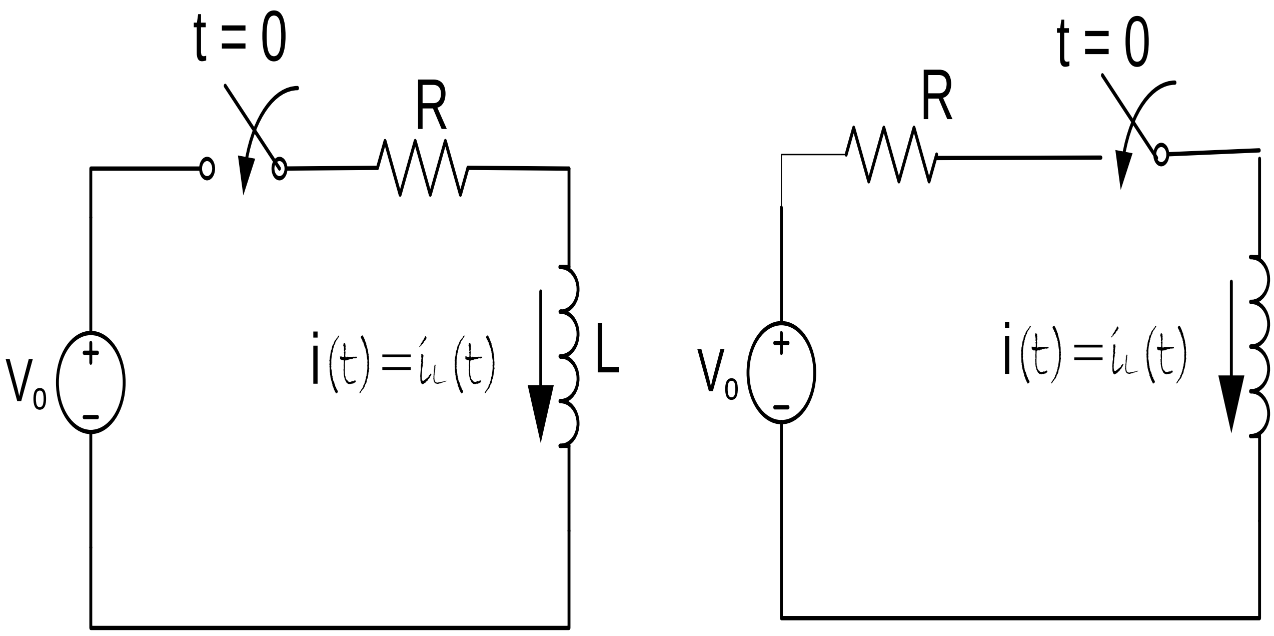
EE 101ME – Electric Circuits

Revision of RL/RC Circuits

RLC Circuits

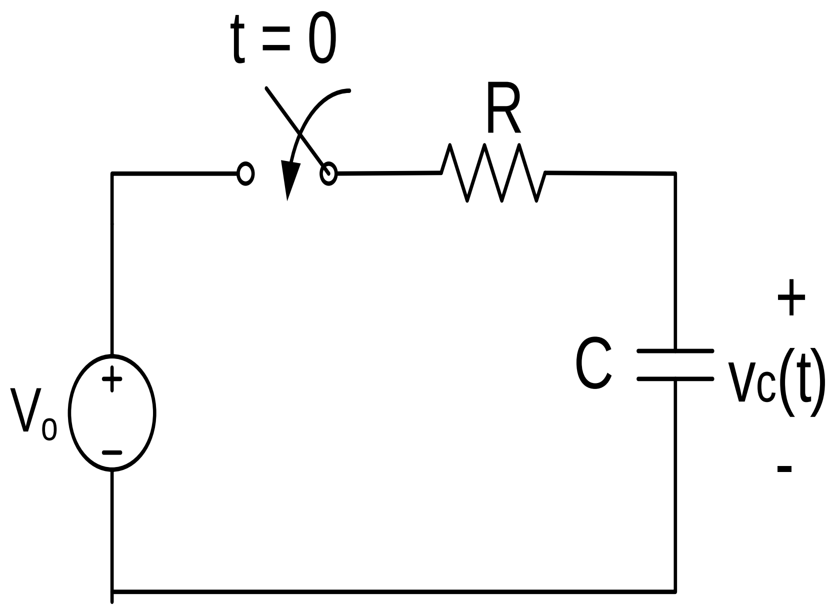
Response of RL Series Circuit



Using Source Transform DC voltage source V_0 and series R can be replaced by

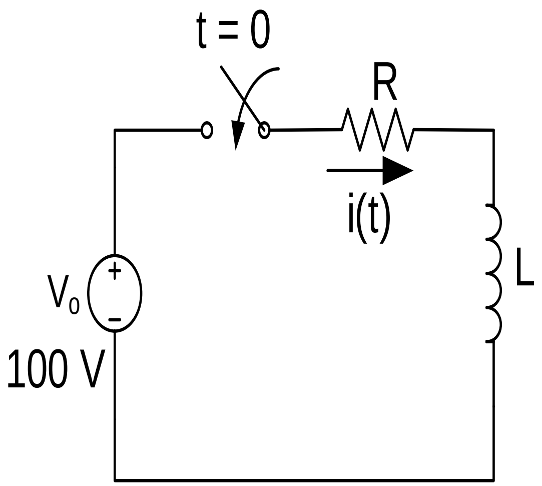
DC current source (V_0/R) and parallel R

Draw an equivalent circuit of the given circuit using a DC current source I_0 and rewrite the expression for $v_c(t)$.



$$v_c(t) = [v_c(0) - I_0 R]\, e^{-t/(RC)} + I_0 R$$

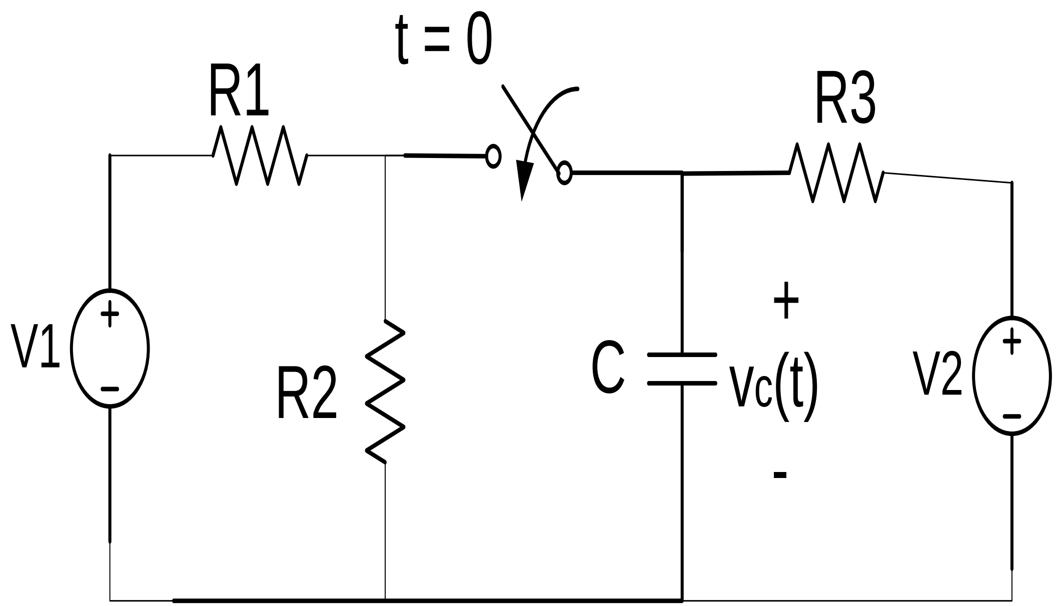
Determine R and L of the following circuit

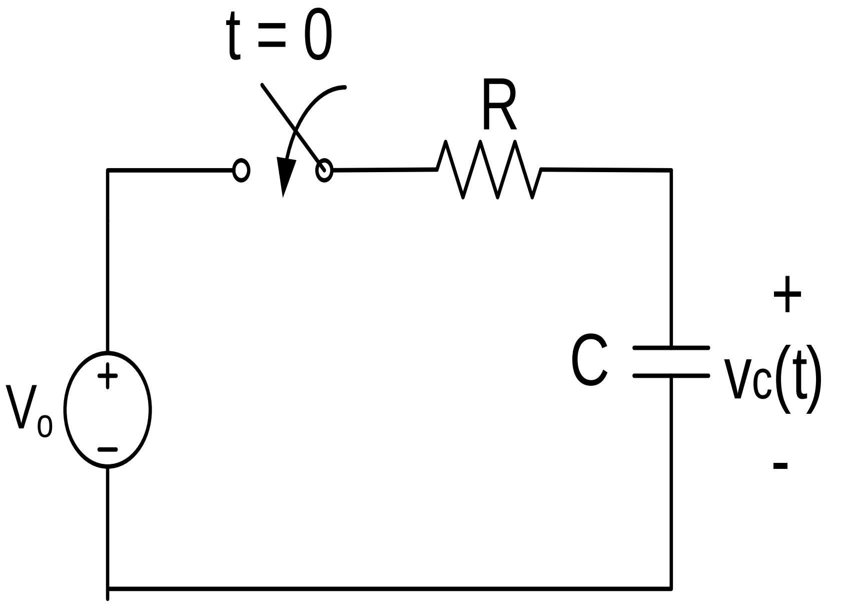


when $i(t)=i_L(t) = 12.5(1- e^{-40t})$ A and $v_L(t) = 100\ e^{-40t}$ V

R = 8 Ohm and L = 0.2 H

The switch in the following circuit is closed at time t = 0 sec. Find expression for Vc(t) for t > 0.





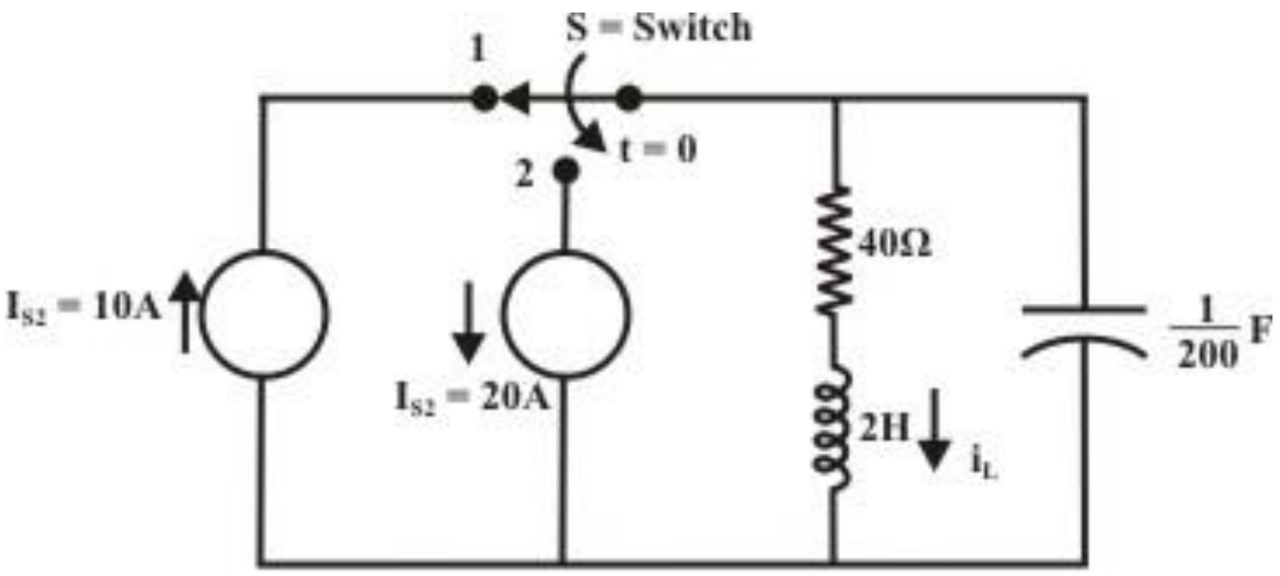
$$v_c(t) = [v_c(0) - V_0\,]\, e^{-t/(RC)} + V_0$$

Vc(0) = V2, R=Rth= R1||R2||R3, V_0=Vth=R(V1/R1 + V2/R3)

Initial Conditions
and
RLC Circuits

Initial Conditions

Switch S is connected to Node 1 for a long time before connecting to Node 2 at time t = 0 sec

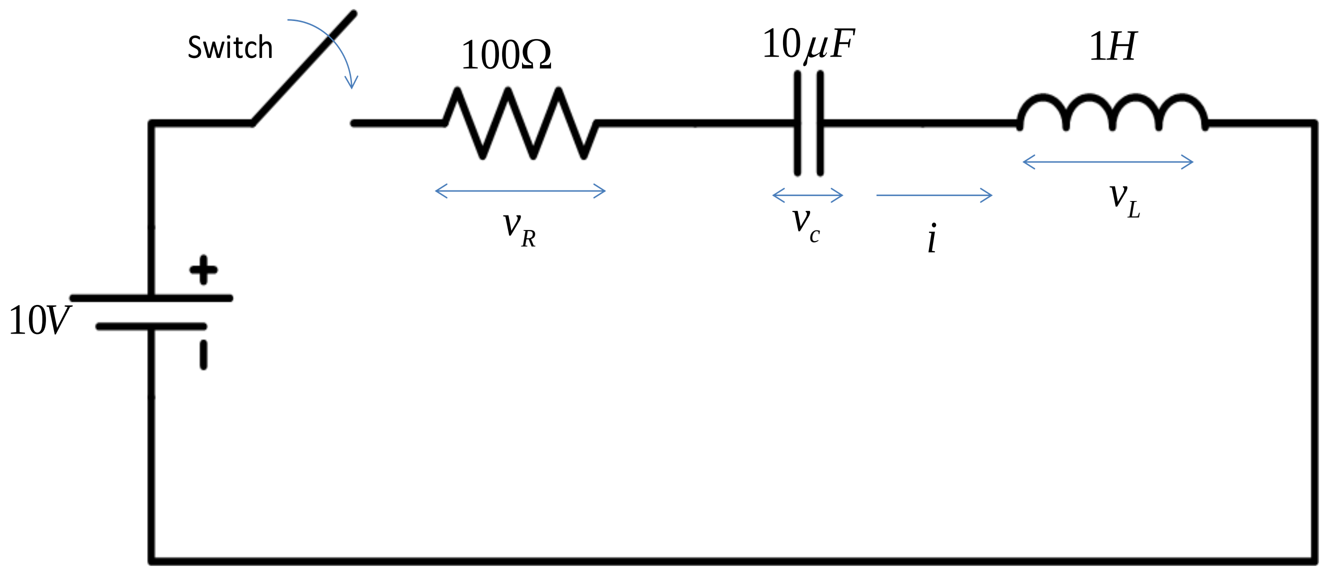


Values for: $(a)\ i_L(0^-);\ (b)\ v_c(0^+);\ (c)\ v_R(0^+);\ (d)\ i_L(\infty)$

Values are 10 A, 400 V, 400 V, -20 A

Before closing the switch, the energy storage elements did not have any stored energy in them. If switch is closed at t=0, find

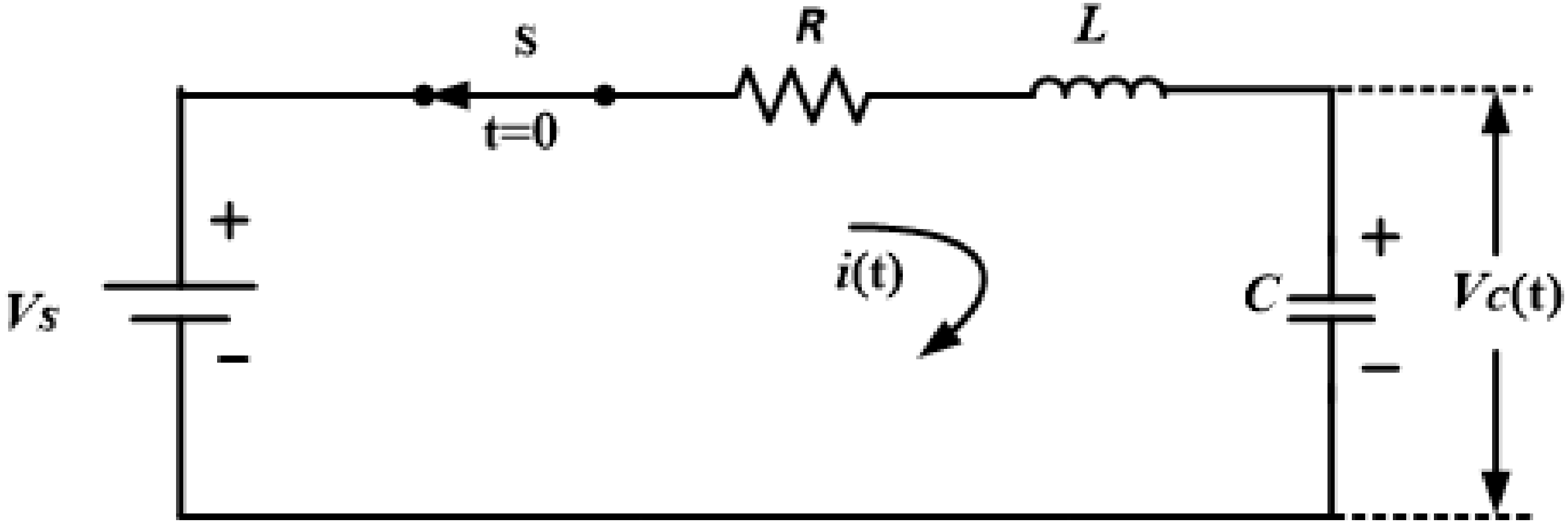
a) $i(0^+)$ b) $\frac{di}{dt}(0^+)$ c) $\frac{d^2i}{dt^2}(0^+)$ d) $V_L(0^+)$ and e) $V_c(0^+)$.



a) 0 A b) 10 A/s c) -1000 A/s^2 d) 10 V e) 0 V

RLC Circuits

Derive an expression for *Vc*(t) for t > 0.



KVL => $L\dfrac{di(t)}{dt} + R\,i(t) + v_c(t) = V_s$ and $i(t) = C\dfrac{dv_c(t)}{dt}$

Substitution yields

$$LC\frac{d^2 v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

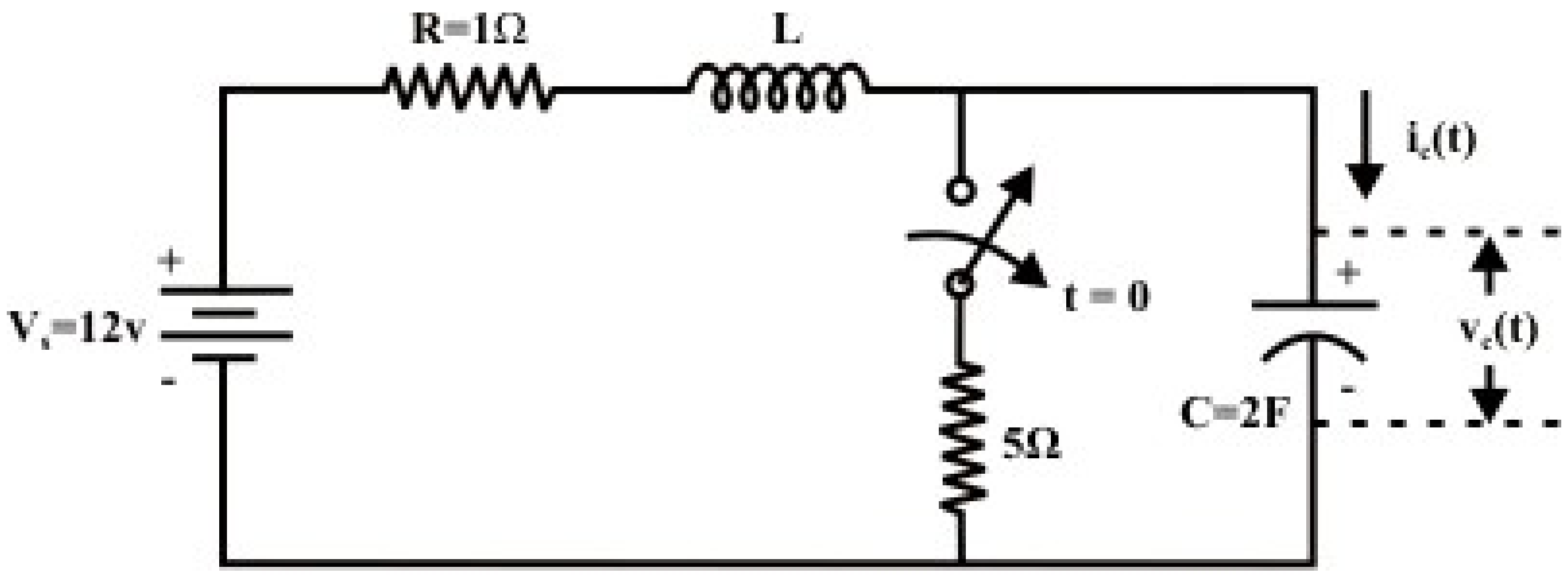
Complete solution

$$v_c(t) = \qquad\qquad = \left(A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} \right) + A$$

Three types of Responses

Example

Switch is closed for a sufficiently long time before opening at t=0. Find the expression for $v_c(t)$ and $i_c(t)$. Take L = 0.2 H.



Solution

Initial conditions for the capacitor voltage and inductor current:

$$v_c(0^+) = v_c(0^-) = 10 \text{ V}$$

$$i_L(0^+) = i_L(0^-) = \frac{12}{1+5} = 2 \text{ A}$$

Roots are:

$$\alpha_1 = -0.563; \ \alpha_2 = -4.436$$

General expression for the capacitor voltage for t > 0 is

$$v_c(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} + A = A_1 e^{-0.563t} + A_2 e^{-4.436t} + A$$

and

$$\frac{dv_{c(t)}}{dt} = \alpha_1 A_1 e^{\alpha_1 t} + \alpha_2 A_2 e^{\alpha_2 t} = -0.563\ A_1 e^{\alpha_1 t} - 4.436 A_2 e^{\alpha_2 t}$$

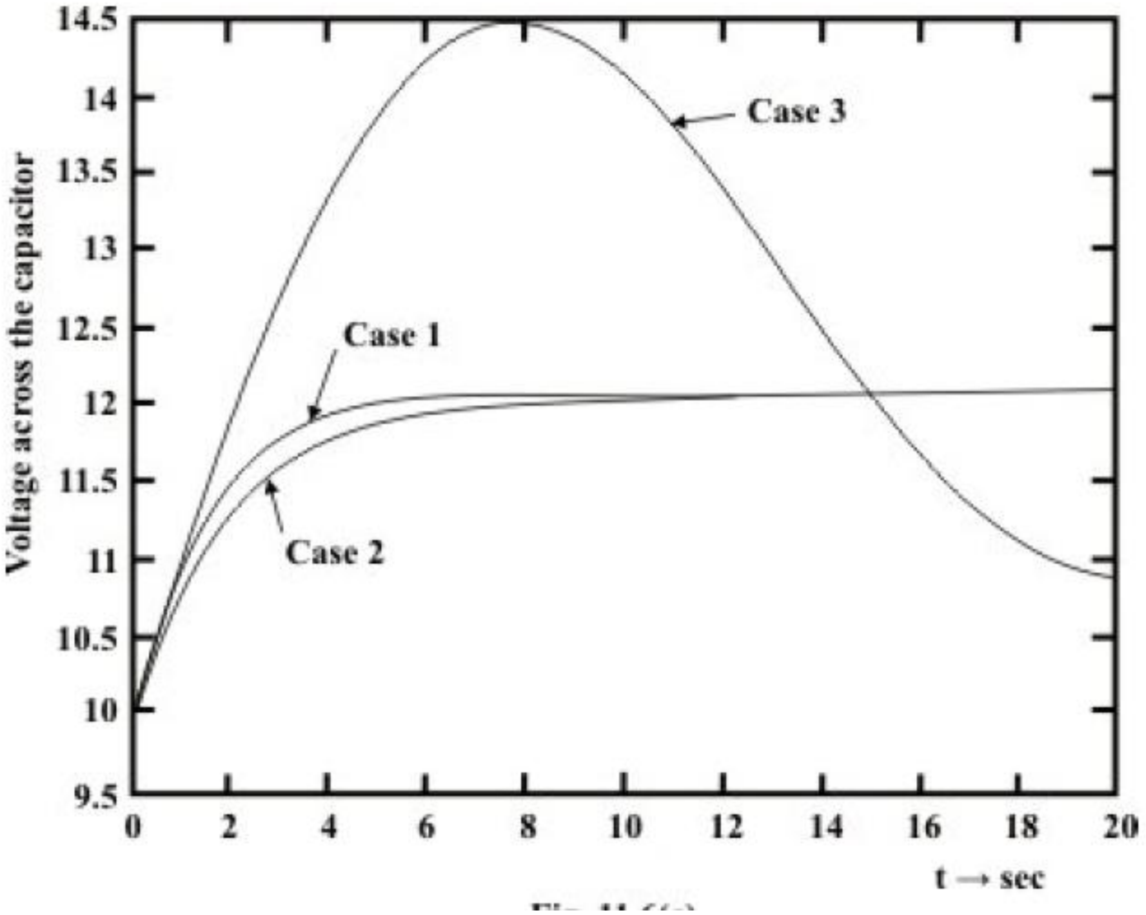
Using initial conditions

A_1 = - 2.032 , A_2 = 0.032 and A = 12

$$v_c(t) = -2.032\, e^{-0.563t} + 0.032\, e^{-4.436t} + 12 \text{ ,V}$$

$$i_c(t) = C\,\frac{dv_{c(t)}}{dt} = 2(1.144e^{-0.563t} - 0.144e^{-4.436t}) \text{ ,A}$$

Draw the plots for capacitor voltage for 20 sec



Case 1:
L = 0. 5 H

Case 2:
L = 0.2 H

Case 3:
L = 0.8 H

END