EE 101ME – Electric Circuits

First order Differential Equation: $dx(t)/dt+ax(t)=b$

Can also be written as: $dx(p)/dp + ax(p) = b$

Multiplying both sides by e^{ap} gives:

$$e^{ap}\, dx(p)/dp + e^{ap}\, ax(p) = e^{ap}\, b$$

$$\Rightarrow \quad d(e^{ap}\, x(p))/dp = e^{ap}\, b$$

$$\Rightarrow \quad d(e^{ap}\, x(p)) = e^{ap}\, b\, dp$$

$$\Rightarrow \int_o^t d(e^{ap}\, x(p)) = \int_o^t e^{ap}\, b\, dp$$

$$\Rightarrow \quad e^{ap}\, x(p) \,\big|_{o}^{t} = (b/a)\, e^{ap} \,\big|_{o}^{t}$$

Substitution of limits of integral => $e^{at}\, x(t) – x(0) = (b/a)\,(e^{at} – 1)$

$$\Rightarrow \quad e^{at}\, x(t) = x(0) + (b/a)\,(e^{at} – 1)$$

Multiplying both sides of above equation by e^{-at}

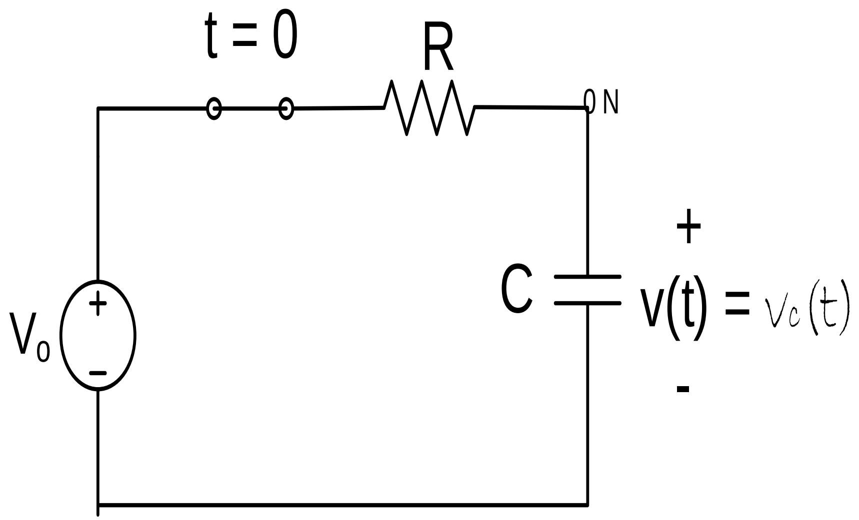
$$\Rightarrow x(t) = e^{-at}\, x(0) + (b/a)\,(1 - e^{-at})$$

$$= (x(0) – b/a)\, e^{-at} + b/a$$

So, for a 1st order Differential Equation: $\dfrac{dx(t)}{dt} + a\, x(t) = b$

Its solution can be written as: $x(t) = (x(0) – b/a)\, e^{-at} + b/a$

RC Series Circuit: Switch is closed at time t = 0



Find expression for the capacitor voltage v(t) for t > 0

Solution:
Voltage at the Node N is v.

Applying KCL at a Node N: (current through the capacitor is equal to current entering the Node N)

$$\Rightarrow \; C\, dv(t)/dt = (V_o - v)/R$$

Applying KCL at a Node N:

$C\,dv(t)/dt = (V_o - v)/R$

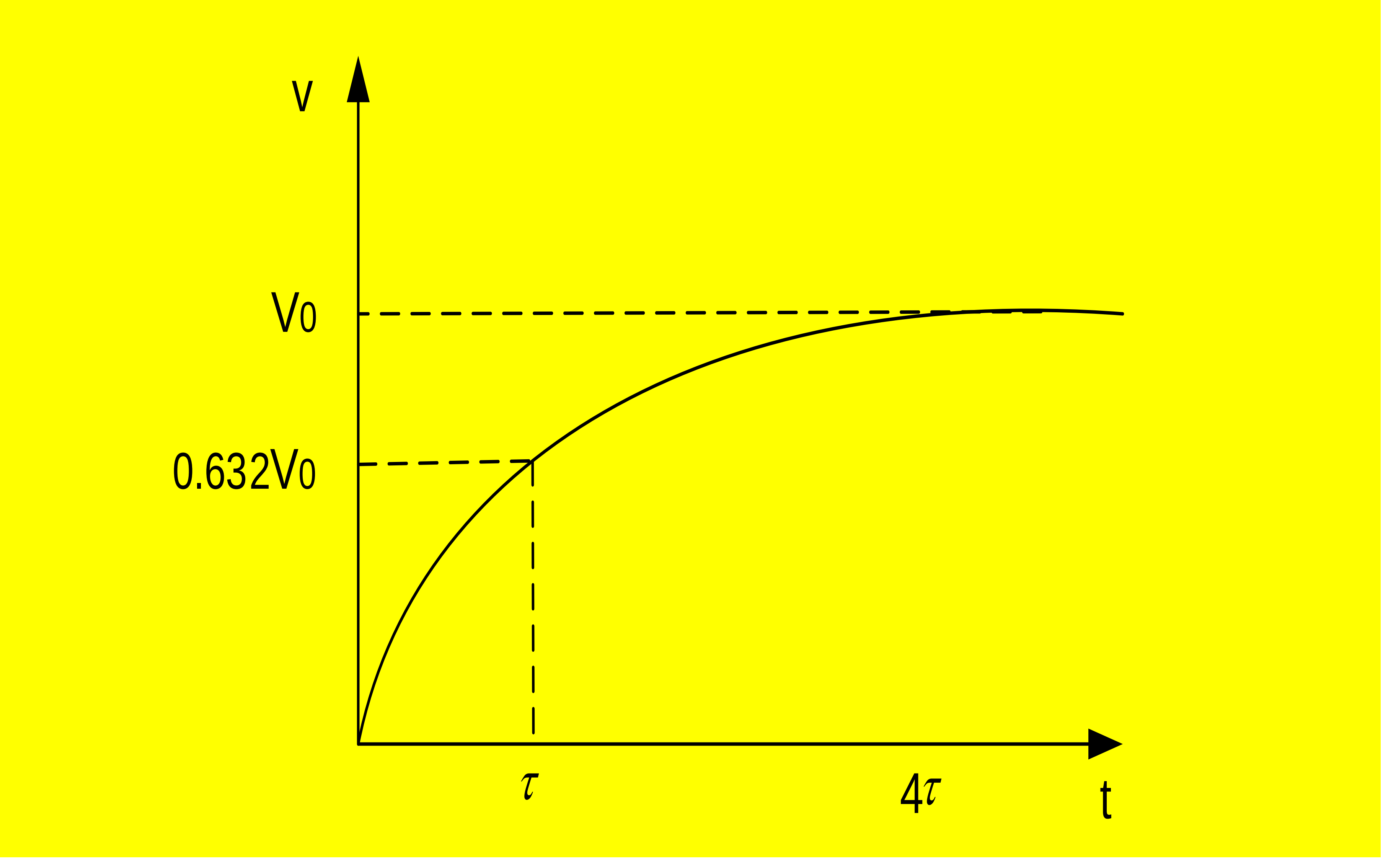
$\Rightarrow dv/dt + (1/RC)\,v = (V_0/RC)$

We know, given $dx(t)/dt + a\,x(t) = b$

its solution is $x(t) = [x(0) - (b/a)]\,e^{-at} + (b/a)$

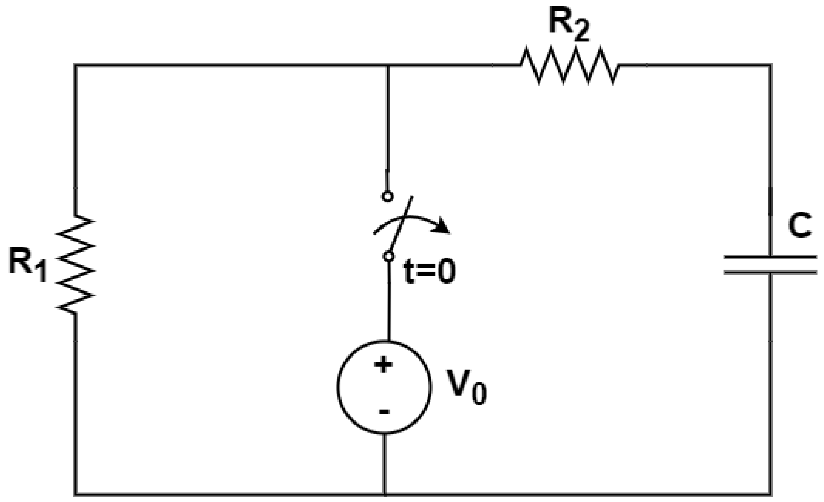
For series RC circuit : After closing the switch $v(0)=v(0^-)=0$
Again, $x(t) = v(t)$, $a = 1/(RC)$ and $b = V_0/(RC)$
Then, $v(t) = v_c(t) = V_0\,(1 - e^{-t/(RC)})$

Plot for the capacitor voltage after closing the switch:



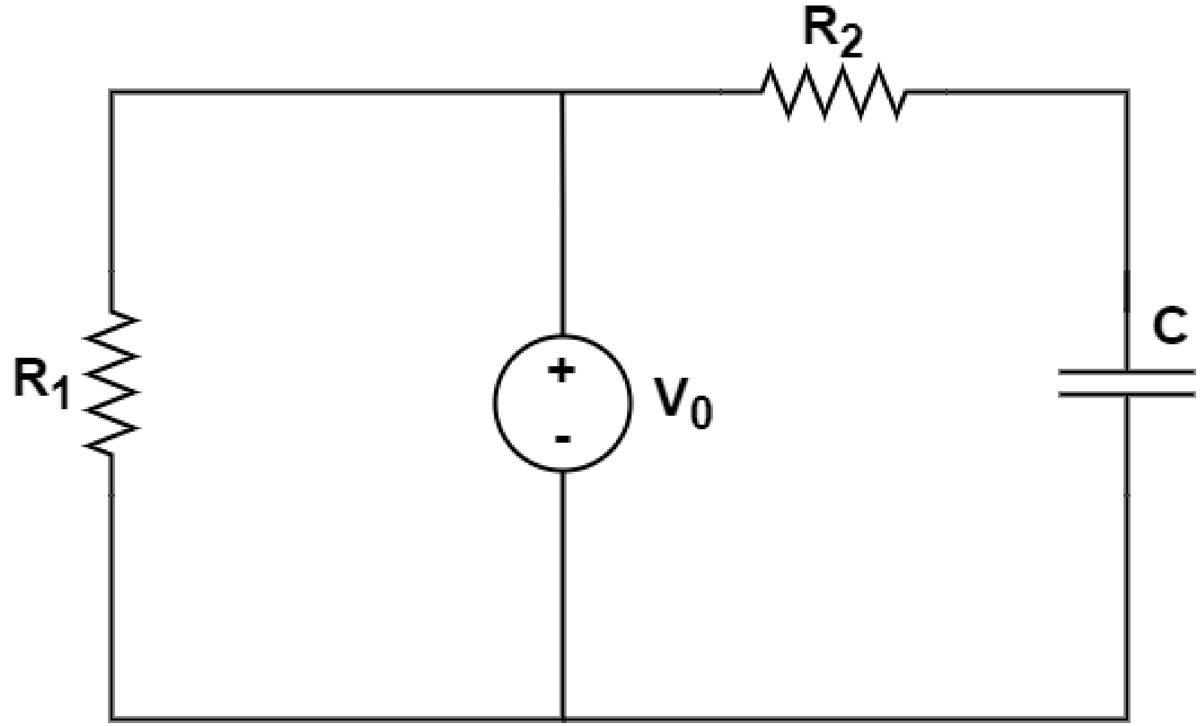
Initial Conditions and RC Circuit

The switch is connected for a long time before opening at time t = 0 sec



Given: R_1= 4 Ohm, R_2= 8 Ohm, C=20 microFarad and V_o= 18 V.

Write the expression for v_c (t) across the capacitor C for t>0.

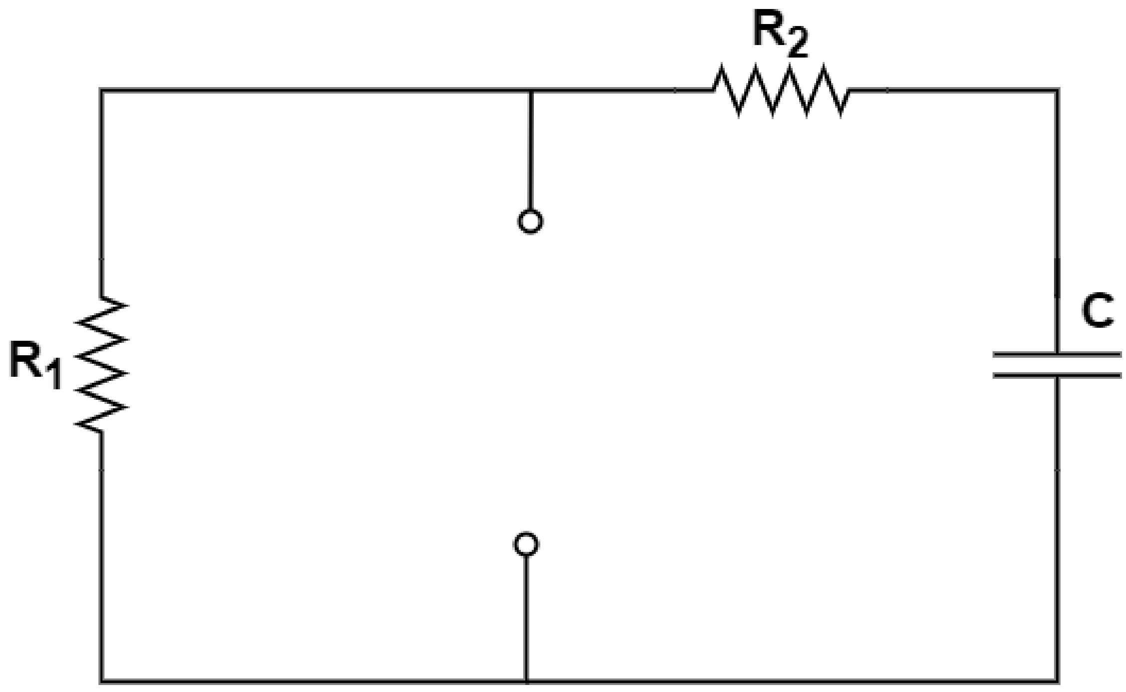


The circuit at $t = 0^-$ before the switch is opened

The capacitor is fully charged to $v_c(0^-) = V_0 = 18V$

since the capacitor current $i_c = C\, dv/dt = 0$ for DC V_0

The switch opens at t = 0. The circuit is shown for $t > 0^+$.



For series RC circuit : After opening the switch $v_c(0)=v(0^-)=18$ V

Note that there is no DC source in the above circuit, so $V_0 = 0$ V

Again, $x(t) = v_c(t)$, $a = 1/(RC)$, $R = R_1+R_2$ and $b = V_0/(RC) = 0$

We know: $x(t) = (x(0) – b/a)\, e^{-at} + b/a$

Then, $v_c(t) = v_c(0)\, e^{– t/(RC)} = 18\, e^{– 4166.67\, t}$ V

END