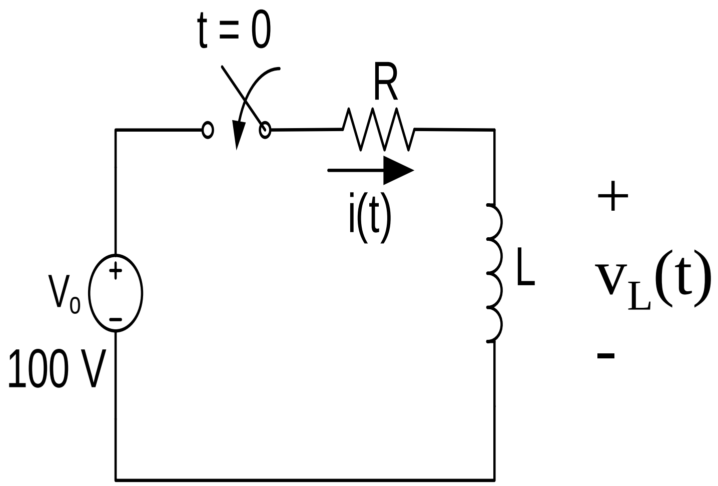
EE1102H – Electric Circuits

Determine R and L of the following circuit

when $i(t)=i_L(t) = 12.5(1- e^{-40t})$ A and $v_L(t) = 100\, e^{-40t}$ V

after the switch is closed at time t = 0.



Soln: After the switch is closed at time t = 0, $R\, i(t) = 100 – v_L(t)$

$\Rightarrow R \times 12.5(1- e^{-40t}) = 100 – 100\, e^{-40t} = 100\, (1- e^{-40t})$

$\Rightarrow R = 100/12.5 = 8$ Ohm

But, time constant L/R = 1/40 => L =8/40 = 0.2 H

Energy stored in an inductor in DC circuits

$E_L = ½ L i_L^2$

Energy stored in an inductor

$$\int v_L i_L dt = \int L \frac{di_L}{dt} i_L \, dt = L \int i_L \, di_L = \frac{1}{2} L i_L^2$$

Energy stored in a capacitor in DC circuits

$E_C = ½ C v_c^2$

Energy stored in a capacitor

$$\int v_c i_c dt = \int v_c \, C \frac{dv_c}{dt} dt = C \int v_c \, dv_c = \frac{1}{2} C v_c^2$$

Chapter Seven

ALTERNATING CURRENT

$$j = 1\angle \pi/2 = 1 e^{j\pi/2} = 1(\cos\frac{\pi}{2} + j\sin \pi/2)$$
$$= j$$
$$\angle\theta = e^{j\theta} = \cos\theta + j\sin\theta$$

Introduction of "j" simplifies ac circuit analysis.

7.1 Introduction

We have so far considered direct current (dc) sources and circuits with dc sources. These currents do not change direction with time. But voltages and currents that vary with time are very common. The electric mains supply in our homes and offices is a voltage that varies like a sine function with time. Such a voltage is called *alternating voltage* (ac voltage) and the current driven by it in a circuit is called the *alternating current* (ac current) . Today, most of the electrical devices we use require ac voltage. This is mainly because most of the electrical energy sold by power companies is transmitted and distributed as alternating current. The main reason for preferring use of ac voltage over dc voltage is that ac voltages can be easily and efficiently converted from one voltage to the other by means of transformers. Further, electrical energy can also be transmitted economically over long distances. AC circuits exhibit characteristics which are exploited in many devices of daily use. For example, whenever we tune our radio to a favourite station, we are taking advantage of a special property of ac circuits – one of many that you will study in this chapter.

The phrases *ac voltage* and *ac current* are contradictory and redundant, respectively, since they mean, literally, *alternating current voltage* and *alternating current current*. Still, the abbreviation *ac* to designate an electrical quantity displaying simple harmonic time dependance has become so universally accepted that we follow others in its use. Further, *voltage* – another phrase commonly used means potential difference between two points.

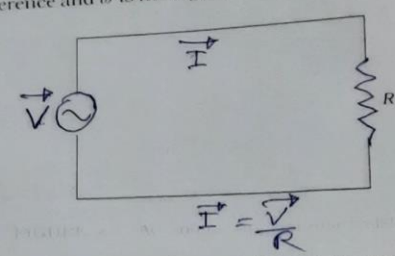
Nicola Tesla (1856 – 1943) Serbian-American scientist, inventor and genius. He conceived the idea of the rotating magnetic field, which is the basis of practically all alternating current machinery, and which helped usher in the age of electric power. He also invented among other things the induction motor, the polyphase system of ac power, and the high frequency induction coil (the Tesla coil) used in radio and television sets and other electronic equipment. The SI unit of magnetic field is named in his honour.

7.2 AC Voltage Applied to a Resistor

Figure 7.1 shows a resistor connected to a source ε of ac voltage. The symbol for an ac source in a circuit diagram is ⊖. We consider a source which produces sinusoidally varying potential difference across its terminals. Let this potential difference, also called ac voltage, be given by

$$v = v_m \sin \omega t \quad \Rightarrow \vec{V} = v_m \angle 0 \qquad (7.1)$$

where v_m is the amplitude of the oscillating potential difference and ω is its angular frequency.



To find the value of current through the resistor, we apply Kirchhoff's loop rule $\sum \varepsilon(t) = 0$ (refer to Section 3.12), to the circuit shown in Fig. 7.1 to get

$$v_m \sin \omega t = i R$$

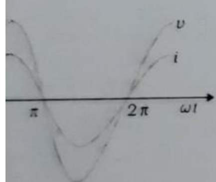
or $$i = \frac{v_m}{R} \sin \omega t$$ in phasor form $\vec{I} = \frac{\vec{V}}{R} = \frac{v_m \angle 0}{R}$

Since R is a constant, we can write this equation as

$$i = i_m \sin \omega t \qquad (7.2)$$

where the current amplitude i_m is given by

$$i_m = \frac{v_m}{R} \qquad (7.3)$$



Equation (7.3) is Ohm's law, which for resistors, works equally well for both ac and dc voltages. The voltage across a pure resistor and the current through it, given by Eqs. (7.1) and (7.2) are plotted as a function of time in Fig. 7.2. Note, in particular that both v and i reach zero, minimum and maximum values at the same time. Clearly, *the voltage and current are in phase with each other.*

We see that, like the applied voltage, the current varies sinusoidally and has corresponding positive and negative values during each cycle. Thus, the sum of the instantaneous current values over one complete cycle is zero, and the average current is zero. The fact that the average current is zero, however, does

It is defined by

Irms → $$I = \sqrt{\bar{i^2}} = \sqrt{\frac{1}{2} i_m^2} = \frac{i_m}{\sqrt{2}}$$

$$= 0.707\, i_m \tag{7.6}$$

In terms of I, the average power, denoted by P is

$$P = \bar{p} = \frac{1}{2} i_m^2 R = I^2 R \tag{7.7}$$

Similarly, we define the *rms voltage* or *effective voltage* by

Vrms → $$V = \frac{v_m}{\sqrt{2}} = 0.707\, v_m \tag{7.8}$$

From Eq. (7.3), we have

$$v_m = i_m R$$

or, $$\frac{v_m}{\sqrt{2}} = \frac{i_m}{\sqrt{2}} R$$

or, $$V = IR \tag{7.9}$$

Equation (7.9) gives the relation between ac current and ac voltage and is similar to that in the dc case. This shows the advantage of introducing the concept of rms values. In terms of rms values, the equation for power [Eq. (7.7)] and relation between current and voltage in ac circuits are essentially the same as those for the dc case.

It is customary to measure and specify rms values for ac quantities. For example, the household line voltage of 220 V is an rms value with a peak voltage of

$$v_m = \sqrt{2}\ V = (1.414)(220\ \text{V}) = 311\ \text{V}$$

In fact, the I or rms current is the equivalent dc current that would produce the same average power loss as the alternating current. Equation (7.7) can also be written as

$$P = V^2 / R = I\,V \quad (\text{since } V = I\,R)$$

Example 7.1 A light bulb is rated at 100W for a 220 V supply. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the rms current through the bulb.

Solution

(a) We are given P = 100 W and V = 220 V. The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220\ \text{V})^2}{100\ \text{W}} = 484\,\Omega$$

(b) The peak voltage of the source is

$$v_m = \sqrt{2}V = 311\ \text{V}$$

(c) Since, $P = I\,V$

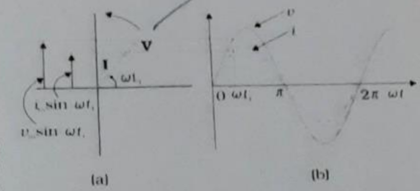
$$I = \frac{P}{V} = \frac{100\ \text{W}}{220\ \text{V}} = 0.454\ \text{A}$$

EXAMPLE 7.1

7.3 Representation of AC Current and Voltage by Rotating Vectors — Phasors

v bold (v phasor) $\vec{V}$

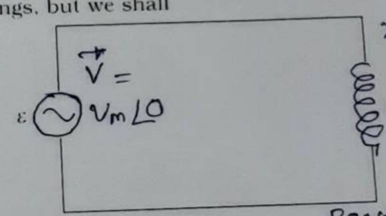
In the previous section, we learnt that the current through a resistor is in phase with the ac voltage. But this is not so in the case of an inductor, a capacitor or a combination of these circuit elements. In order to show phase relationship between voltage and current in an ac circuit, we use the notion of *phasors*. The analysis of an ac circuit is facilitated by the use of a phasor diagram. A phasor is a vector which rotates about the origin with angular speed ω, as shown in Fig. 7.4. The vertical components of phasors **V** and **I** represent the sinusoidally varying quantities v and i. The magnitudes of phasors **V** and **I** represent the amplitudes or the peak values v_m and i_m of these oscillating quantities. Figure 7.4(a) shows the voltage and current phasors and their relationship at time t_1 for the case of an ac source connected to a resistor i.e., corresponding to the circuit shown in Fig. 7.1. The projection of voltage and current phasors on vertical axis, i.e., $v_m \sin\omega t$ and $i_m \sin\omega t$, respectively represent the value of voltage and current at that instant. As they rotate with frequency ω, curves in Fig. 7.4(b) are generated.



From Fig. 7.4(a) we see that phasors **V** and **I** for the case of a resistor are in the same direction. This is so for all times. This means that the phase angle between the voltage and the current is zero.

7.4 AC Voltage Applied to an Inductor

Figure 7.5 shows an ac source connected to an inductor. Usually, inductors have appreciable resistance in their windings, but we shall assume that this inductor has negligible resistance. Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be $v = v_m \sin\omega t$. Using the Kirchhoff's loop rule, $\sum \varepsilon(t) = 0$, and since there is no resistor in the circuit,



$$v - L\frac{di}{dt} = 0 \qquad (7.10)$$

where the second term is the self-induced Faraday emf in the inductor; and L is the self-inductance of

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Though voltage and current in ac circuit are represented by phasors – rotating vectors, they are not vectors themselves. They are scalar quantities. It so happens that the amplitudes and phases of harmonically varying scalars combine mathematically in the same way as do the projections of rotating vectors of corresponding magnitudes and directions. The *rotating vectors* that *represent* harmonically varying scalar quantities are introduced only to provide us with a simple way of adding these quantities using a rule that we already know.

example: v can be $v_1 = v_{m1}\sin(\omega t + \pi/6) \Rightarrow$ Phasor $\vec{V_1} = v_{m1} \angle \pi/6$

the inductor. The negative sign follows from Lenz's law [Chapter 6]. Combining Eqs. (7.1) and (7.10), we have

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{v}{L} = \frac{v_m}{L}\sin\omega t \tag{7.11}$$

Equation (7.11) implies that the equation for $i(t)$, the current as a function of time, must be such that its slope $\mathrm{d}i/\mathrm{d}t$ is a sinusoidally varying quantity, with the same phase as the source voltage and an amplitude given by v_m/L. To obtain the current, we integrate $\mathrm{d}i/\mathrm{d}t$ with respect to time:

$$\int \frac{\mathrm{d}i}{\mathrm{d}t}\mathrm{d}t = \frac{v_m}{L}\int \sin(\omega t)\mathrm{d}t$$

and get,

$$i = -\frac{v_m}{\omega L}\cos(\omega t) + \text{constant}$$

The integration constant has the dimension of current and is time-independent. Since the source has an emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero, so that no constant or time-independent component of the current exists. Therefore, the integration constant is zero.

Using

$$-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$$, we have

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \tag{7.12}$$

where $i_m = \frac{v_m}{\omega L}$ is the amplitude of the current. The quantity ωL is analogous to the resistance and is called *inductive reactance*, denoted by X_L:

$$X_L = \omega L \tag{7.13}$$

The amplitude of the current is, then

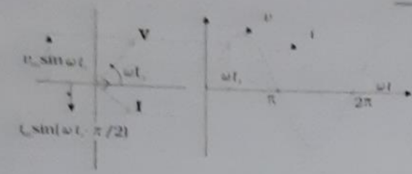
$$i_m = \frac{v_m}{X_L} \tag{7.14}$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit. The inductive reactance is directly proportional to the inductance and to the frequency of the current.

A comparison of Eqs. (7.1) and (7.12) for the source voltage and the current in an inductor shows that the current lags the voltage by $\pi/2$ or one-quarter (1/4) cycle. Figure 7.6 (a) shows the voltage and the current phasors in the present case at instant t_1. The current phasor **I** is $\pi/2$ behind the voltage phasor **V**. When rotated with frequency ω counter-clockwise, they generate the voltage and current given by Eqs. (7.1) and (7.12), respectively and as shown in Fig. 7.6(b).

Interactive animation on Phasor diagrams of ac circuits containing, R, L, C and RLC series circuits

Same phasor diagram $\vec{I}$ lags $\vec{V}$ by $\pi/2$



(a) (b)

We see that the current reaches its maximum value later than the voltage by one-fourth of a period $\left[\frac{T}{4} = \frac{\pi/2}{\omega}\right]$. You have seen that an inductor has reactance that limits current similar to resistance in a dc circuit. Does it also consume power like a resistance? Let us try to find out.

The instantaneous power supplied to the inductor is

$$p_L = i\,v = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \times v_m \sin(\omega t)$$

$$= -i_m v_m \cos(\omega t)\sin(\omega t)$$

$$= -\frac{i_m v_m}{2}\sin(2\omega t)$$

So, the average power over a complete cycle is

$$P_L = \left\langle -\frac{i_m v_m}{2}\sin(2\omega t)\right\rangle$$

$$= -\frac{i_m v_m}{2}\left\langle \sin(2\omega t)\right\rangle = 0,$$

since the average of sin $(2\omega t)$ over a complete cycle is zero.

Thus, the *average power supplied to an inductor over one complete cycle is zero.*

Example 7.2 A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Solution The inductive reactance,

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3}\ \Omega$$

$$= 7.85\Omega$$

The rms current in the circuit is

$$I = \frac{V}{X_L} = \frac{220\ \text{V}}{7.85\ \Omega} = 28\text{A}$$

EXAMPLE 7.2

7.5 AC Voltage Applied to a Capacitor

Figure 7.7 shows an ac source ε generating ac voltage $v = v_m \sin \omega t$ connected to a capacitor only, a purely capacitive ac circuit.

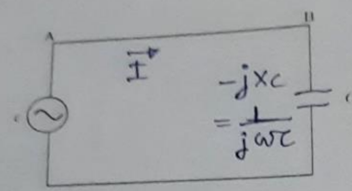


FIGURE 7.7

When a capacitor is connected to a voltage source in a dc circuit, current will flow for the short time required to charge the capacitor. As charge accumulates on the capacitor plates, the voltage across them increases, opposing the current. That is, a capacitor in a dc circuit will limit or oppose the current as it charges. When the capacitor is fully charged, the current in the circuit falls to zero.

When the capacitor is connected to an ac source, as in Fig. 7.7, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle. Let q be the charge on the capacitor at any time t. The instantaneous voltage v across the capacitor is

$$v = \frac{q}{C} \tag{7.15}$$

From the Kirchhoff's loop rule, the voltage across the source and the capacitor are equal,

$$v_m \sin \omega t = \frac{q}{C}$$

To find the current, we use the relation $i = \frac{dq}{dt}$

$$i = \frac{d}{dt}(v_m C \sin \omega t) = \omega C v_m \cos(\omega t)$$

Using the relation, $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, we have

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right) \tag{7.16}$$

where the amplitude of the oscillating current is $i_m = \omega C v_m$. We can rewrite it as

$$i_m = \frac{v_m}{(1/\omega C)}$$

Comparing it to $i_m = v_m/R$ for a purely resistive circuit, we find that $(1/\omega C)$ plays the role of resistance. It is called *capacitive reactance* and is denoted by X_c,

$$X_c = 1/\omega C \tag{7.17}$$

so that the amplitude of the current is

$$i_m = \frac{v_m}{X_C} \tag{7.18}$$

$$I = \frac{V}{X_C} = \frac{220\text{ V}}{212\ \Omega} = 1.04\text{ A}$$

The peak current is

$$i_m = \sqrt{2}I = (1.41)(1.04\text{ A}) = 1.47\text{ A}$$

This current oscillates between +1.47A and −1.47 A, and is ahead of the voltage by $\pi/2$.

If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

Example 7.4

Example 7.5 A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig. 7.9.

FIGURE 7.9

The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increases; (b) decreases; (c) is unchanged, as the iron rod is inserted. Give your answer with reasons.

Solution As the iron rod is inserted, the magnetic field inside the coil magnetizes the iron increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the glow of the light bulb decreases.

Example 7.5

7.6 AC Voltage Applied to a Series LCR Circuit

Figure 7.10 shows a series LCR circuit connected to an ac source ε. As usual, we take the voltage of the source to be $v = v_m \sin \omega t$. $= v_m \angle 0$

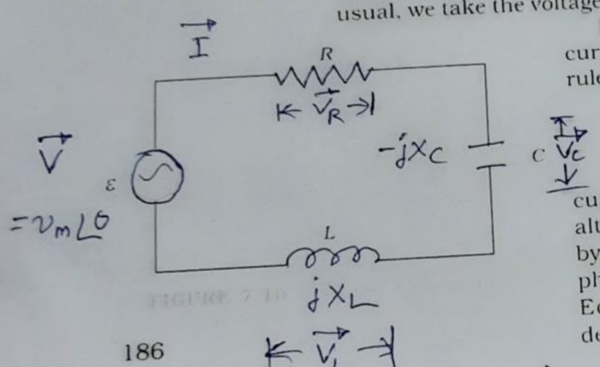


FIGURE 7.10

If q is the charge on the capacitor and i the current, at time t, we have, from Kirchhoff's loop rule:

$$L\frac{di}{dt} + iR + \frac{q}{C} = v \tag{7.20}$$

We want to determine the instantaneous current i and its phase relationship to the applied alternating voltage v. We shall solve this problem by two methods. First, we use the technique of phasors and in the second method, we solve Eq. (7.20) analytically to obtain the time-dependence of i.

$Z = R + jX_L - jX_C = R - j(X_C - X_L) \Rightarrow \vec{I} = \frac{\vec{V}}{Z}$

7.6.1 Phasor-diagram solution

From the circuit shown in Fig. 7.10, we see that the resistor, inductor and capacitor are in series. Therefore, the ac current in each element is the same at any time, having the same amplitude and phase. Let it be

$$i = i_m \sin(\omega t + \phi) \quad (7.21)$$

$\Rightarrow$ phasor $\vec{I} = i_m \angle \phi$

where ϕ is the phase difference between the voltage across the source and the current in the circuit. On the basis of what we have learnt in the previous sections, we shall construct a phasor diagram for the present case.

Let **I** be the phasor representing the current in the circuit as given by Eq. (7.21). Further, let $\mathbf{V_L}$, $\mathbf{V_R}$, $\mathbf{V_C}$, and **V** represent the voltage across the inductor, resistor, capacitor and the source, respectively. From previous section, we know that $\mathbf{V_R}$ is parallel to **I**, $\mathbf{V_C}$ is $\pi/2$ behind **I** and $\mathbf{V_L}$ is $\pi/2$ ahead of **I**. $\mathbf{V_L}$, $\mathbf{V_R}$, $\mathbf{V_C}$ and **I** are shown in Fig. 7.11(a) with apppropriate phase-relations.

$\vec{V}_R = \vec{I}R$; $\vec{V}_L = \vec{I}jX_L = j\vec{I}X_L = 1\angle\pi/2 \times i_m\angle\phi \times \omega L = i_m\omega L \angle \pi/2 + \phi \Rightarrow i_m\omega L \sin(\omega t + \frac{\pi}{2} + \phi)$

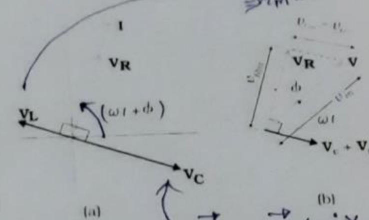


FIGURE 7.11

$\vec{V}_C = \vec{I}\cdot(-jX_C) = -j \cdot i_m\angle\phi \cdot \frac{1}{\omega C} = \frac{i_m}{\omega C}\angle\phi - \pi/2 \Rightarrow \frac{i_m}{\omega C}\sin(\omega t + \phi - \pi/2)$

The length of these phasors or the amplitude of $\mathbf{V_R}$, $\mathbf{V_C}$ and $\mathbf{V_L}$ are:

$$v_{Rm} = i_m R,\ v_{Cm} = i_m X_C,\ v_{Lm} = i_m X_L \quad (7.22)$$

The voltage Equation (7.20) for the circuit can be written as

$$v_L + v_R + v_C = v \quad (7.23)$$

The phasor relation whose vertical component gives the above equation is

$$\mathbf{V_L} + \mathbf{V_R} + \mathbf{V_C} = \mathbf{V} \quad (7.24)$$

This relation is represented in Fig. 7.11(b). Since $\mathbf{V_C}$ and $\mathbf{V_L}$ are always along the same line and in opposite directions, they can be combined into a single phasor $(\mathbf{V_C} + \mathbf{V_L})$ which has a magnitude $|v_{Cm} - v_{Lm}|$. Since **V** is represented as the hypotenuse of a right-triangle whose sides are $\mathbf{V_R}$ and $(\mathbf{V_C} + \mathbf{V_L})$, the pythagorean theorem gives:

$$v_m^2 = v_{Rm}^2 + (v_{Cm} - v_{Lm})^2$$

Substituting the values of v_{Rm}, v_{Cm}, and v_{Lm} from Eq. (7.22) into the above equation, we have

$$v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$= i_m^2 \left[R^2 + (X_C - X_L)^2 \right]$$

or, $$i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}} \quad [7.25(a)]$$

By analogy to the resistance in a circuit, we introduce the *impedance* Z in an ac circuit:

$$i_m = \frac{v_m}{Z} \quad [7.25(b)]$$

$$\text{where } Z = \sqrt{R^2 + (X_C - X_L)^2} \quad (7.26)$$

$Z = R - j(X_C - X_L)$

$$\vec{I} = \frac{\vec{V}}{R+j(X_L-X_C)} \times \frac{R-j(X_L-X_C)}{R-j(X_L-X_C)} = \frac{\vec{V}(R+j(X_C-X_L))}{R^2+(X_L-X_C)^2}$$

$$= \frac{\vec{V}}{\sqrt{R^2+(X_C-X_L)^2}} \angle \tan^{-1}\{(X_C-X_L)/R\} = \frac{\vec{V}\angle\phi}{Z}$$

$$= \frac{V_m \angle\phi}{Z} = i_m \angle\phi$$

Since phasor **I** is always parallel to phasor $\mathbf{V_R}$, the phase angle ϕ is the angle between $\mathbf{V_R}$ and **V** and can be determined from Fig. 7.12:

$$\tan\phi = \frac{v_{Cm} - v_{Lm}}{v_{Rm}}$$

Using Eq. (7.22), we have

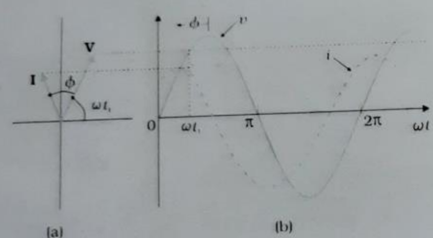
$$\tan\phi = \frac{X_C - X_L}{R} \tag{7.27}$$

Equations (7.26) and (7.27) are graphically shown in Fig. (7.12). This is called *Impedance diagram* which is a right-triangle with Z as its hypotenuse.

Equation 7.25(a) gives the amplitude of the current and Eq. (7.27) gives the phase angle. With these, Eq. (7.21) is completely specified.

If $X_C > X_L$, ϕ is positive and the circuit is predominantly capacitive. Consequently, the current in the circuit leads the source voltage. If $X_C < X_L$, ϕ is negative and the circuit is predominantly inductive. Consequently, the current in the circuit lags the source voltage.

Figure 7.13 shows the phasor diagram and variation of v and i with ωt for the case $X_C > X_L$.



Thus, we have obtained the amplitude and phase of current for an *LCR* series circuit using the technique of phasors. But this method of analysing ac circuits suffers from certain disadvantages. First, the phasor diagram say nothing about the initial condition. One can take any arbitrary value of t (say, t_1, as done throughout this chapter) and draw different phasors which show the relative angle between different phasors. The solution so obtained is called the *steady-state solution*. This is not a general solution. Additionally, we do have a *transient solution* which exists even for $v = 0$. The general solution is the sum of the transient solution and the steady-state solution. After a sufficiently long time, the effects of the transient solution die out and the behaviour of the circuit is described by the steady-state solution.

7.6.2 Resonance

An interesting characteristic of the series *RLC* circuit is the phenomenon of resonance. The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's *natural frequency*. If such a system is driven by an energy source at a frequency that is near the natural frequency, the amplitude of oscillation is found to be large. A familiar example of this phenomenon is a child on a swing. The swing has a natural frequency for swinging back and forth like a pendulum. If the child pulls on the

AC Circuits

Why not only ac?

Storage, transmission loss, waste of power

Why not only dc?

Generation, dc/ac/dc conversion

Hybrid systems using ac+dc

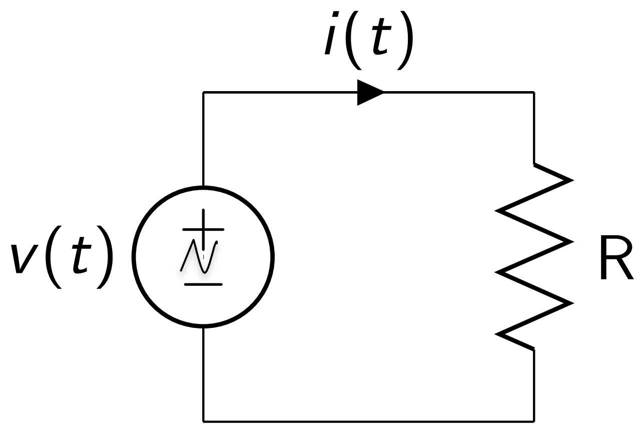
Projector, audio system

- Why RMS of ac?

Average captures no information

Peak captures limited information

RMS current and voltage

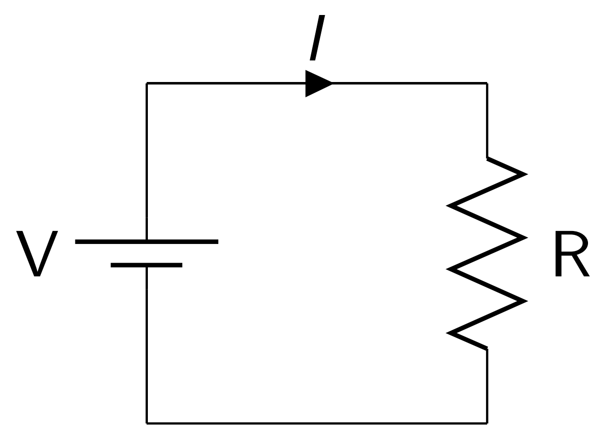


$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,\mathrm{dt}}$$

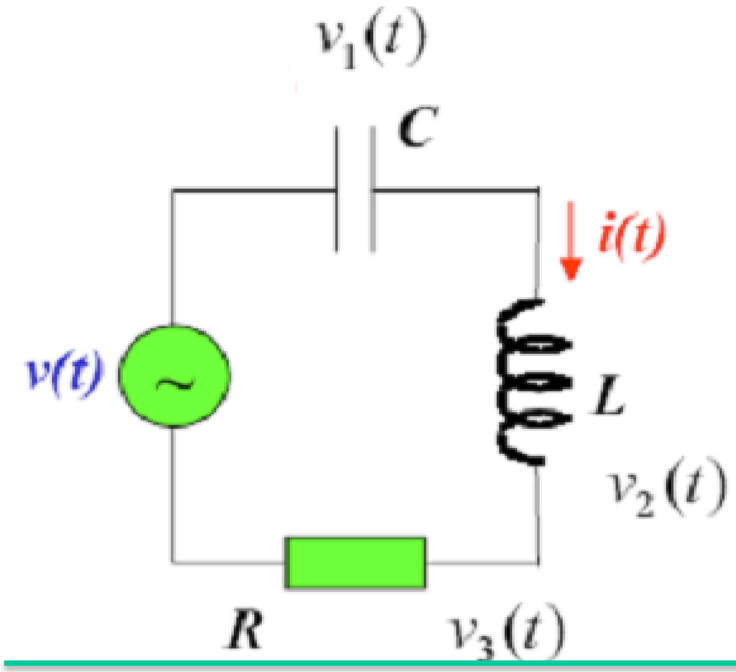
$$V = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,\mathrm{dt}}$$

$$P = \frac{1}{T}\int_0^T i^2(t)R\,\mathrm{dt}$$

$$= I^2 R$$



$$P = I^2 R$$



Find $i(t)$ if $v(t) = 15\cos\omega_0 t$ V

- $R = 15\Omega; C = 800\mu F; L = 0.2H$
- $\omega_o = 50 rad/s$

- Then $Z_R = 15\Omega$

$$Z_C = \frac{1}{j\omega_o C} = -j25\Omega$$

$$Z_L = j\omega_o L = j10\Omega$$

$$Z = 15 - j15\ \Omega$$

In phasors: $V = 15\angle 0$

Then, $I = V/Z$

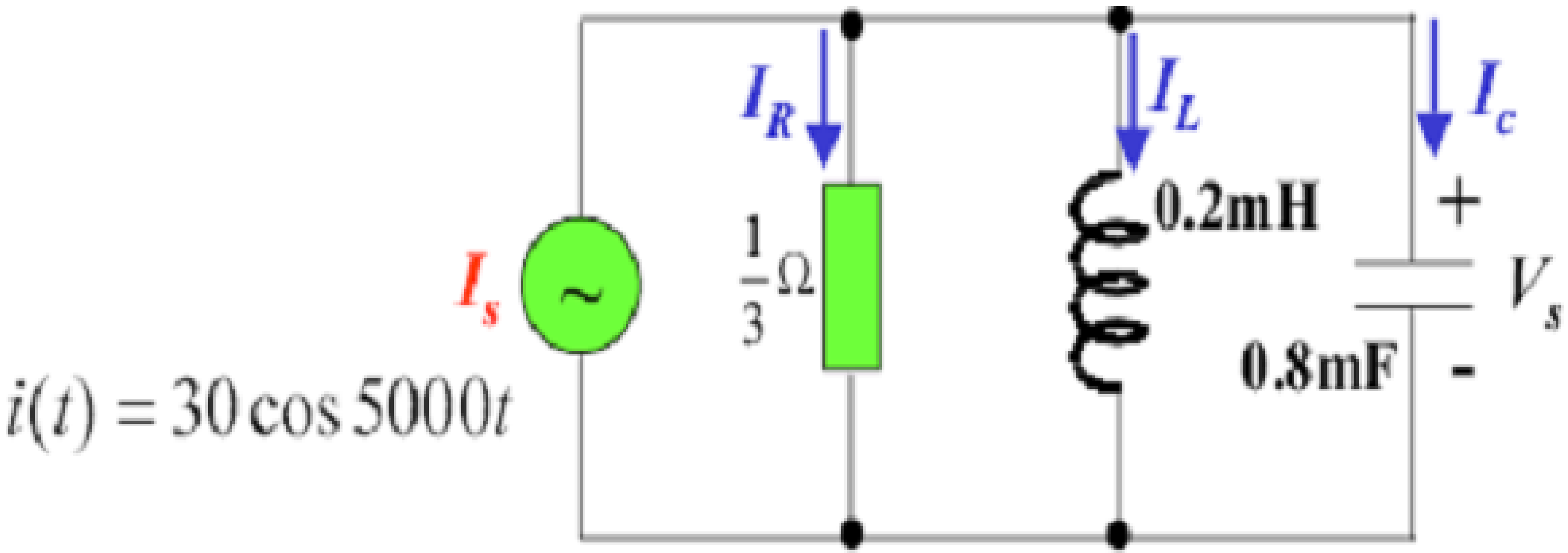
$= 15\angle 0 / 15\sqrt{2}\angle -45^0$

- If $v(t) = 15\cos\omega_o t$ V
- Then $i(t) = \frac{1}{\sqrt{2}}\cos(\omega_o t + \frac{\pi}{4})$ A

Find I_R, I_L, I_C and I_S as functions of V_s.

Draw phasor diagram of the circuit.

Determine V_s and i_L(t).

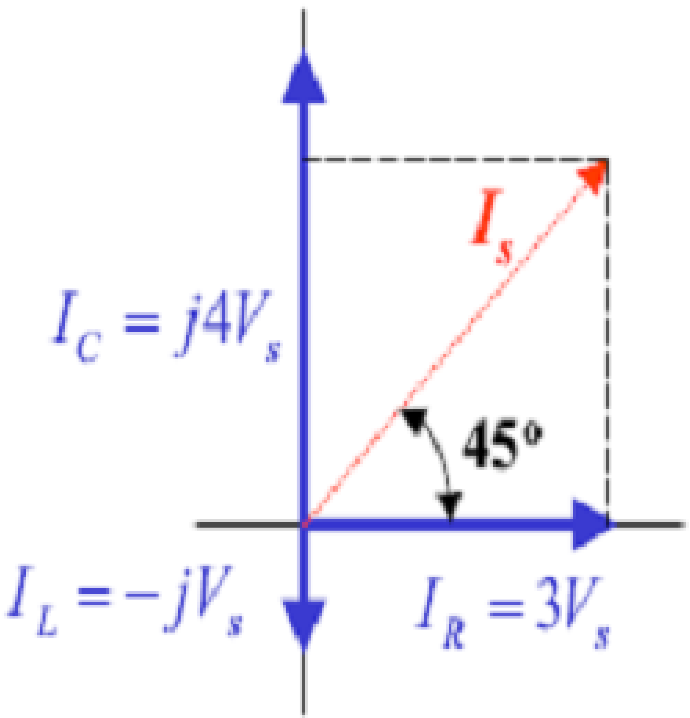


$Y_R = 3S$ $\qquad I_R = 3V_s$

(*S* unit is Mho, inverse of Ohm)

$Y_L = \frac{1}{j\omega_o L} = -jS$ $\qquad I_L = -jV_s$

$Y_C = j\omega_o C = j4S$ $\qquad I_C = j4V_s$



(Phasor Diagram)

In phasors: $I_s = I_R + I_L + I_C = (3 + j3)V_s$

$V_s = I_s / (3+j3) = 30 / (3+j3) = \frac{10}{\sqrt{2}} \angle -(\frac{\pi}{4})$

$I_L = -j\ V_s = \frac{10}{\sqrt{2}} \angle -(3\pi/4)$

In time domain:

$$v_s(t) = \frac{10}{\sqrt{2}}\cos(5000t - \frac{\pi}{4})$$

$$i_L(t) = \frac{10}{\sqrt{2}}\cos(5000t - \frac{3\pi}{4})$$

END