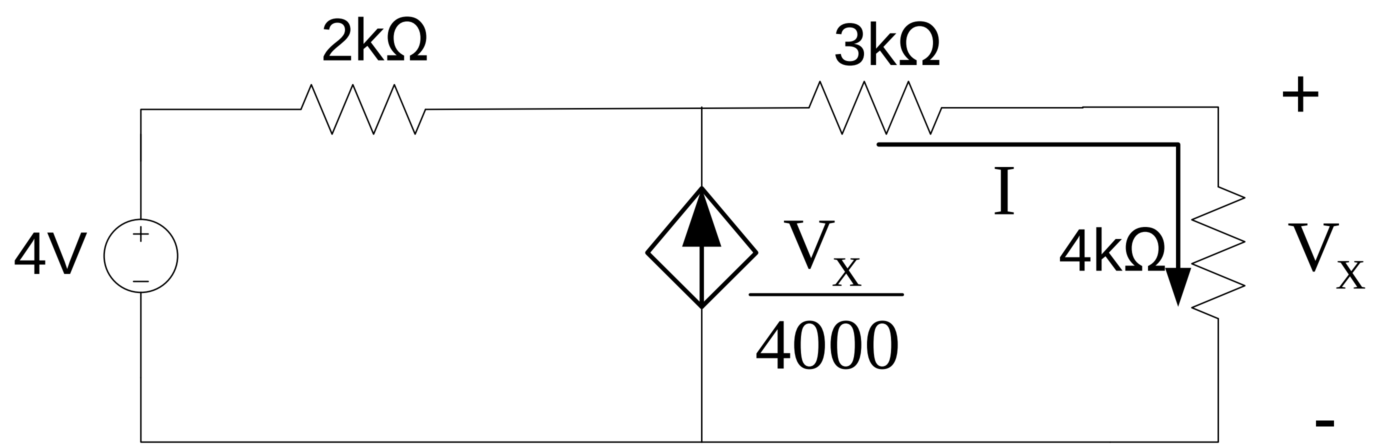
EE 101ME – Electric Circuits

Revisiting Thevenin's Theorem

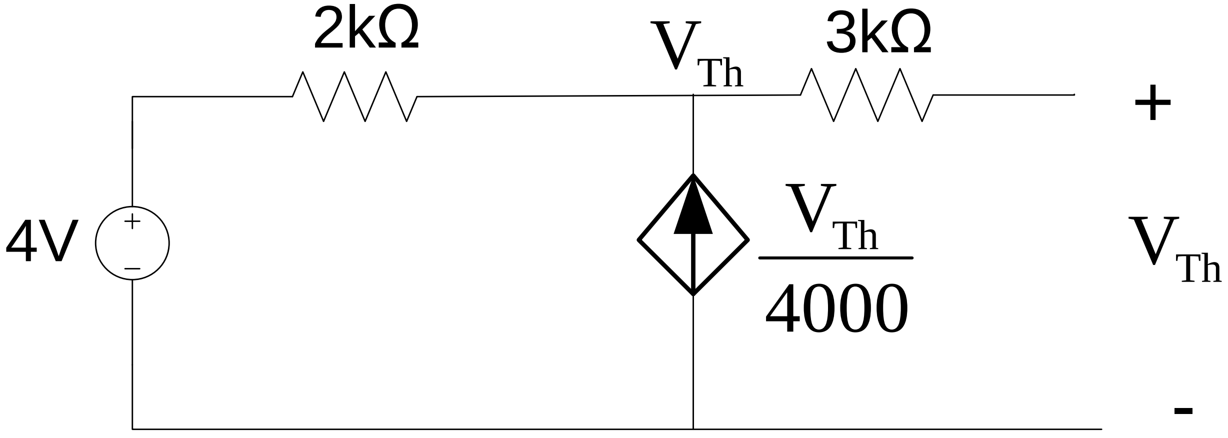
Find I (current through 4kΩ) applying Thevenin's Theorem



Solution

V_{Th}

1. Open Circuit the 4kΩ resistance



Applying nodal analysis,

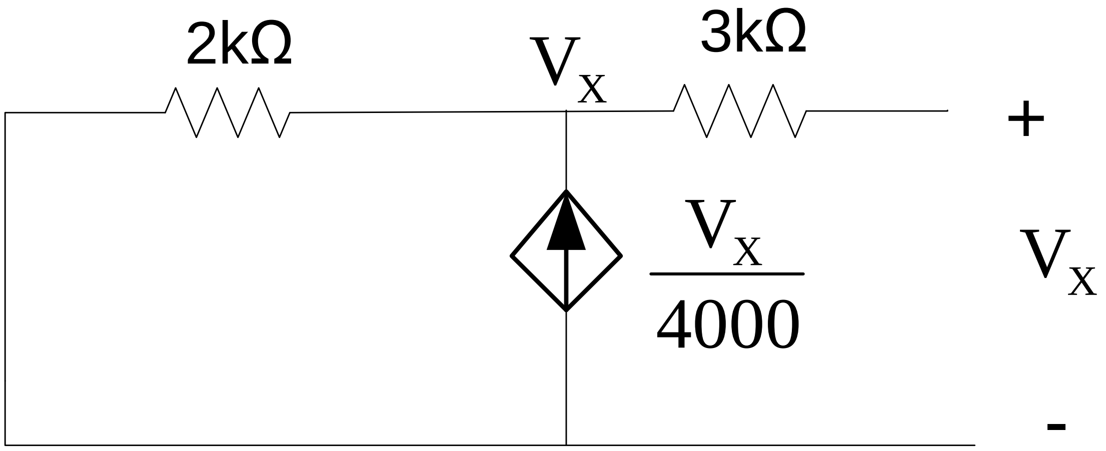
$$\frac{V_{Th}-4}{2000}=\frac{V_{Th}}{4000}$$

$$\text{or,}\quad 2V_{Th}-8=V_{Th}$$

$$V_{Th}=8V$$

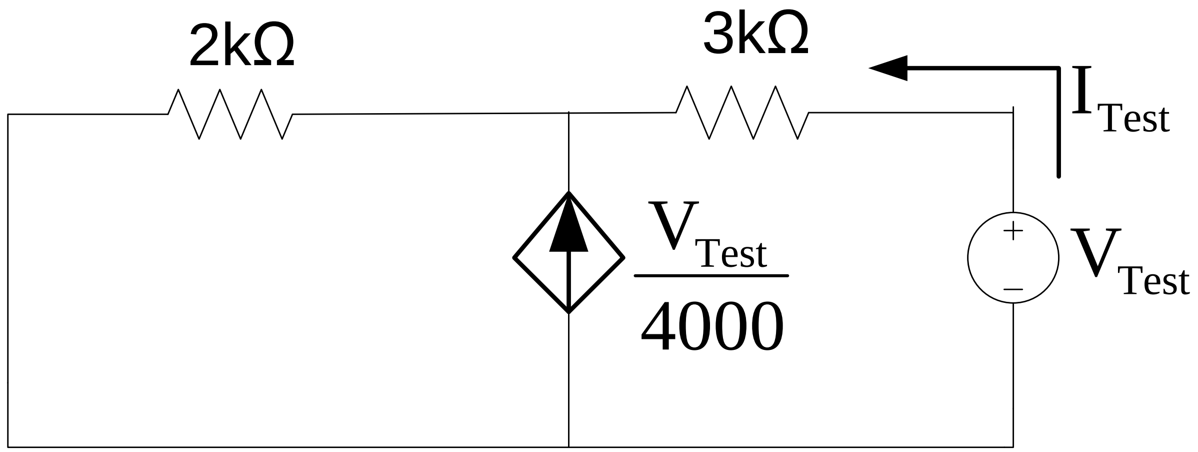
R_{eq}

1. Deactivate the independent sources



2. Do NOT deactivate dependent sources

1. Apply a test voltage V_{Test}



Applying nodal analysis,

$$I_{Test} + \frac{V_{Test}}{4000} = \frac{\left(V_{Test} - I_{Test} \times 3000\right)}{2000}$$

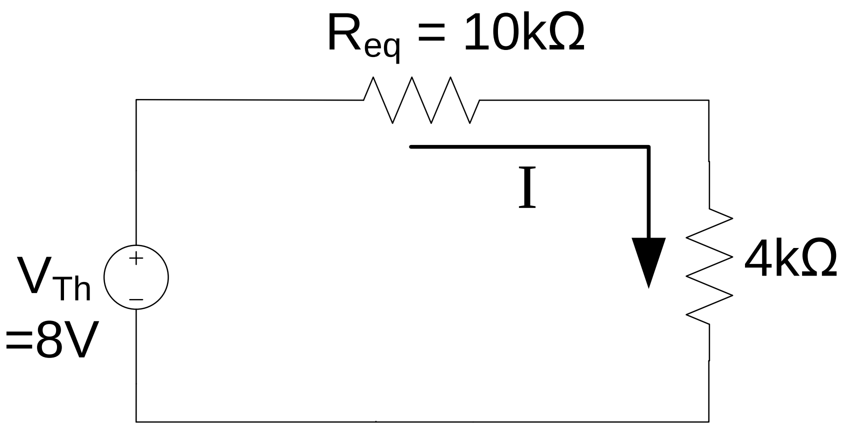
$$4000I_{Test} + V_{Test} = 2V_{Test} - 6000I_{Test}$$

$$V_{Test} = 10000I_{Test}$$

$$\frac{V_{Test}}{I_{Test}} = 10000 = 10k\Omega$$

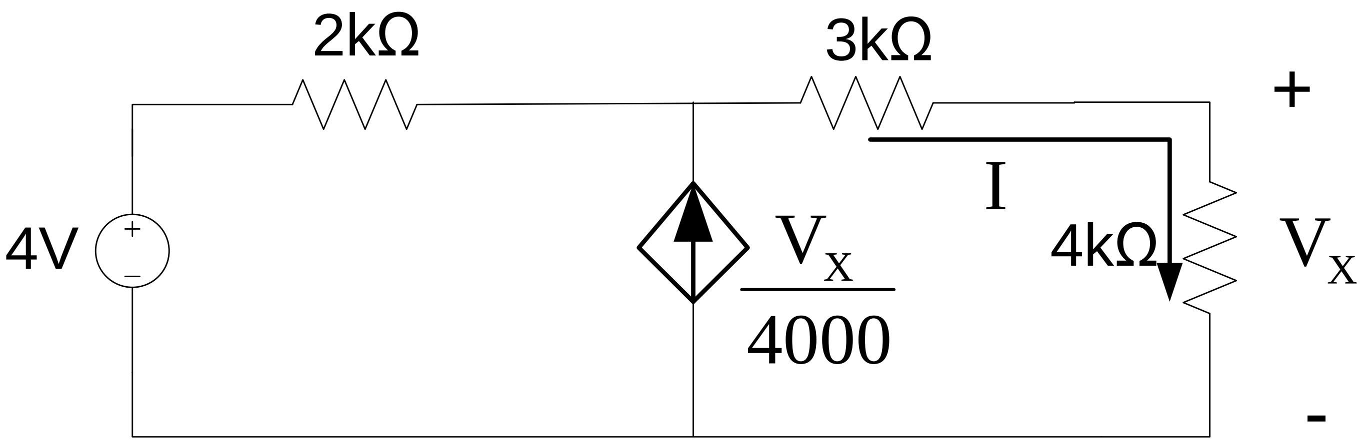
$R_{eq} = 10k\Omega$

Thevenin's Equivalent Circuit



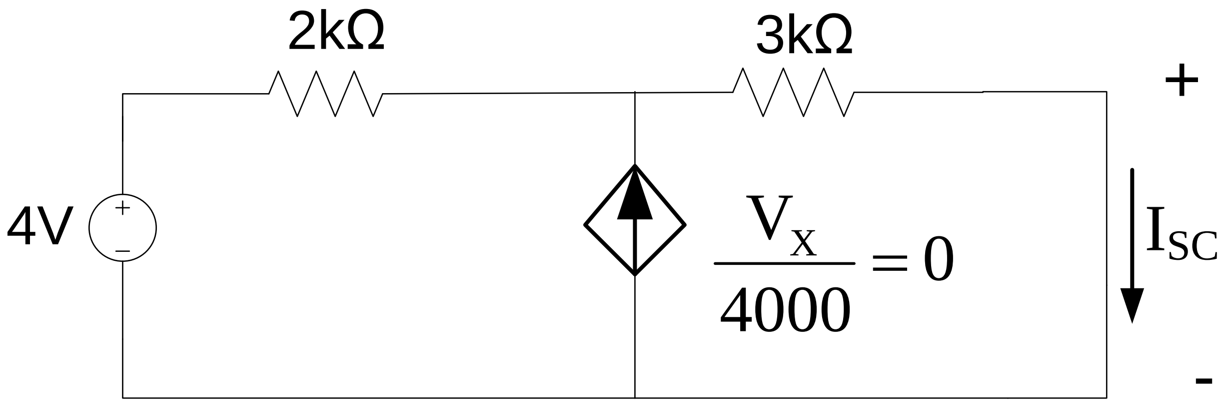
$$I = \frac{8}{10+4} = 0.571\text{mA}$$

Alternative Method to find $R_{eq} = R_{th}$

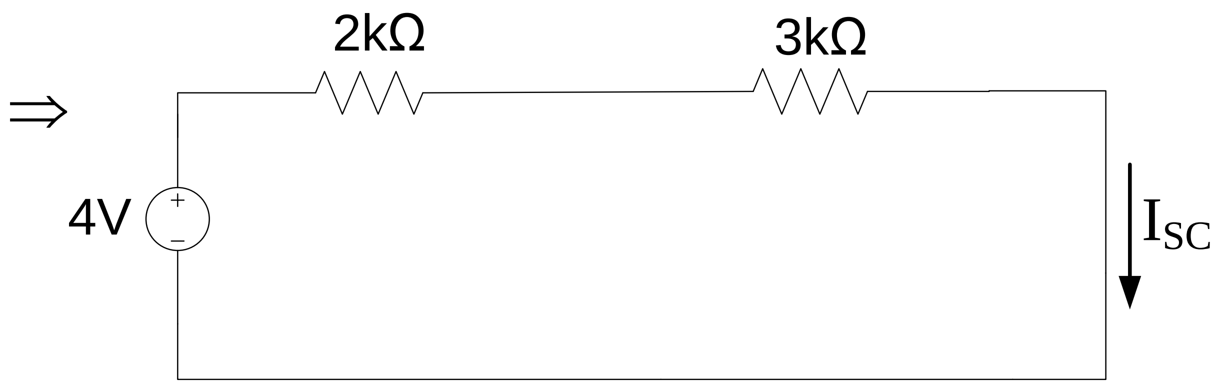


We know $V_{Th} = 8V$

Find Short Circuit Current I_{sc}

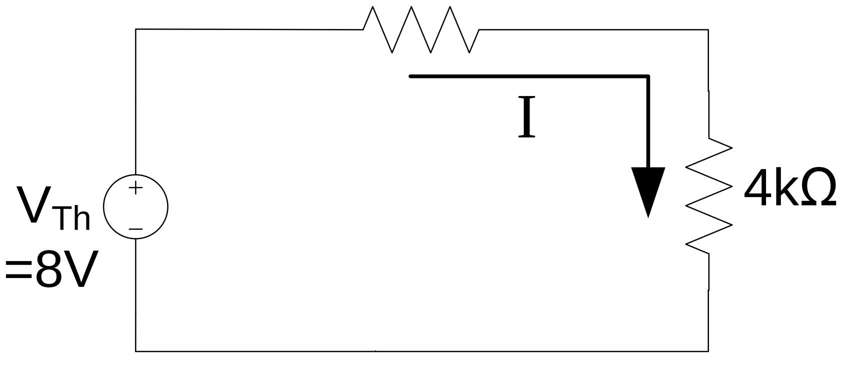


$$V_X = 0$$



$$I_{SC} = \frac{4}{2k + 3k} = 0.8mA$$

$$R_{eq} = \frac{V_{Th}}{I_{SC}} = \frac{8V}{0.8mA} = 10k\Omega$$



$$I = \frac{8}{10+4} = 0.571mA$$

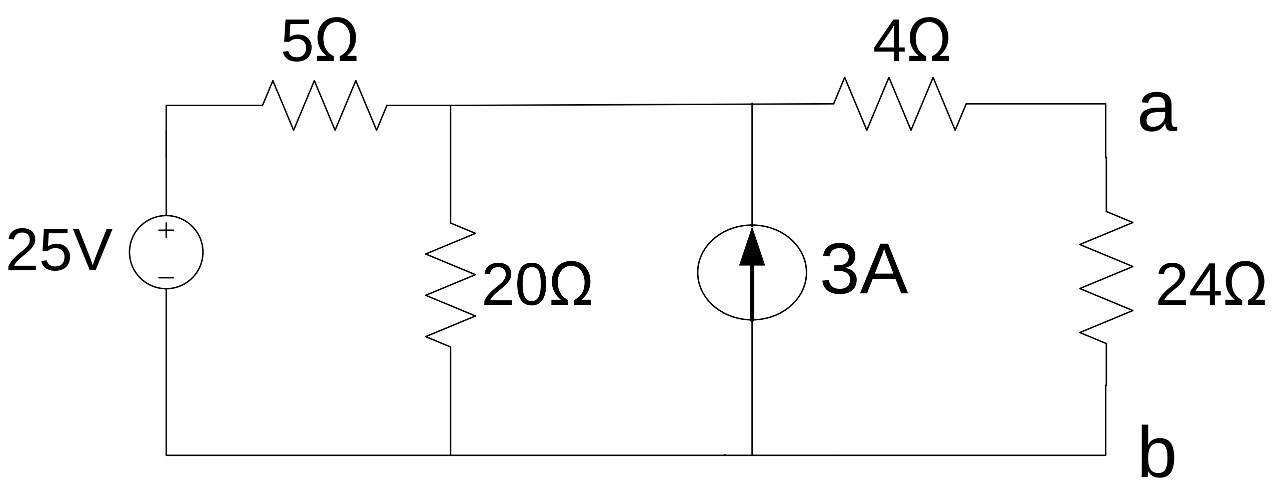
Thevenin's Equivalent Circuit

Norton's Theorem

Norton's Theorem

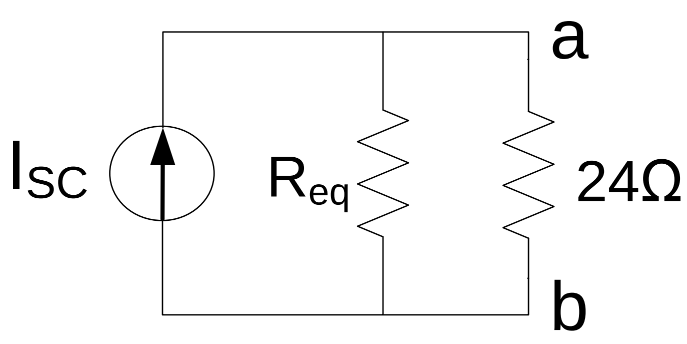
Theorem : Any two -node linear network may be replaced by a current source equal to the short circuited current between the nodes in parallel with the resistance as seen by a load at this port (terminal/node pair)

Q. Find voltage V_{ab} across terminal a-b (across 24Ω load) using Norton's theorem.

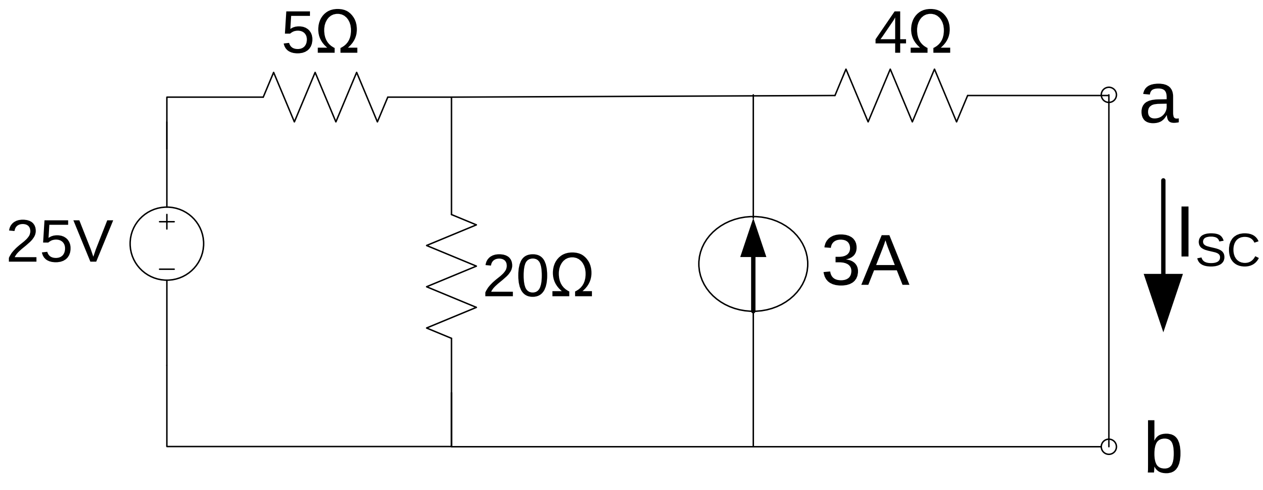


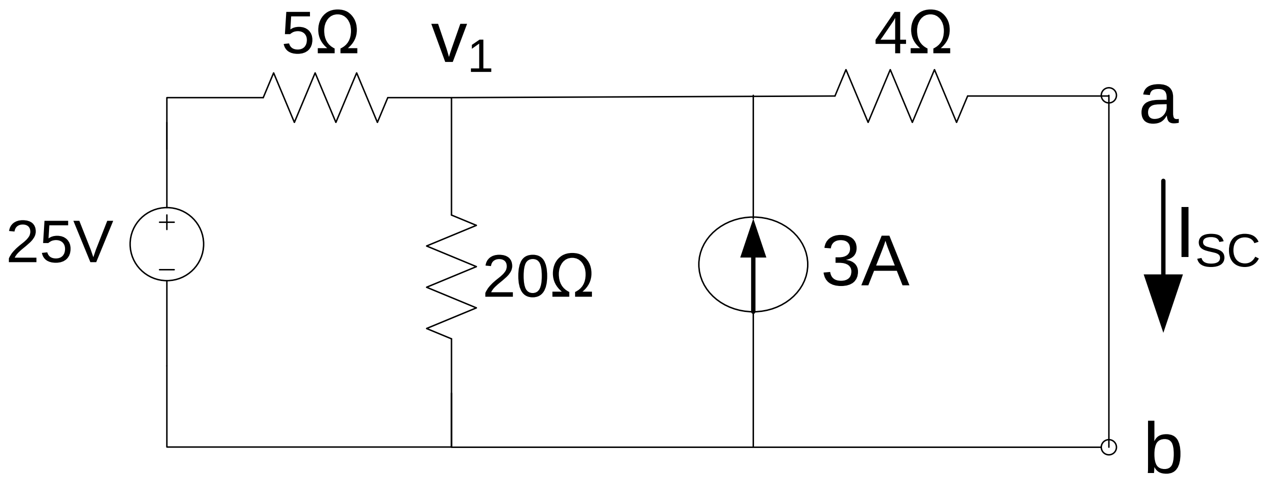
Norton's Equivalent Circuit

$\Rightarrow$



1. Short Circuit a-b and find I_{SC}





Applying nodal analysis,

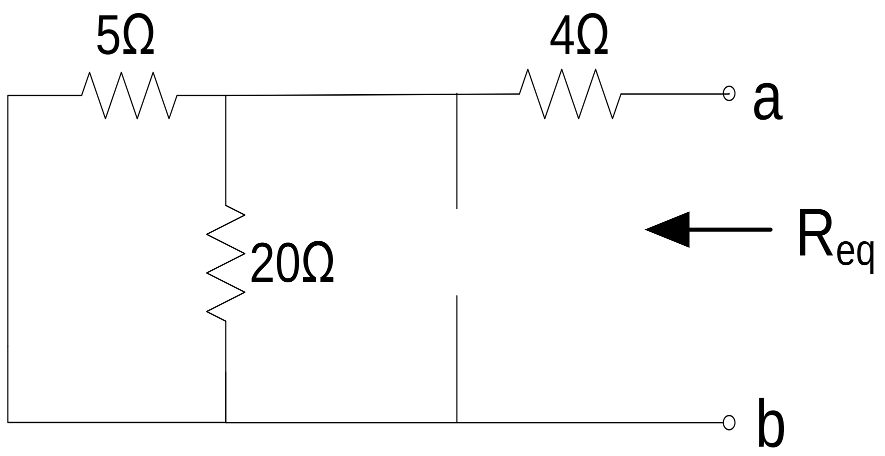
$$\frac{v_1 - 25}{5} + \frac{v_1}{20} + \frac{v_1}{4} = 3$$

$$v_1 = 16V$$

$$I_{SC} = \frac{v_1}{4} = \frac{16}{4} = 4A$$

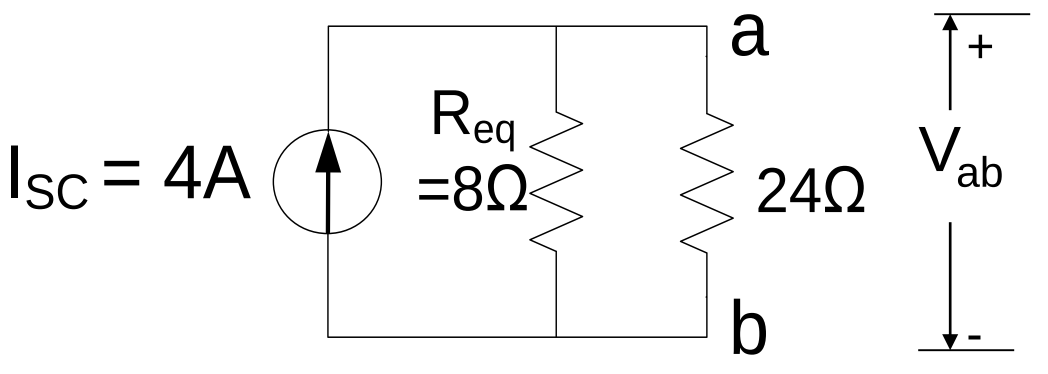
R_{eq}

- ➢Deactivate all independent sources
- ✓Voltage sources – short circuit
- ✓Current sources – Open circuit



$$R_{eq} = 4 + 20 \| 5 = 4 + 4 = 8\Omega$$

$\Rightarrow$



Norton's Equivalent Circuit

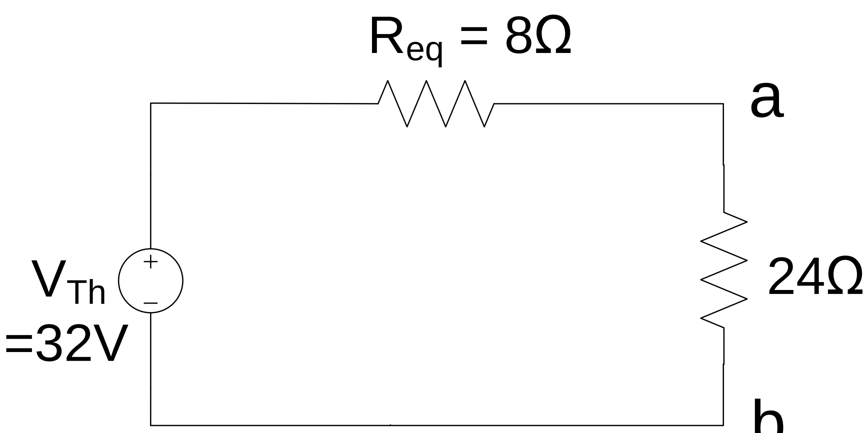
Voltage V_{ab} across terminal a-b (across 24Ω load) is

$$V_{ab} = 4 \times \frac{8}{8+24} \times 24 = 24V$$

Thevenin's Equivalent Circuit

By Source Transformation

Norton's Equivalent Circuit

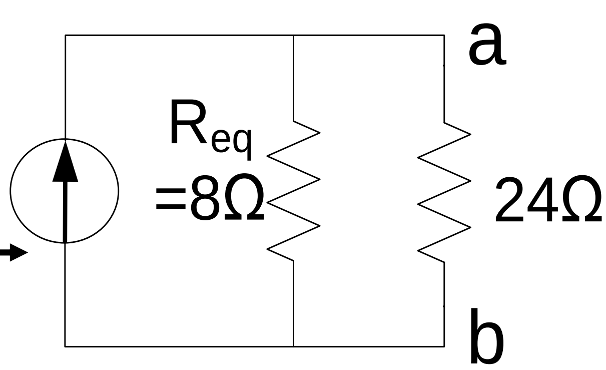


Thevenin's Equivalent Circuit

⇒

$I_{SC} = 4A$

Source Transformation



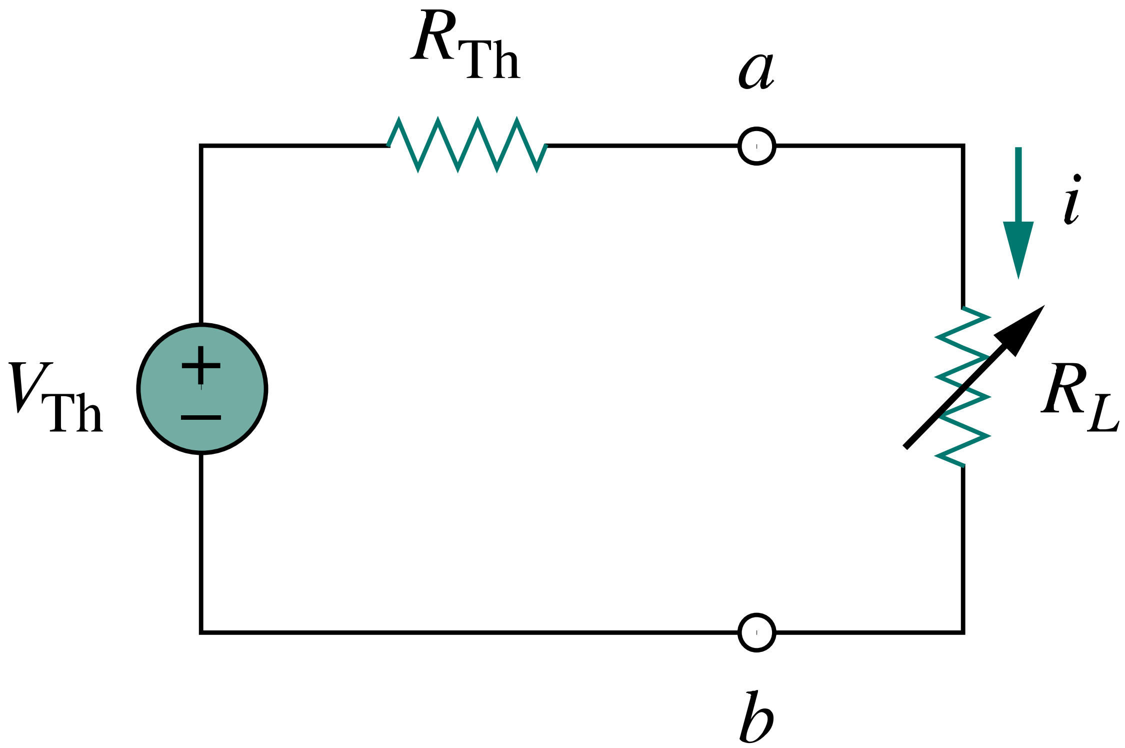
Norton's Equivalent Circuit

$$I_{SC} = \frac{V_{Th}}{R_{eq}}$$

$$R_{eq} = \frac{V_{Th}}{I_{SC}}$$

Maximum Power Transfer

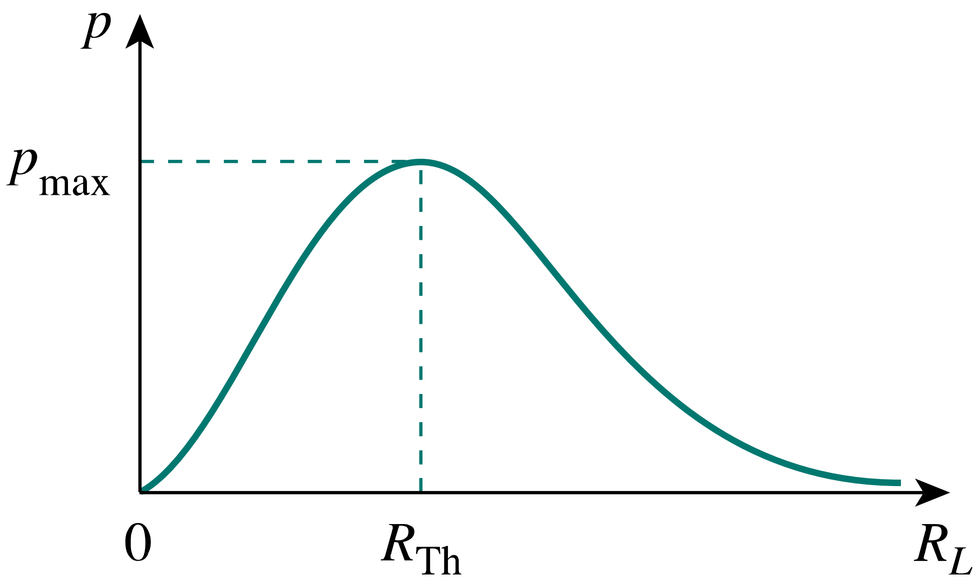
Maximum Power Transfer



Power delivered to load

$$p = i^2 R_L = \left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} \right)^2 R_L$$

Plot between p and R_L



Expression for P_{max}

$$\frac{dp}{dR_L} = V_{\mathrm{Th}}^2 \left[\frac{(R_{\mathrm{Th}} + R_L)^2 - 2R_L(R_{\mathrm{Th}} + R_L)}{(R_{\mathrm{Th}} + R_L)^4} \right]$$

$$= V_{\mathrm{Th}}^2 \left[\frac{(R_{\mathrm{Th}} + R_L - 2R_L)}{(R_{\mathrm{Th}} + R_L)^3} \right] = 0$$

which gives

$$\boxed{R_L = R_{\mathrm{Th}}}$$

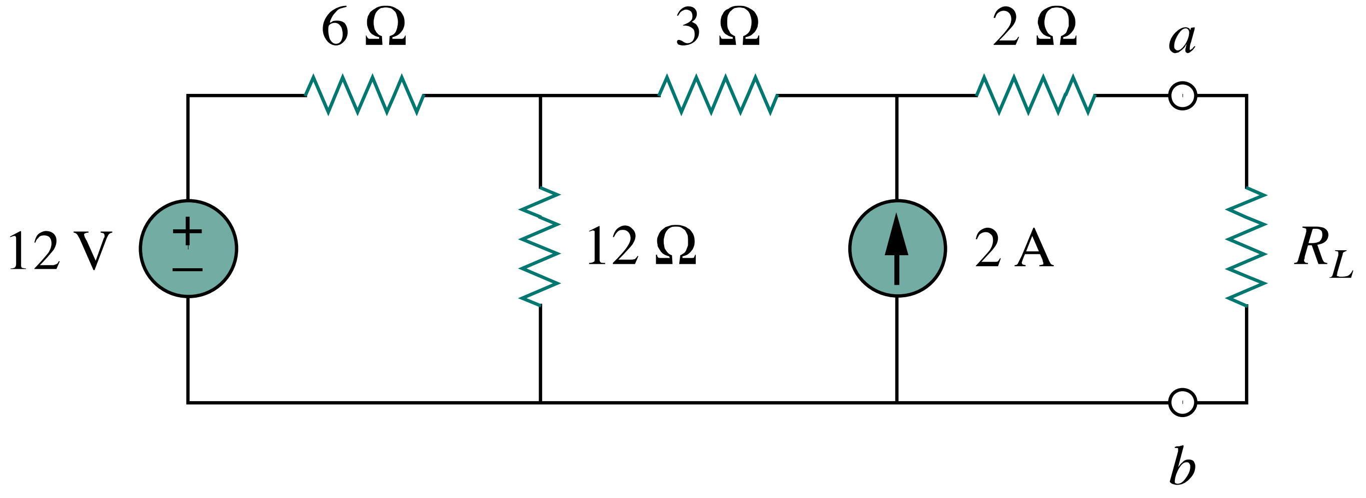
for delivering maximum power to the load resistor.

- Then, expression for maximum power

$$p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}}$$

Example

- Find maximum power that can be transferred to the load resistor R_L .



Find R_{TH} and V_{TH}

- Circuits to find the equivalent quantities

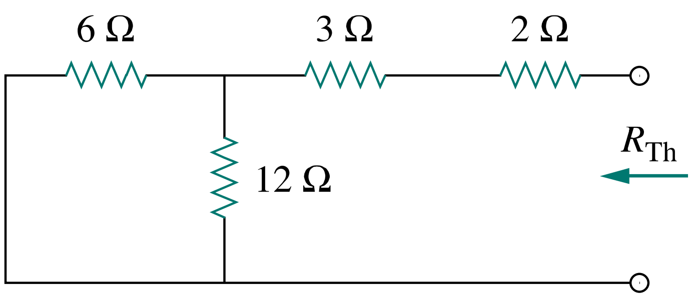


Fig.1

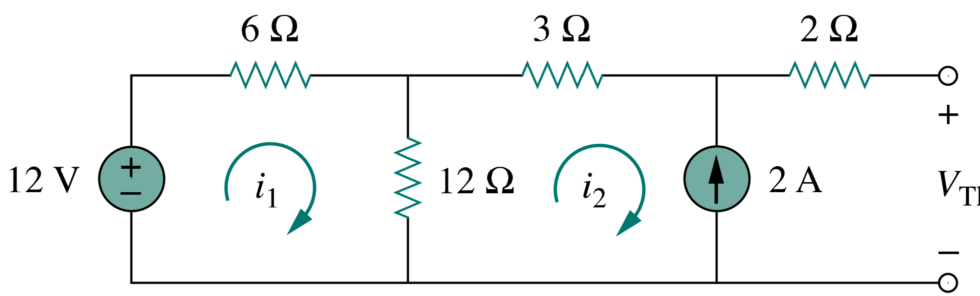


Fig.2

From Fig.1 it is easy to find $R_{th} = 9\ \Omega$

Let's use mesh analysis of Fig.2. Loop current $i_2 = -2$ A

Applying KVL to the leftmost loop of Fig.2, we get $-12 + 6i_1 + 12(i_1+2) = 0$

Solving the above equation we get $i_1 = -2/3$ A

KVL of the outer loop enables us to write

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \quad \Longrightarrow \quad V_{\text{Th}} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{\text{Th}} = 9\ \Omega$$

and the maximum power is

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

END