EE 1102H – Electric Circuits

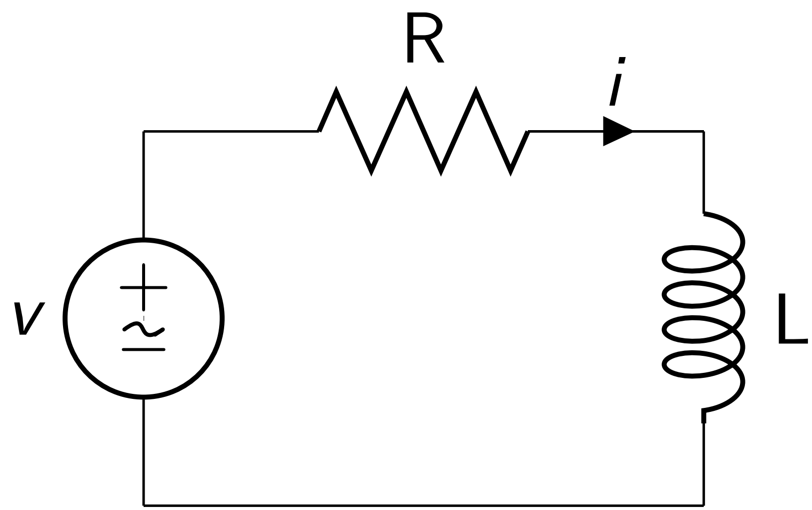
Reviews of Instantaneous Power, Average Power

Apparent Power

Power Factor (PF)

PF improvement

Instantaneous Power



Let $v = V_m \sin(\omega t + \theta)$

Then $i = I_m \sin(\omega t + \phi)$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \text{ and } \phi = -\tan^{-1}\frac{\omega L}{R} + \theta$$

$$\Rightarrow \theta - \phi = tan^{-1}\frac{\omega L}{R}$$

$$p(t) = v(t)i(t)$$

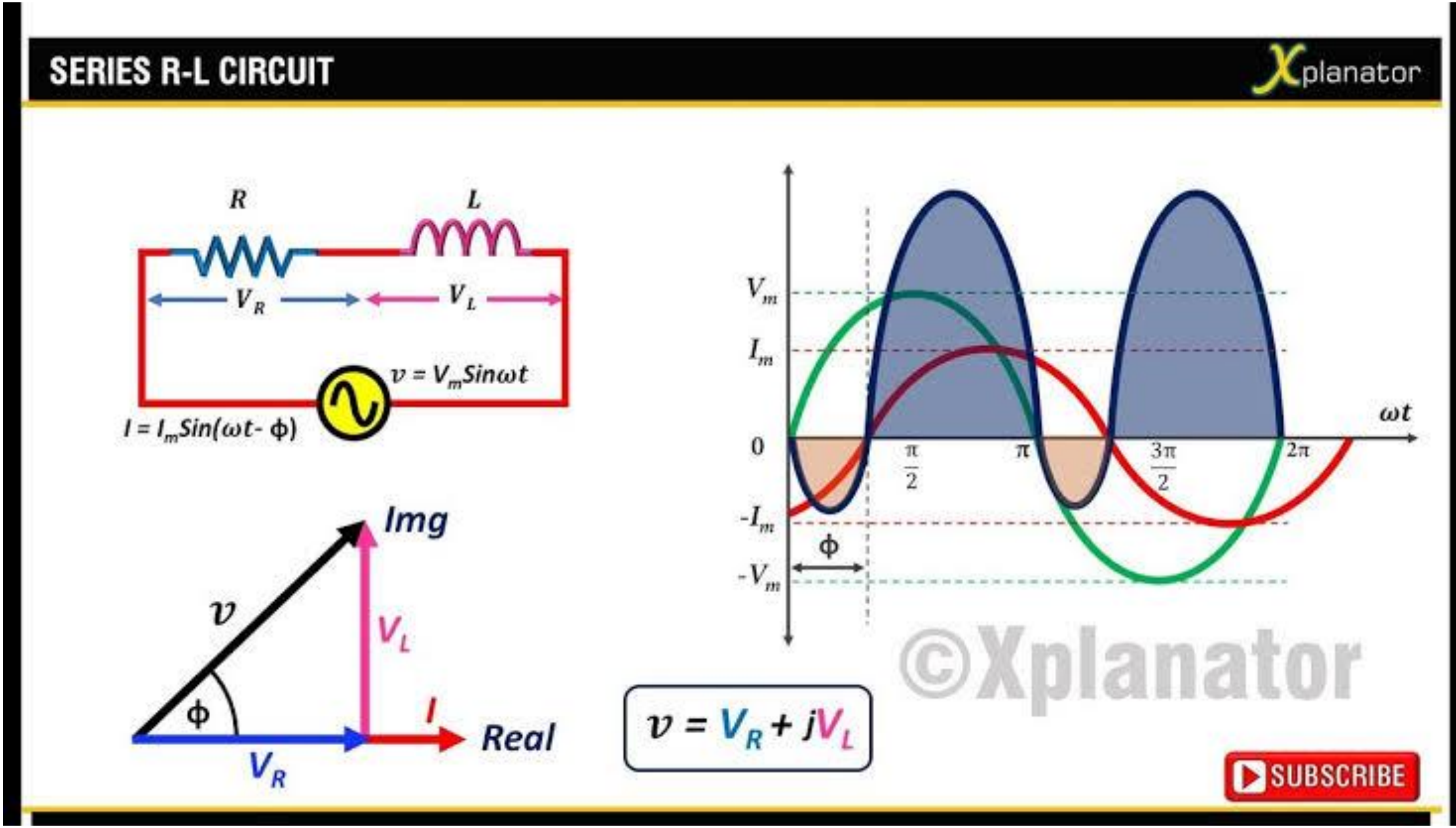
$$p(t) = V_m \sin(\omega t + \theta) I_m \sin(\omega t + \phi)$$

$$p(t) = \frac{V_m I_m}{2}(\cos(\theta - \phi) - \cos(2\omega t + \theta + \phi))$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta - \phi) - \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)$$

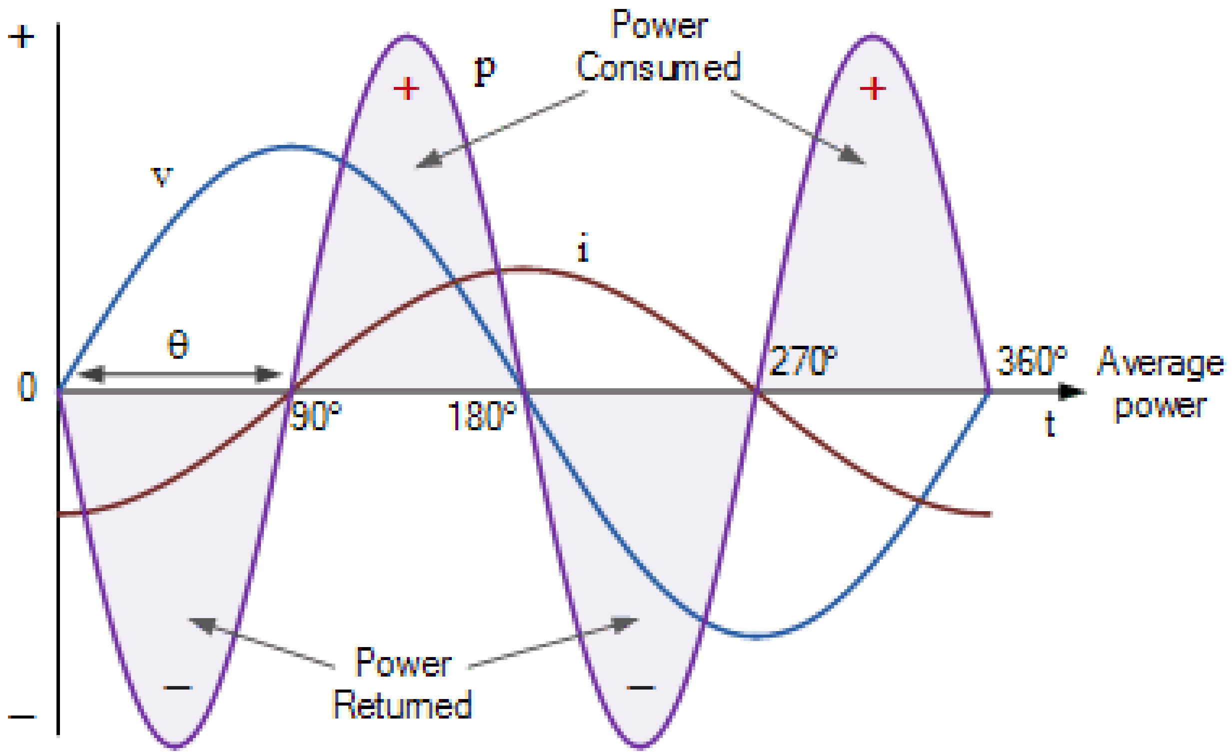
Average Power = $\frac{1}{T}\int_0^T p(t)dt = P$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$



$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

Power in Pure Inductor



Average Power

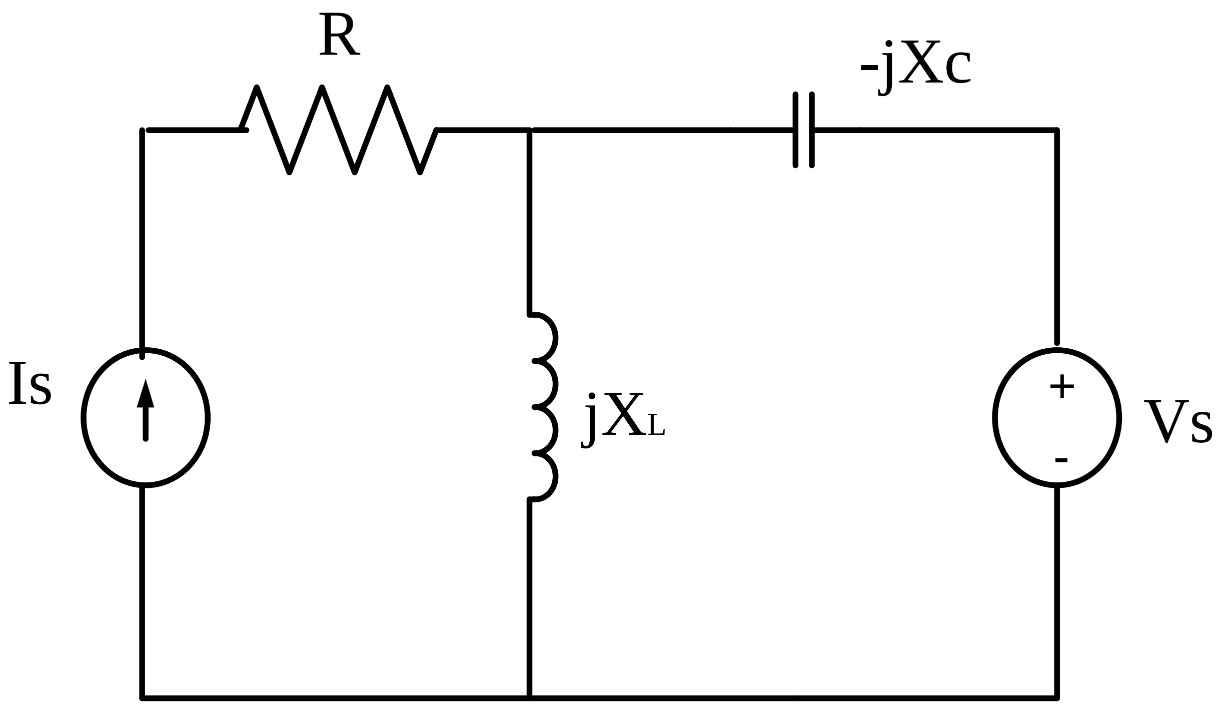
Resistor: V_m and I_m are in phase.

$$P = \frac{1}{2} V_m I_m \cos(0) = \frac{1}{2} V_m I_m$$

Ideal Inductor : V_m leads I_m by 90°.

$$P = \frac{1}{2} V_m I_m \cos(90^\circ) = 0$$

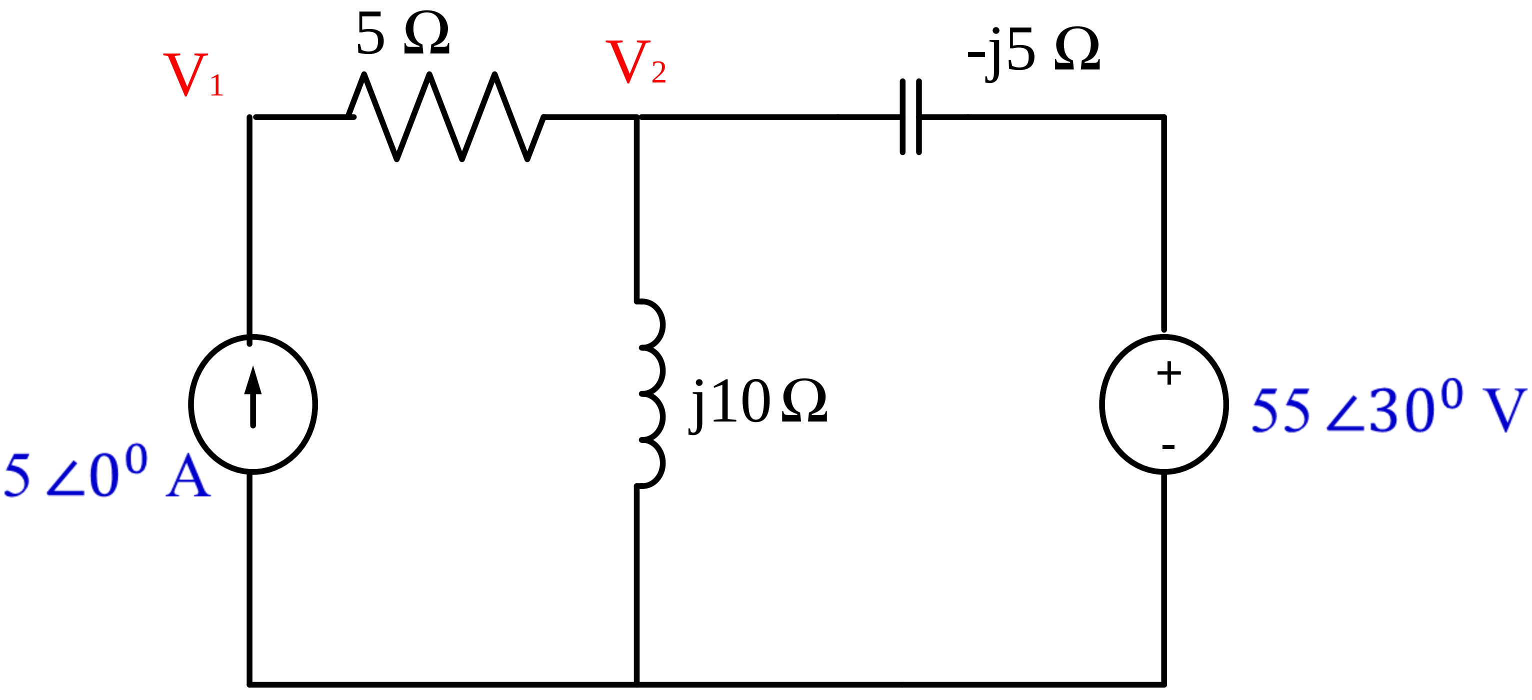
Find power delivered by the current source



Given Is = $5\angle 0^0$ A; Vs=$55\angle 30^0$ V; R = 5 Ohm

X_L = 10 Ohm and X_c = 5 Ohm

Solution



$$5\angle 0^0 + \frac{55\angle 30^0 - V_2}{-j5} = \frac{V_2}{j10}$$

$$\Rightarrow j50 - 2(55\angle 30^0 - V_2) = V_2$$

$$\Rightarrow V_2 = 95.263 + j5$$

But,

$$V_1 = 5X5 + V_2 = 120.263 + j5$$
$$= 120.367 \angle 2.381^0$$

Therefore, power delivered by the Current source =

$V_1 I_s \cos(\angle V_1 - \angle I_s)$

$= \frac{1}{2} V_m I_m \cos(2.381^0 - 0)$

$= (\frac{1}{2}) \times 120.367 \times 5 \times \cos(2.381^0) = 300.66$ W

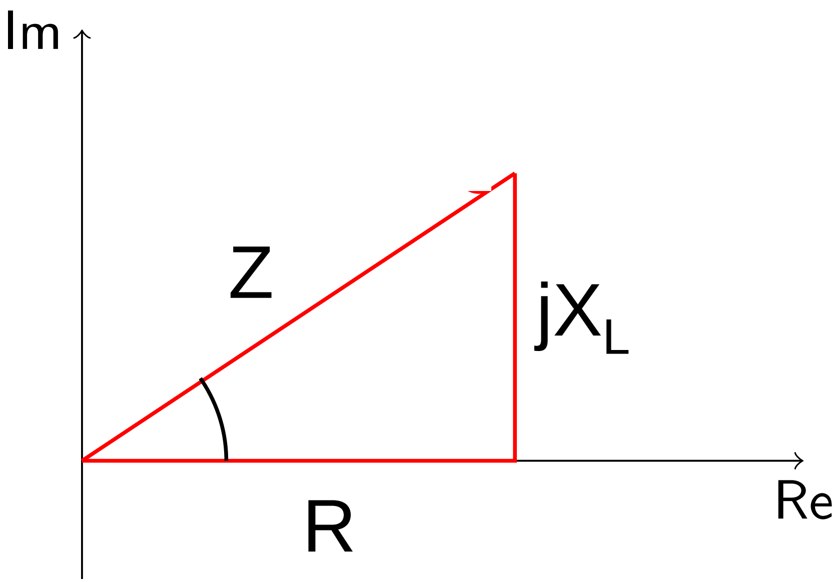
Average or Real Power

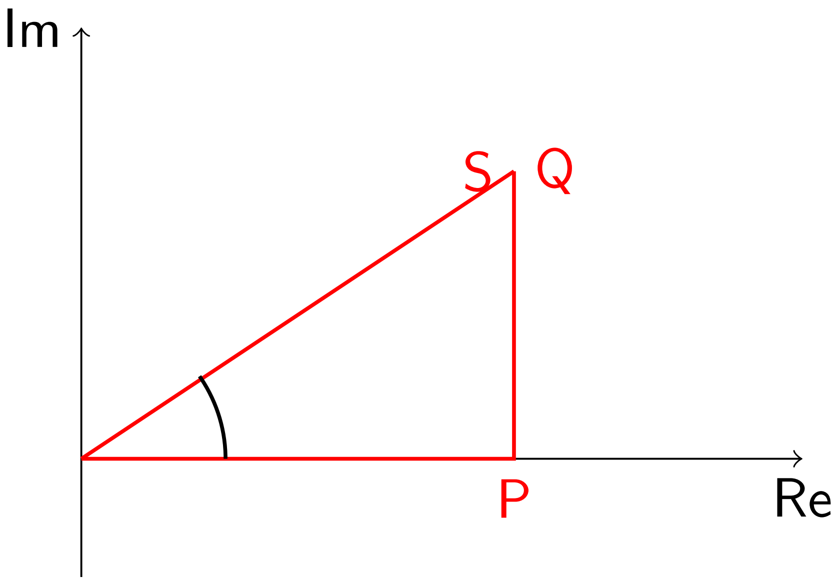
$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = VI \cos(\theta - \phi)$$

where

$$V = \frac{V_m}{\sqrt{2}}, \quad I = \frac{I_m}{\sqrt{2}}$$

Impedance and Power Triangle





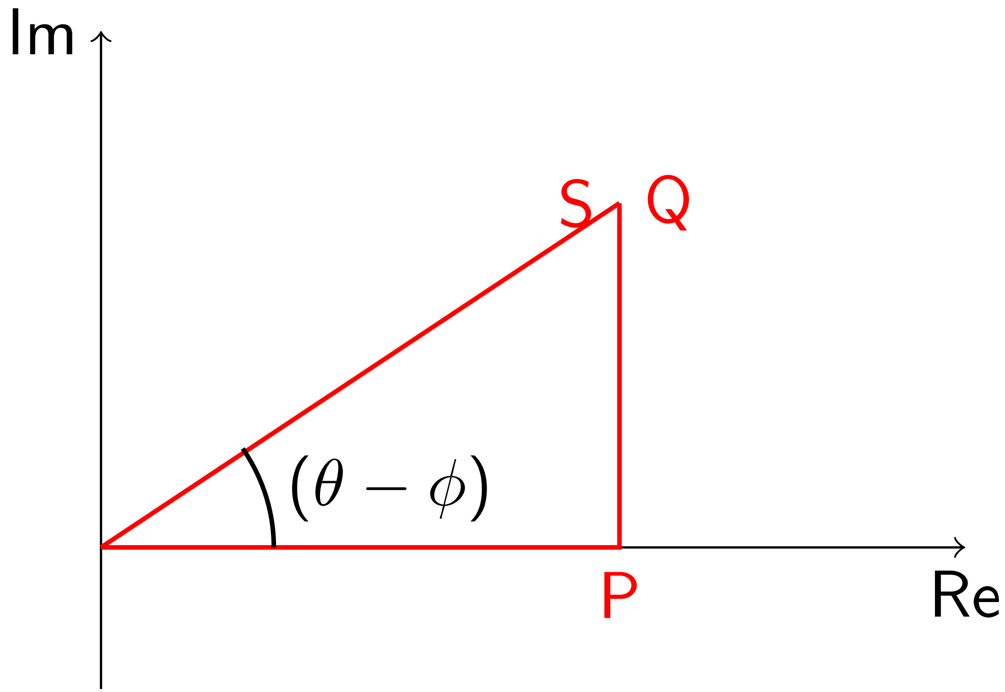
Impedance $Z = R + j X_L$

$P = I^2 R$ in W,

$Q = I^2 X_L$ in VAR

$S = VI$ in VA

Power Triangle



$$\mathbf{S} = VI\cos(\theta - \phi) + \jmath VI\sin(\theta - \phi)$$

P = Real power in W, Q = Reactive power in VAR

Complex Power

$$\mathbf{S} = VI\cos(\theta - \phi) + jVI\sin(\theta - \phi)$$

$$= VI\angle(\theta - \phi) = V\angle\theta \; I\angle -\phi$$

Phasor voltage and current in RMS are

$$\mathbf{V} = V\angle\theta, \quad \mathbf{I} = I\angle\phi$$

Then the complex power is

$$\mathbf{S} = \mathbf{V}\mathbf{I}^*$$

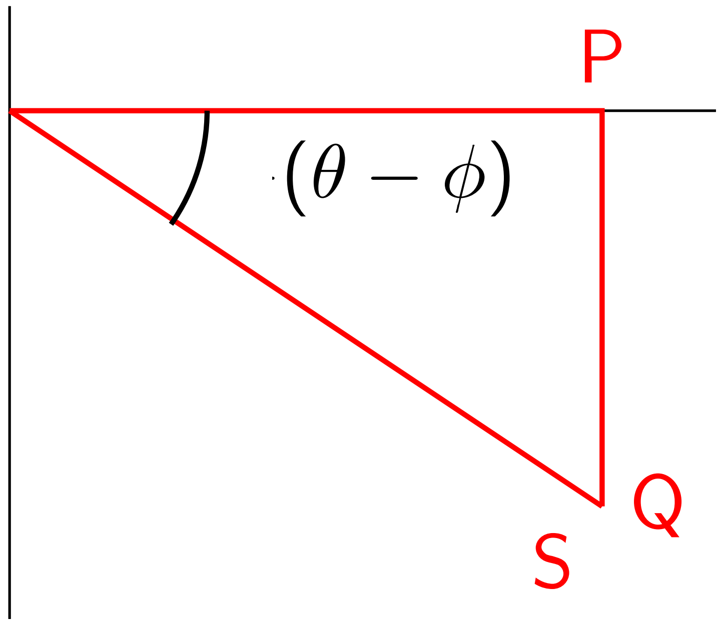
Apparent Power

Magnitude of complex power is called the apparent power

$$|S| = VI$$

Test

- Draw the power triangle of a RC load connected to a sinusoidal source



Power factor

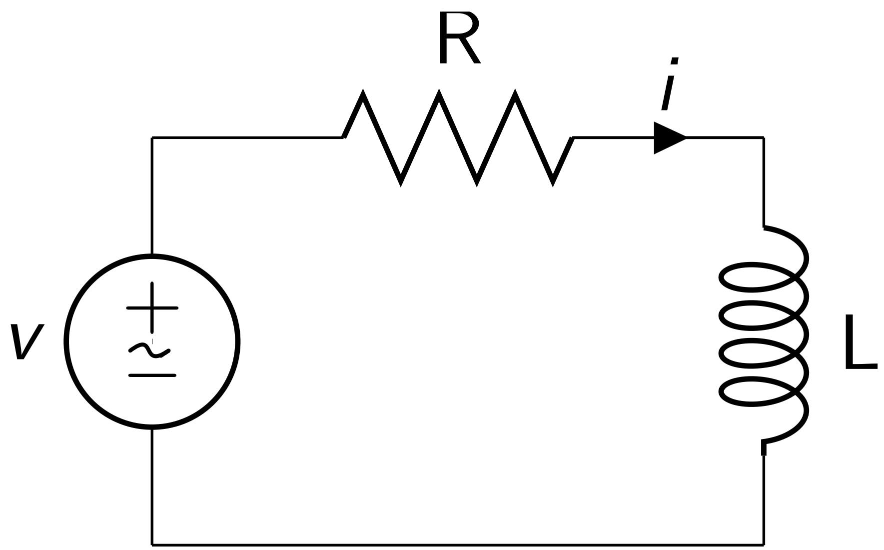
pf is the ratio of real power to the apparent power

$$\text{pf} = \frac{VI\cos(\theta - \phi)}{VI} = \cos(\theta - \phi)$$

Maximum value of pf is 1

$(\theta - \phi) < 0$ pf is leading

Estimate average, reactive powers and pf



Given, $v(t) = 240\sqrt{2}\sin(1000t)$ V, R = 4Ω and L = 3mH

Answers:

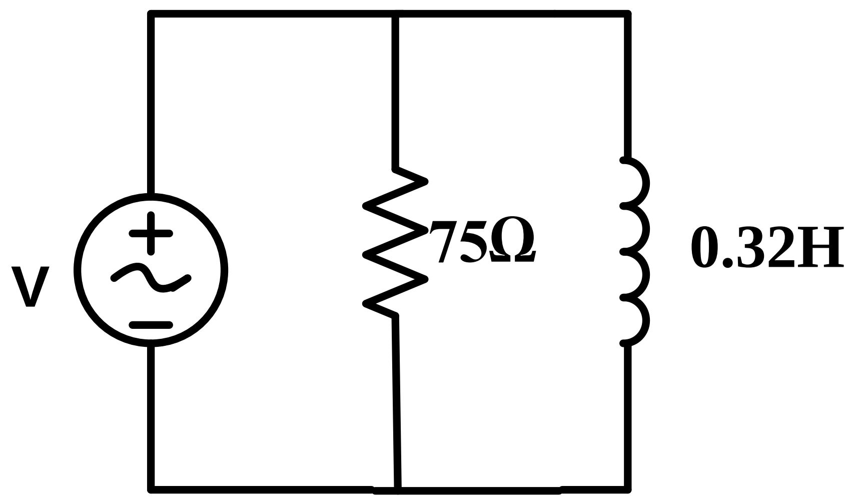
Real/Active Power : 9216 W,

Reactive Power: 6912 VAR,

Power Factor: 0.8 (lagging)

Example 2:

Consider the following circuit which has a voltage source of $v = 240\sqrt{2}\,\text{Cos}(100\pi t)$ V.



Find its real power, reactive power and power factor.

Given, **V** = $240\sqrt{2} \angle 0^0 = V_m \angle\theta$ and **Z** = $(75 \parallel j32\pi) = 48 + j\,36 =$ $60 \angle 36.87^0$ (approx.)

Then, **I** = **V**/**Z** = $4\sqrt{2} \angle\text{-}36.87^0 = I_m \angle\phi$ and $(\theta\text{-}\phi) = 36.87^0$.
The power factor is $\cos(\theta\text{-}\phi) = \cos(36.87^0)$ = **0.8 lagging**

(as the pf angle is +ve)

(1) Real power = P = $(½)\, V_m\, I_m \cos(\theta\text{-}\phi) = (½)\, 240\sqrt{2} \times 4\sqrt{2} \times \cos(36.87^0) = 768$ W

Also, using RMS values of voltage and current, $P = V_{rms} \times I_{rms} \times \cos(36.87^0) = 240 \times 4 \times 0.8 = 768$ W

Also, $P = V_{rms}^2/R = 240^2/75 = 768$ W

(2) Reactive power = Q

$= V_{rms}^2 / X_L$

$= V_{rms} \times I_{rms} \times \sin(36.87^0) = 576$ VAR

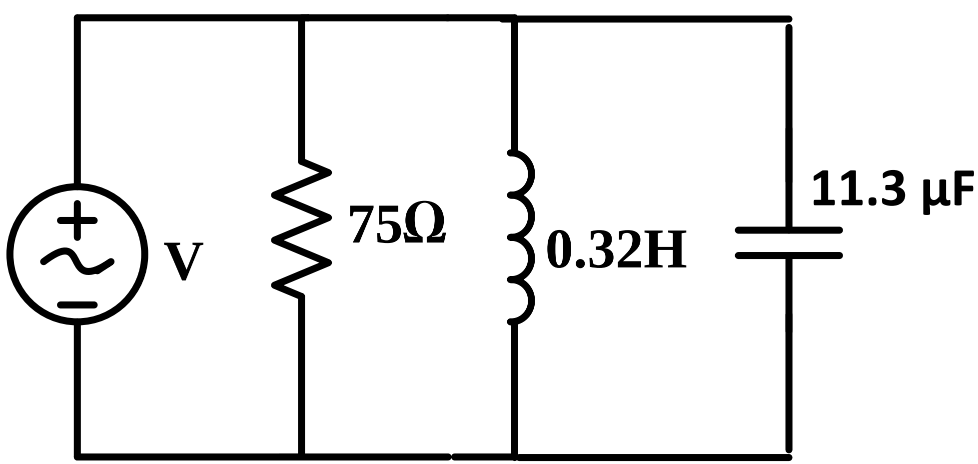
(Note: $X_L = 100\ \Omega$)

Also, reactive power = $P \tan(\theta-\phi) = 576$ VAR

Example 3:

Consider the circuit of Example 2, which has a voltage source of $240\sqrt{2}\,\text{Cos}(100\pi t)$ V.

Find its power factor if a capacitor of C = 11.3 μF is connected in parallel with the voltage source.



Reactive power of the capacitor, $Q_c = - V_{rms}^2/X_c = - 204.04$ VAR

Net reactive power experienced by the source
$= 576 – 204.04 = 371.96$ VAR

Net real power experienced by the source = 768 W

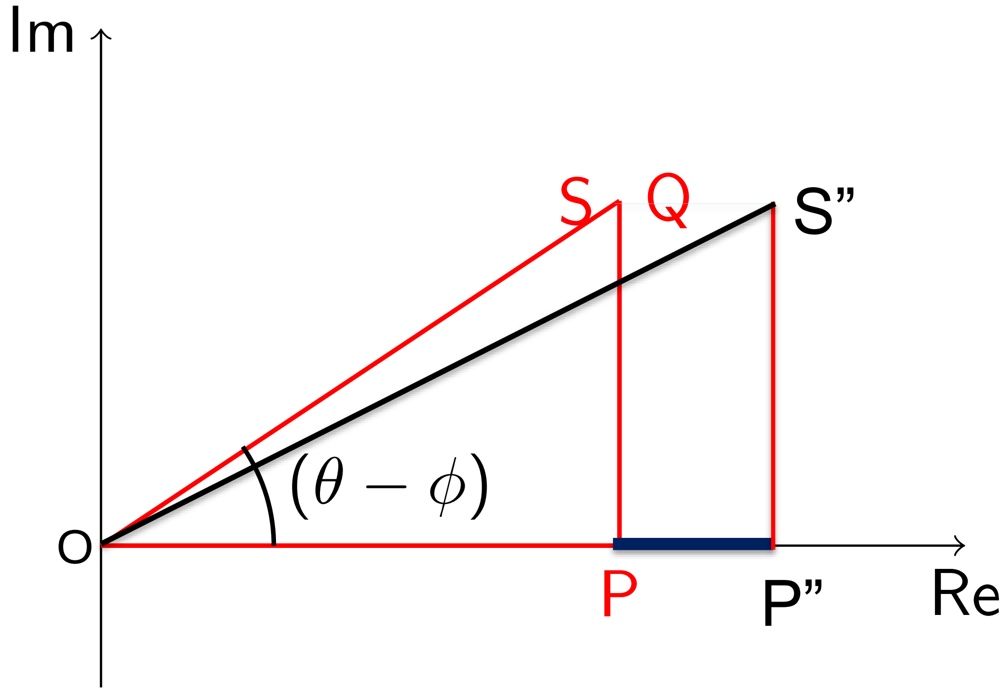
Using the power triangle,

$\tan(\theta-\phi) = 371.96/768$

$\Rightarrow (\theta-\phi) = 25.842^0$ (pf angle is positive)

Therefore, power factor = $\cos(25.842^0) = 0.9$ lagging

Power Factor Improvement by adding Real power load (R_L)



P +P"= Real power in W, Q = Reactive power in VAR

S"= Apparent power in VA

Improved Power factor

pf is the ratio of real power to the apparent power

$$= \frac{P+P''}{|S''|}$$

Previous power triangle angle is $(\theta - \emptyset)$.

New power triangle has angle $(\theta'' - \emptyset'')$ which is lesser than the previous one

=> improvement in the power factor

END