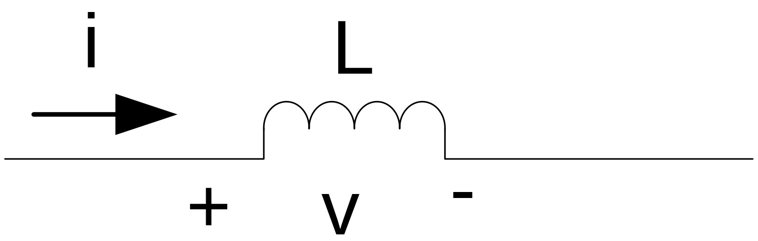
EE101ME – Electric Circuits

RL and RC Circuits

The Inductor



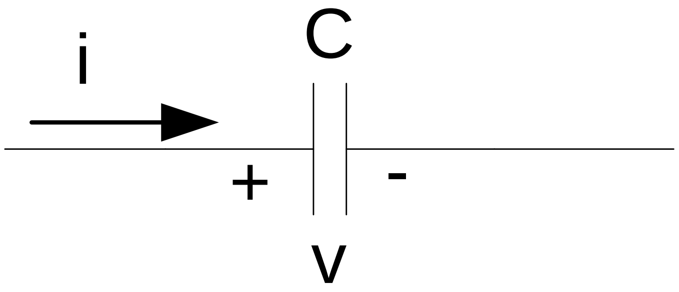
$$v = L\frac{di}{dt}$$

If current i is constant, voltage v is 0

So, inductor behaves as a short circuit to dc input current

Again, current through an inductor cannot change instantaneously (immediately)

The Capacitor



$$i = C\frac{dv}{dt}$$

If voltage v is constant, then i is 0

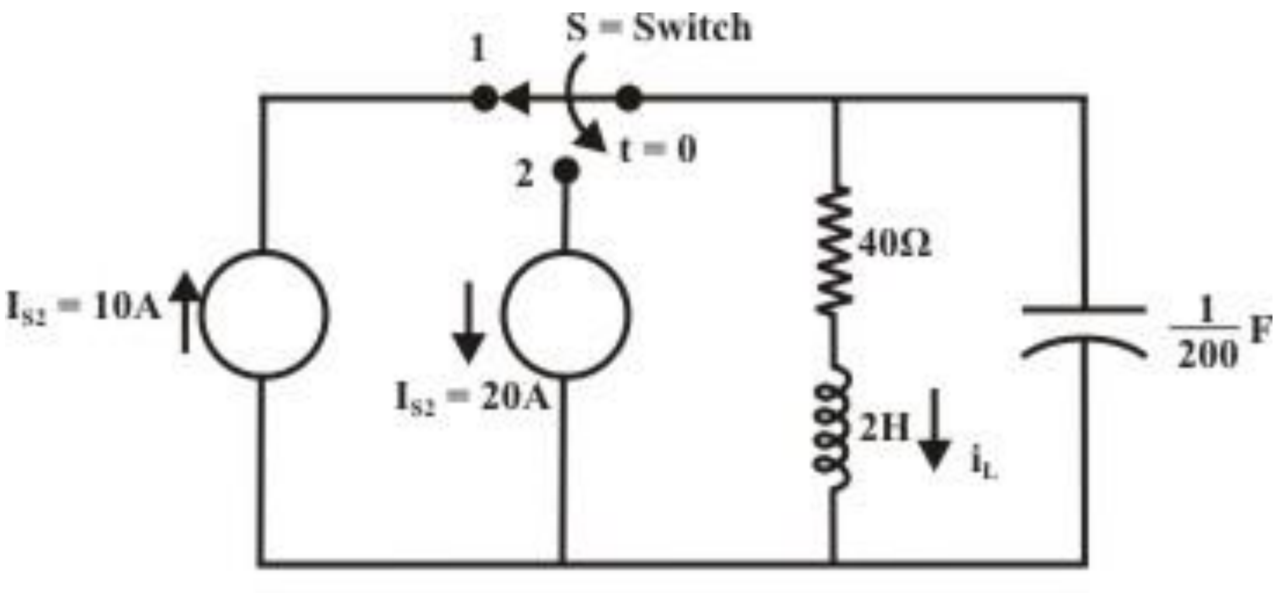
<u>So, capacitor behaves as an open circuit to dc input voltage</u>

<u>Again, voltage across a capacitor cannot change instantaneously (immediately)</u>

Initial Conditions of RLC Circuit

Initial Conditions

Switch S is connected to Node 1 for a long time before connecting to Node 2 at time t = 0 sec

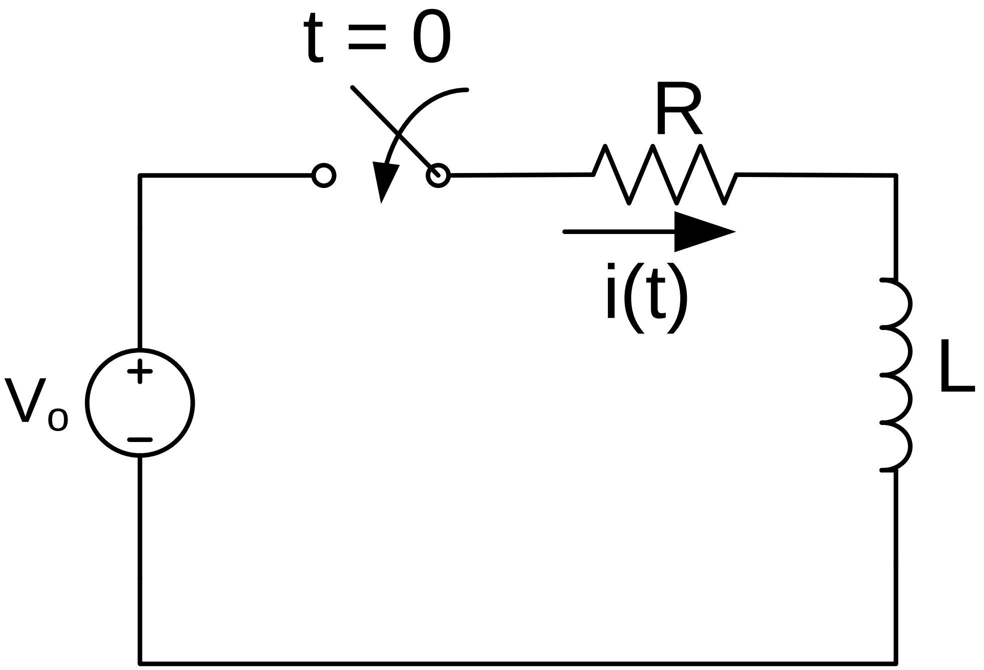


Values for:

$$(a)\ i_L(0^-);\ \ (b)\ \ v_c(0^+);\ \ (c)\ \ v_R(0^+);\ \ (d)\ i_L(\infty)$$

Values are 10 A, 400 V, 400 V, -20 A

Response of RL Series Circuit



Find i(t) for t > 0

After switching:

$$Ri + L\frac{di}{dt} = V_0 \quad \text{for } t > 0$$

Solution -

Complimentary Function

+ Particular Integral

Solution -

Natural Response (i_n) from

$$Ri + L\frac{di}{dt} = 0$$

Forced Response (i_f) from

$$Ri + L\frac{di}{dt} = V_0$$

Complete Solution is

$$i = i_n + i_f$$

For Natural Response

$$Ri + L\frac{di}{dt} = 0$$

$$\text{or, } \frac{di}{dt} + \frac{R}{L}i = 0$$

$$or, \; i_n = Ae^{-\frac{R}{L}t}$$

For Forced Response

$$Ri + L\frac{di}{dt} = V_0$$

$$\textit{But} \quad L\frac{di}{dt} = 0$$

$$\text{So,} \quad i_f = \frac{V_0}{R}$$

Overall Response

$$i(t) = Ae^{-\left(\frac{R}{L}\right)t} + \frac{V_0}{R} \qquad t > 0$$

For Finding A : Apply the condition

$$i(t=0) = i(t=0^-) = i(t=0^+)$$

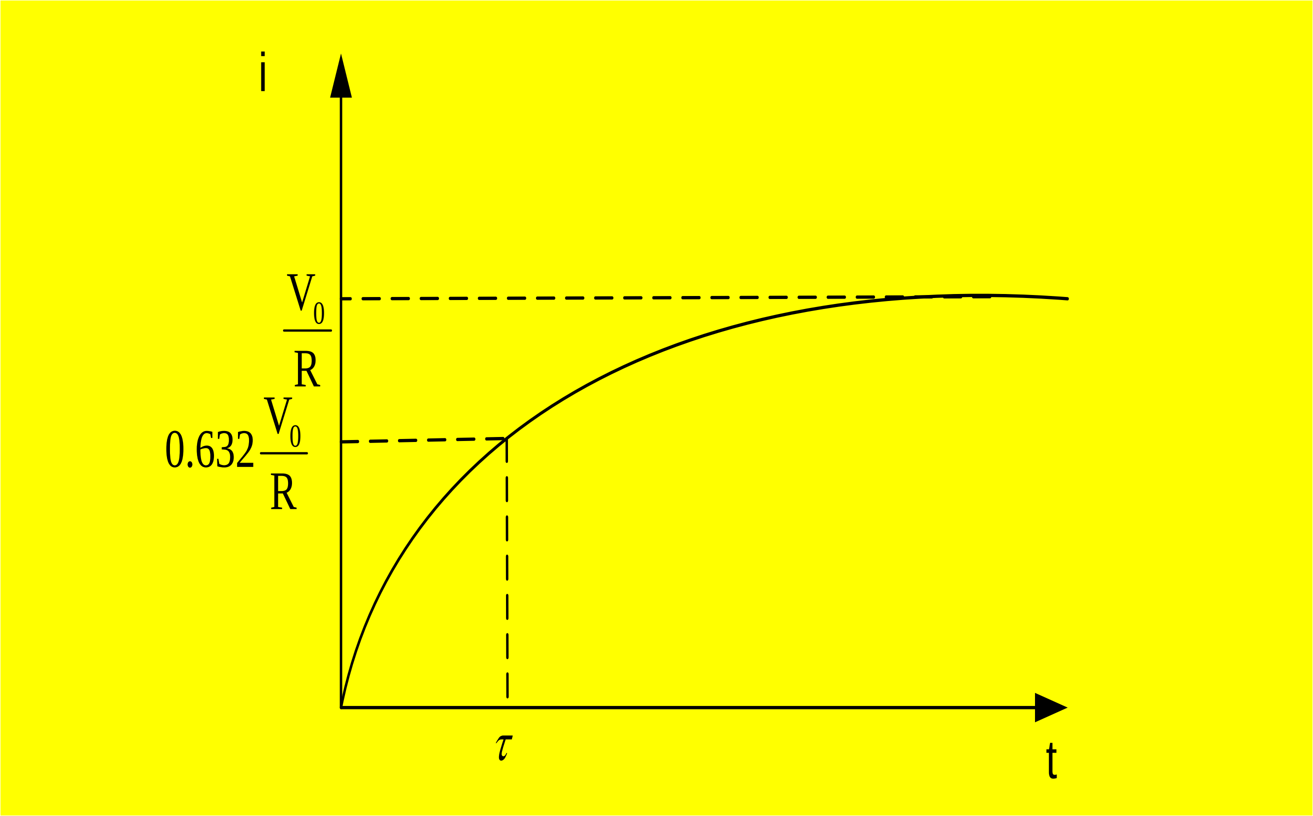
$$0 = A + \frac{V_0}{R}$$

$$A = -\frac{V_0}{R}$$

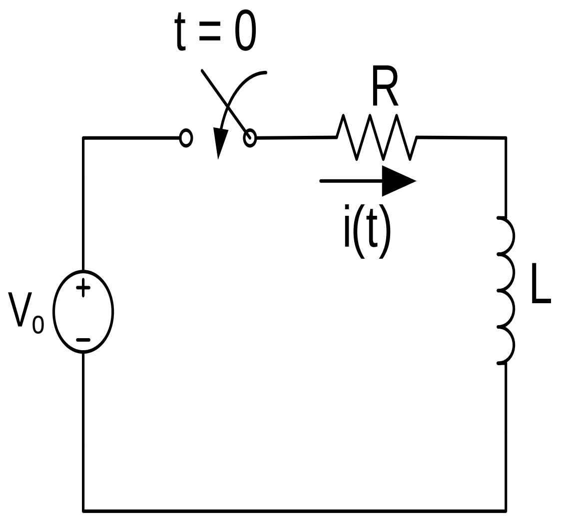
$$i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t} \right) \quad t > 0$$

$$or, \;\; i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{t}{\tau}} \right) \quad t > 0 \qquad \tau = \frac{L}{R} \;\; \text{Time Constant}$$

Response of R-L Circuit or plot of the inductor current:



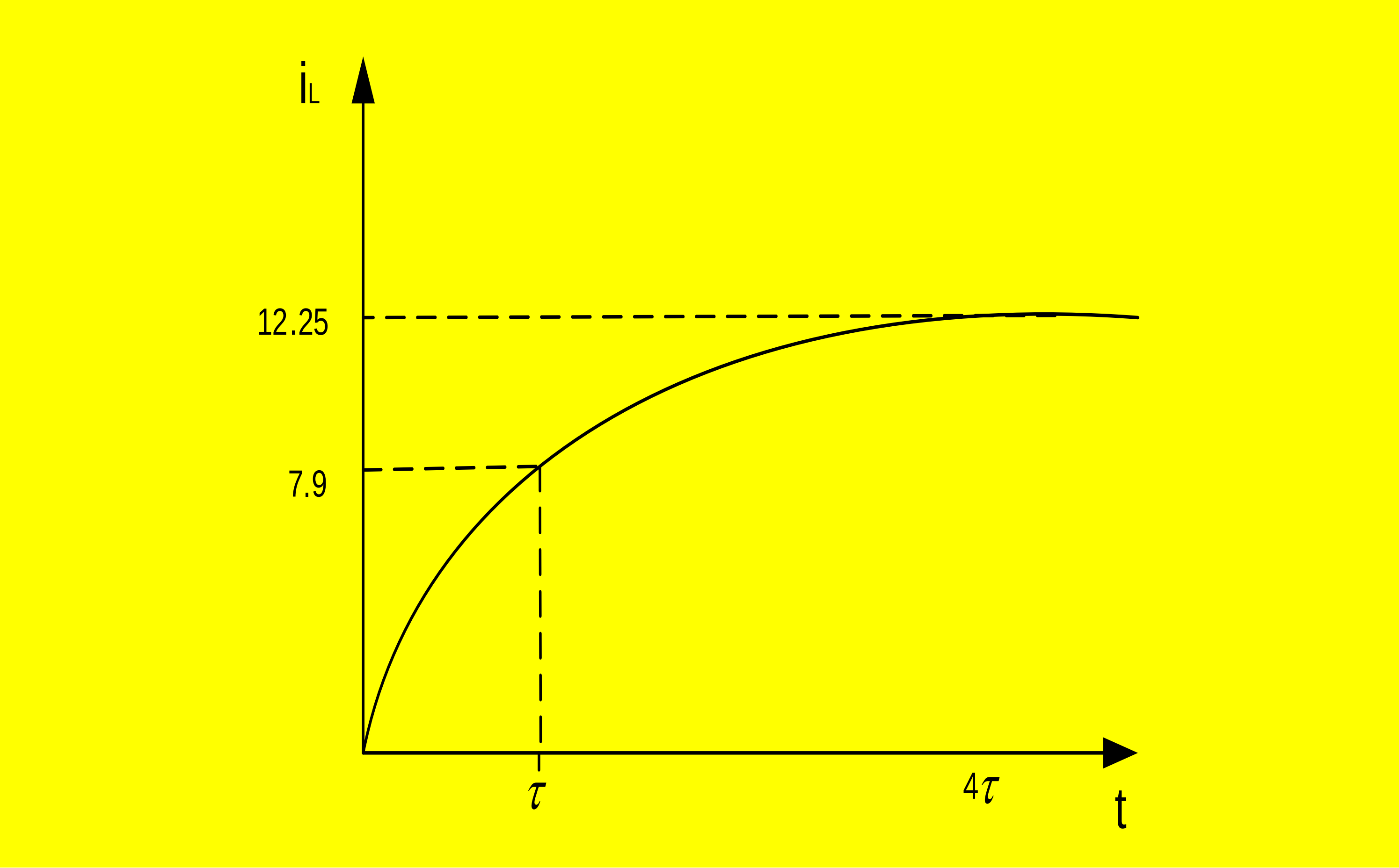
Find expression for the inductor current after switching and then plot $i_L(t)$ for $t \leq 0.1$ sec.



Vo = 100 V, R = 8Ω, L = 0.2 H

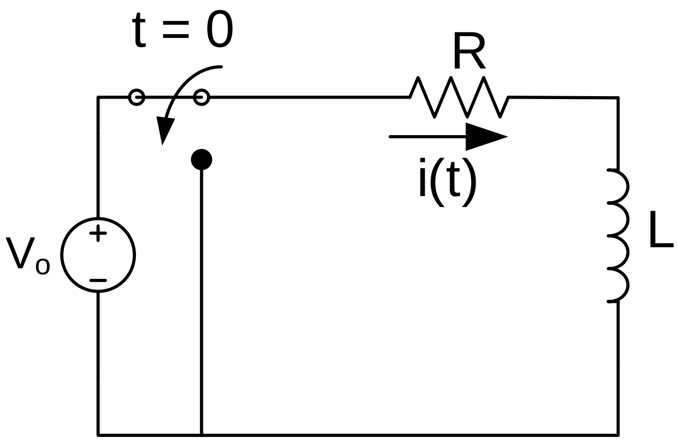
$$i_L(t) = [0 - 12.5]\, e^{-40t} + 12.5 = 12.5(1 - e^{-40t})$$

Response of R-L Circuit or plot of the inductor current:



(i) Find expression for the inductor current after switching.

(ii) At which time the inductor current becomes 50% of its initial value.

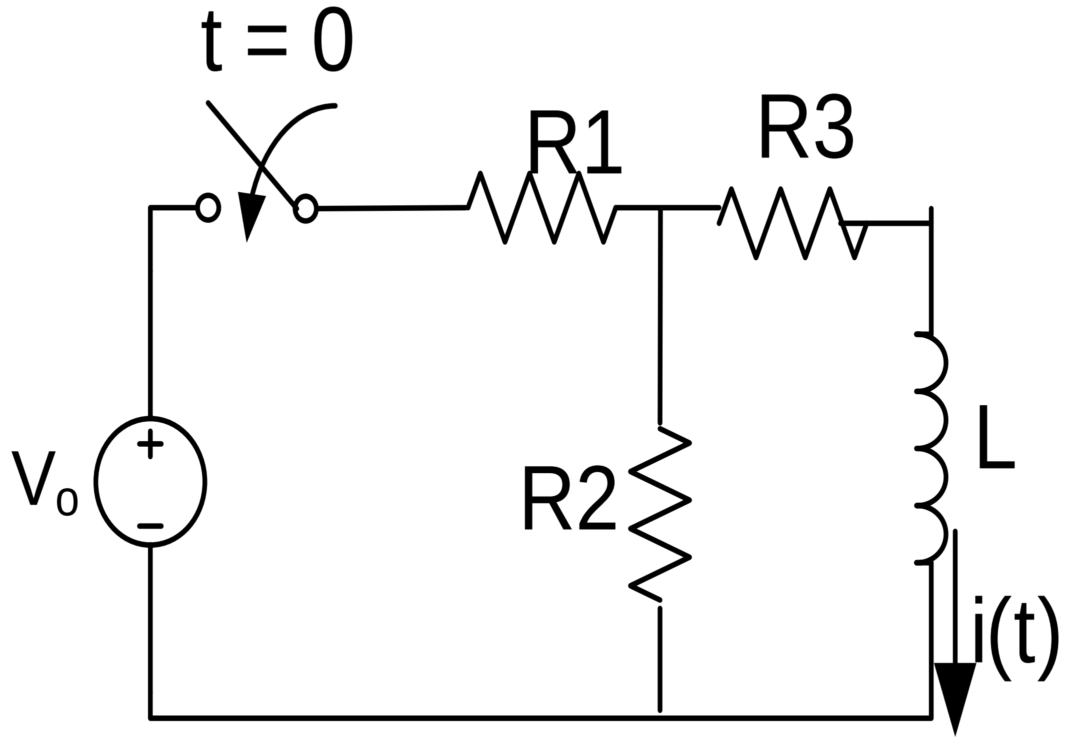


V_o = 100 V, R = 8Ω, L = 0.2 H

(i) $i_L(t) = 12.5\, e^{-40t}$

(ii) t = 0.01733 sec

Find expression for the inductor current after closing the switch at time t = 0.

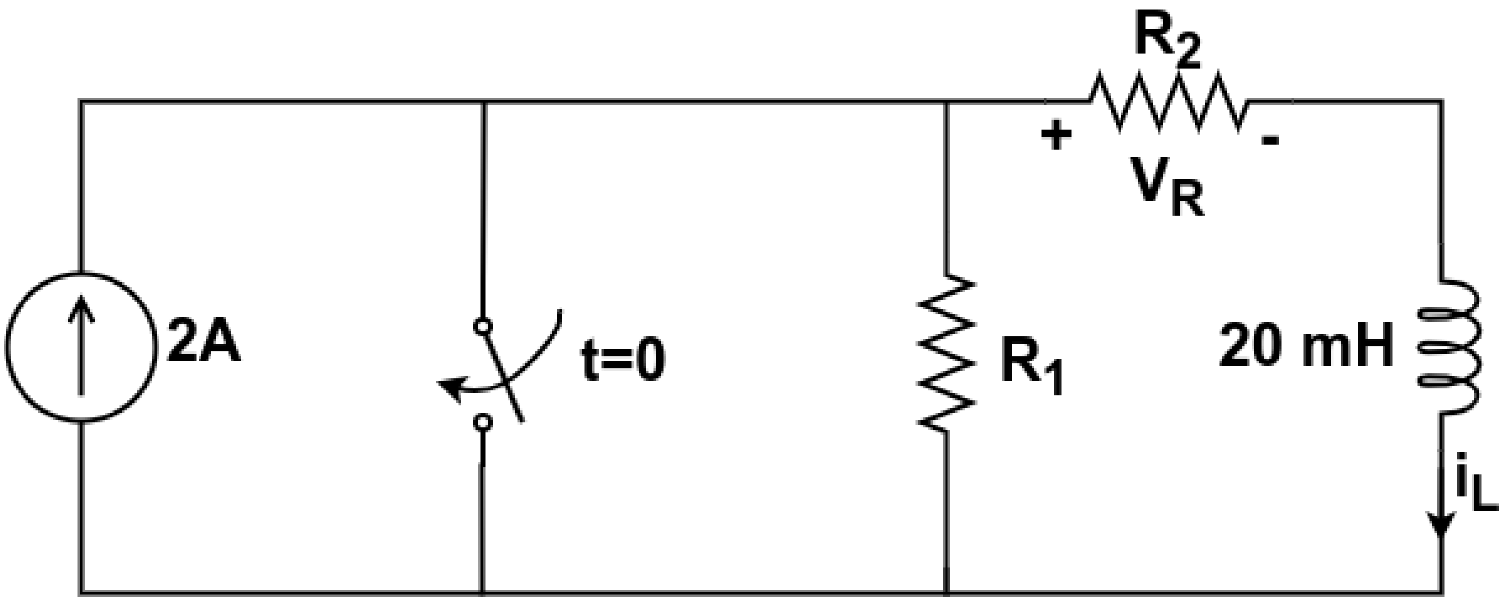


$i_L(t) = 0.333(1- e^{-1.5t})$

Vo = 1 V, R1= R2=R3=1Ω, L=1H

RL Circuit

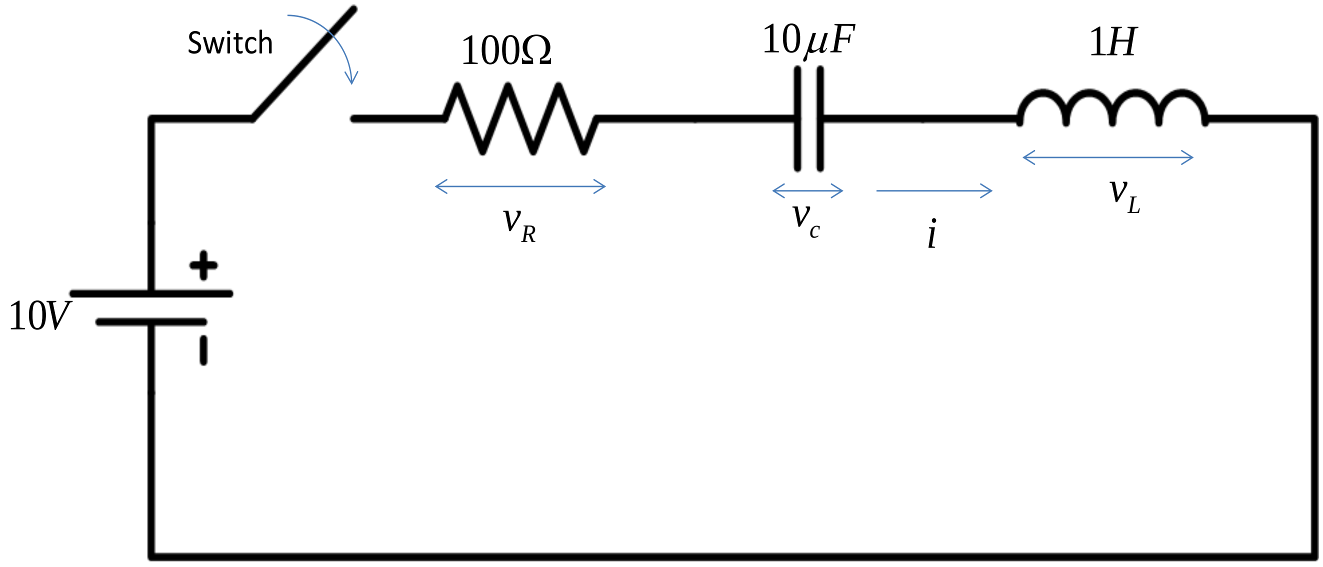
Given $R_1 = 7.8204$ Ohm and $R_2 = 13.8629$ ohm,
find $v_R(0^+)$ and $v_R(1\text{ ms})$. The switch closes at time t = 0.



$v_R(0^+) = 10$ V and $v_R(1\text{ ms}) = 5$ V

Before closing the switch, the energy storage elements did not have any stored energy in them. If switch is closed at t=0, find

a) $i(0^+)$ b) $\frac{di}{dt}(0^+)$ c) $\frac{d^2i}{dt^2}(0^+)$ d) $V_L(0^+)$ and e) $V_c(0^+)$.



a) 0 A b) 10 A/s c) -1000 A/s^2 d) 10 V e) 0 V

END