EE 1102H – Electric Circuits

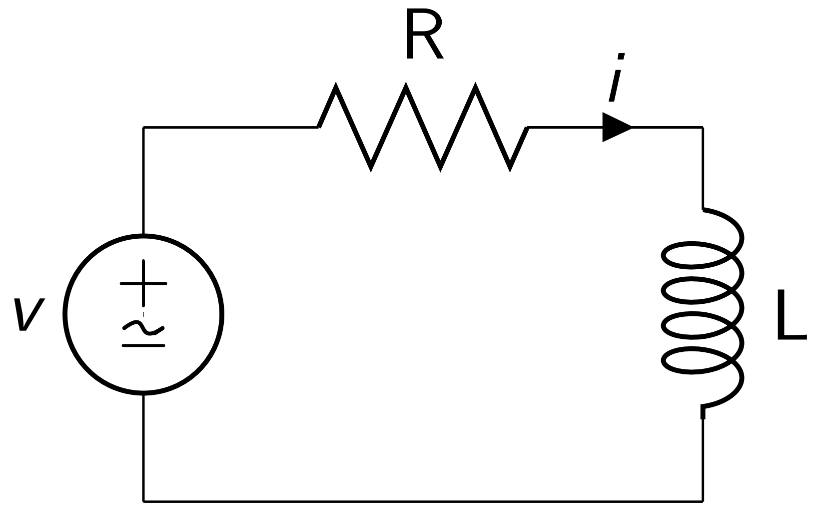
Instantaneous Power, Average Power

Complex Power,

Apparent Power

and Power Factor

Instantaneous Power



Let $v = V_m \sin(\omega t + \theta)$

Then $i = I_m \sin(\omega t + \phi)$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \text{ and } \phi = -\tan^{-1}\frac{\omega L}{R} + \theta$$

$$p(t) = v(t)i(t)$$

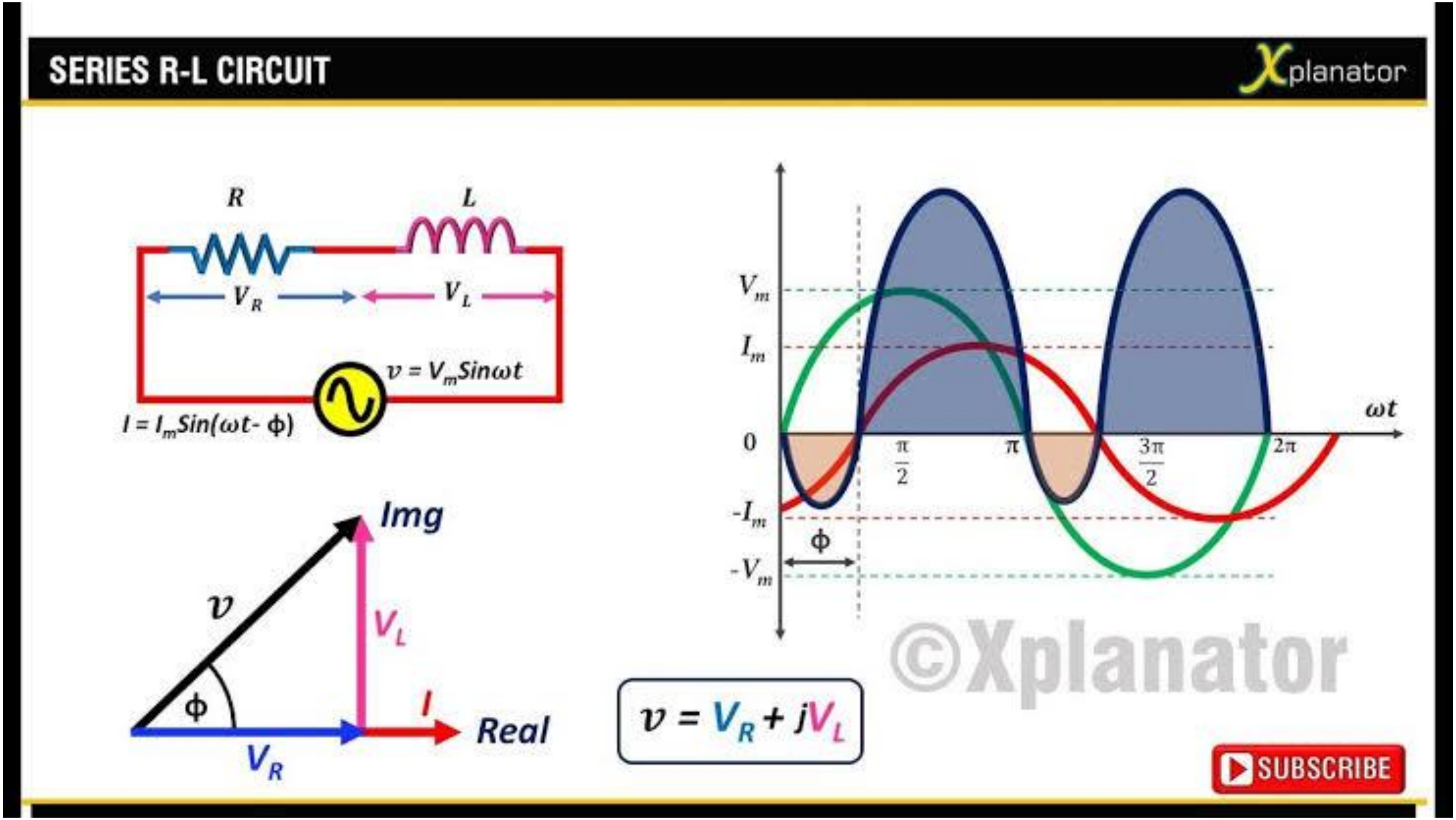
$$p(t) = V_m \sin(\omega t + \theta) I_m \sin(\omega t + \phi)$$

$$p(t) = \frac{V_m I_m}{2}(\cos(\theta - \phi) - \cos(2\omega t + \theta + \phi))$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta - \phi) - \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)$$

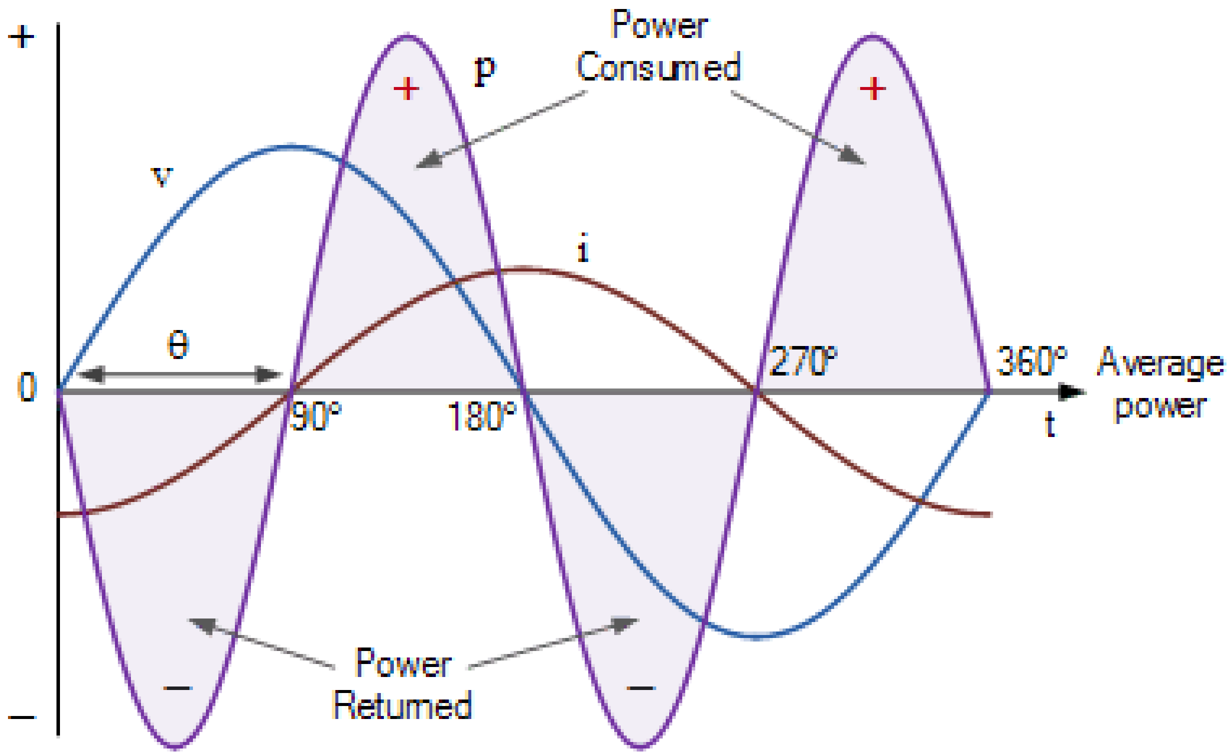
Average Power = $\frac{1}{T}\int_0^T p(t)dt = P$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$



$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

Power in Pure Inductor



Average Power

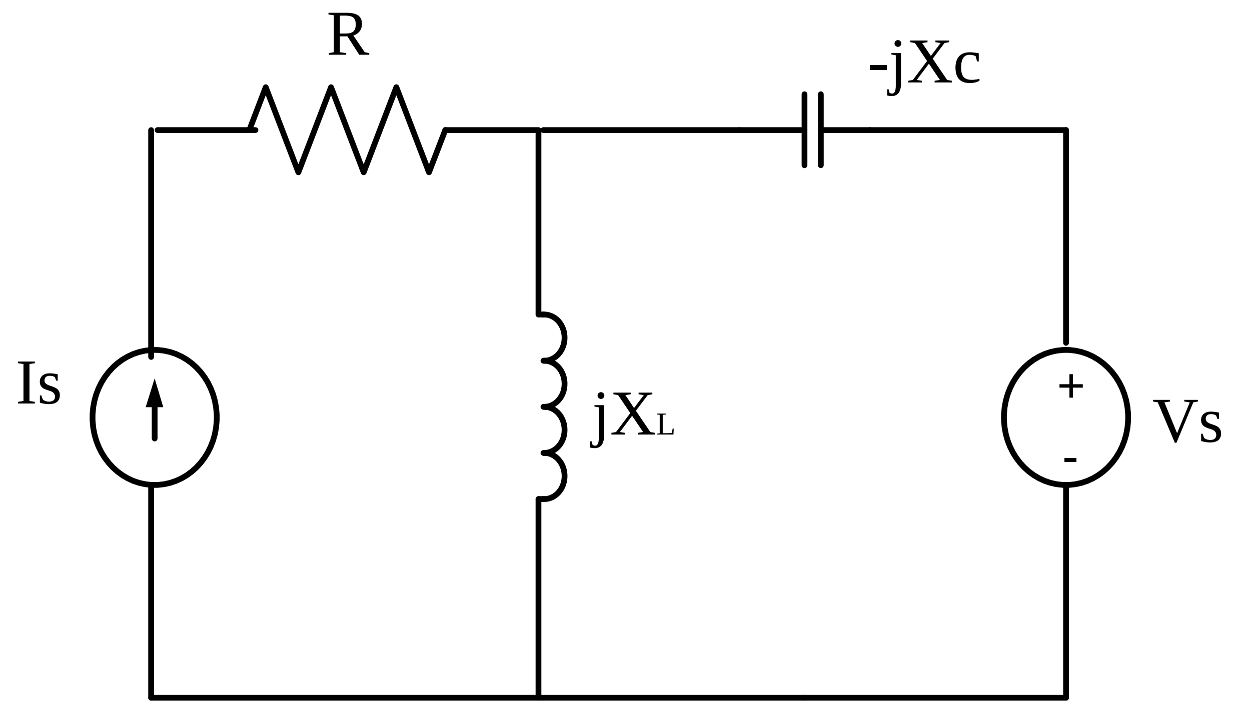
Resistor: V_m and I_m are in phase.

$$P = \frac{1}{2} V_m I_m \cos(0) = \frac{1}{2} V_m I_m$$

Ideal Inductor : V_m leads I_m by 90°.

$$P = \frac{1}{2} V_m I_m \cos(90^\circ) = 0$$

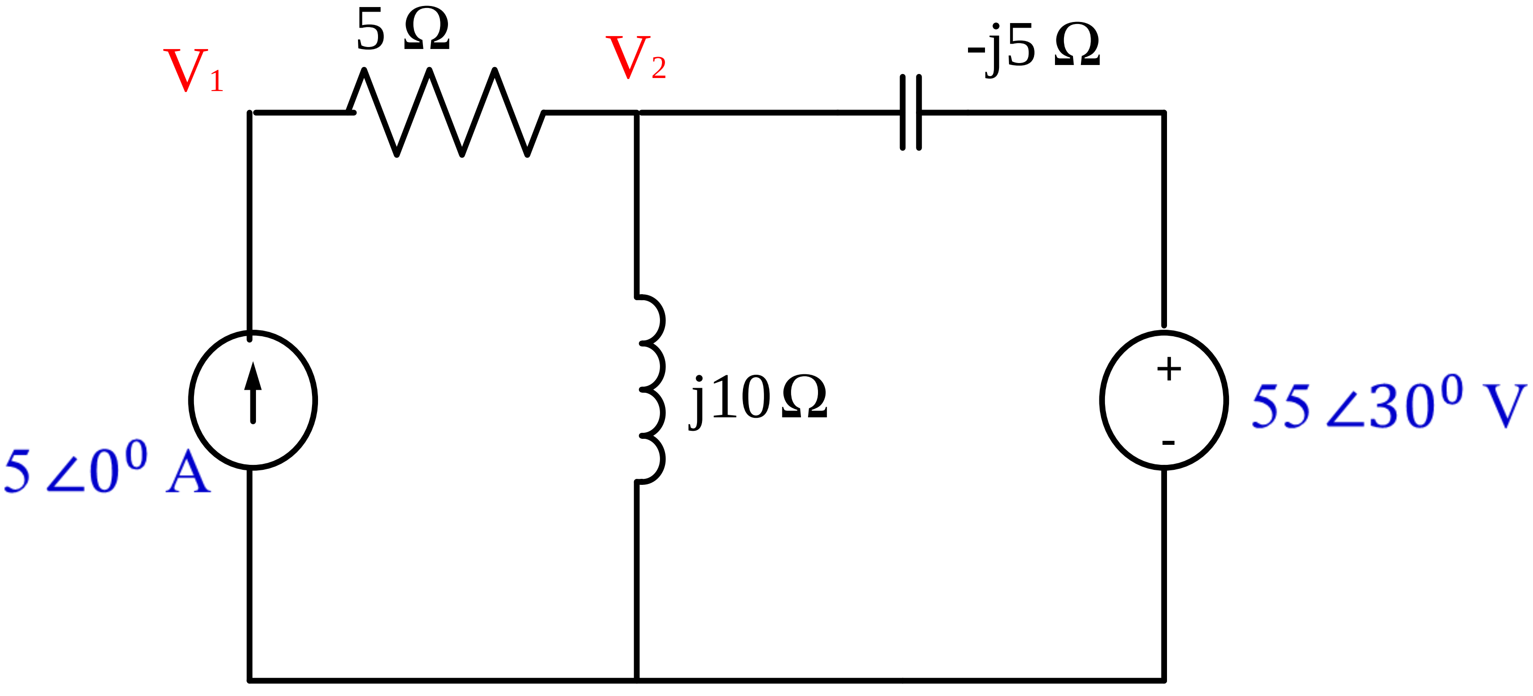
Find power delivered by the current source



Given Is = $5\angle 0^0$ A; Vs=$55\angle 30^0$ V; R = 5 Ohm

X_L = 10 Ohm and X_c = 5 Ohm

Solution



$$5\angle 0^0 + \frac{55\angle 30^0 - V_2}{-j5} = \frac{V_2}{j10}$$

$$\Rightarrow j50 - 2(55\angle 30^0 - V_2) = V_2$$

$$\Rightarrow V_2 = 95.263 + j5$$

But,

$$V_1 = 5X5 + V_2 = 120.263 + j5$$
$$= 120.367 \angle 2.381^0$$

Therefore, power delivered by the Current source =

$V_1 \; I_s \; \text{Cos} \; (\angle V_1 - \angle I_s)$

$= ½ \; V_m \; I_m \; \text{Cos} \; (2.381^0 - 0)$

$= (½) \; x120.367 \; x5x \; \text{Cos}(2.381^0) = 300.66 \; W$

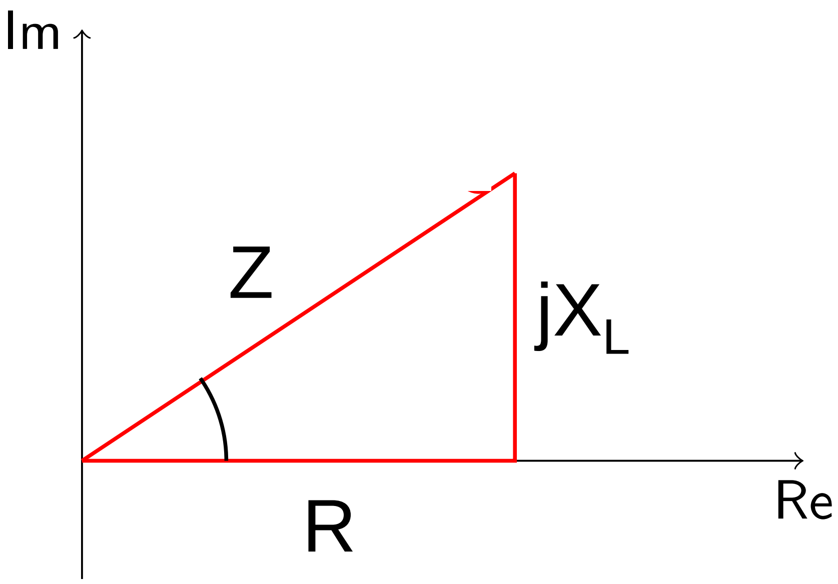
Average or Real Power

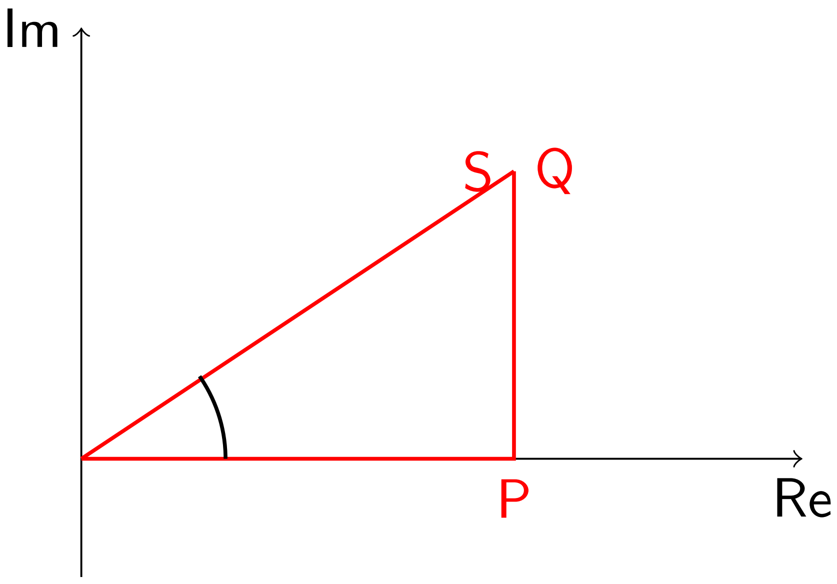
$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = VI \cos(\theta - \phi)$$

where

$$V = \frac{V_m}{\sqrt{2}}, \quad I = \frac{I_m}{\sqrt{2}}$$

Impedance and Power Triangle

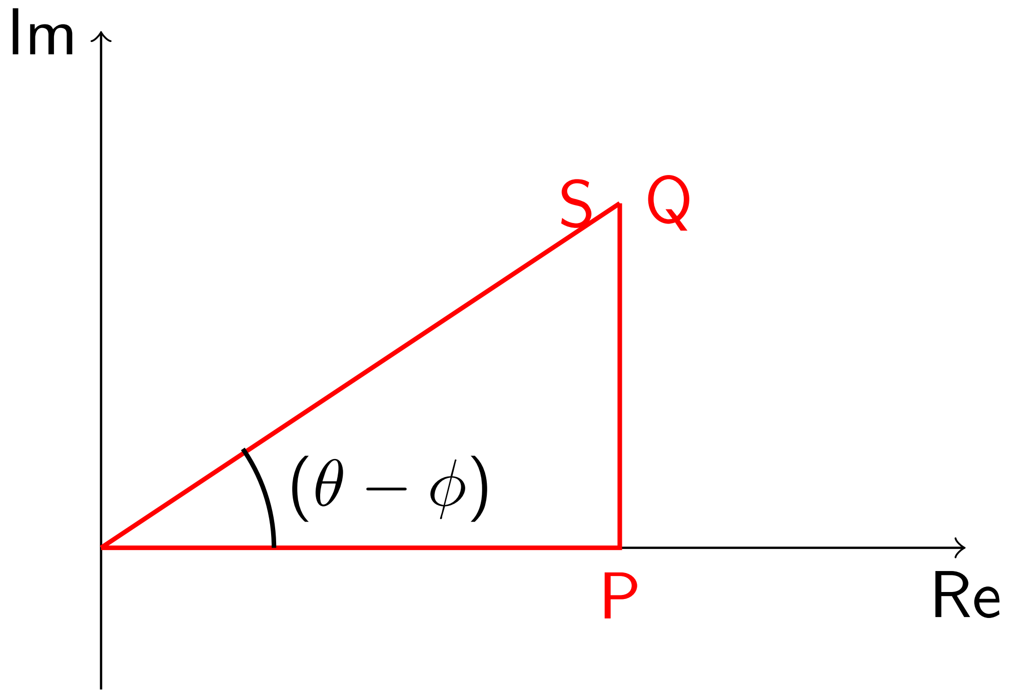




Impedance $Z = R + j\,X_L$

$P = I^2 R$ in W,
$Q = I^2 X_L$ in VAR
$S = VI$ in VA

Power Triangle



$$\mathbf{S} = VI\cos(\theta - \phi) + \jmath VI\sin(\theta - \phi)$$

P = Real power in W, Q = Reactive power in VAR

Complex Power

$$\mathbf{S} = VI\cos(\theta - \phi) + jVI\sin(\theta - \phi)$$

$$= VI\angle(\theta - \phi) = V\angle\theta \; I\angle - \phi$$

Phasor voltage and current in RMS are

$$\mathbf{V} = V\angle\theta, \quad \mathbf{I} = I\angle\phi$$

Then the complex power is

$$\mathbf{S} = \mathbf{V}\mathbf{I}^*$$

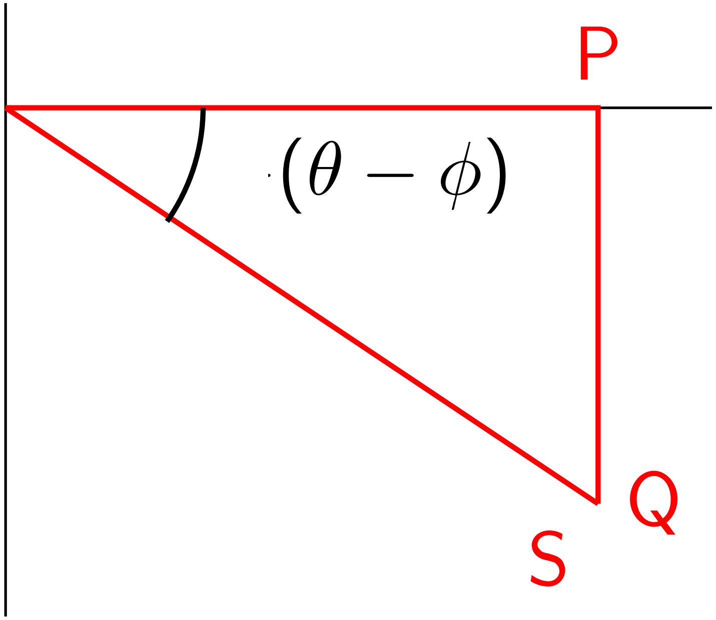
Apparent Power

Magnitude of complex power is called the apparent power

$$|S| = VI$$

Test

- Draw the power triangle of a RC load connected to a sinusoidal source



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