EE1103H & 1104H

– Electric Circuits

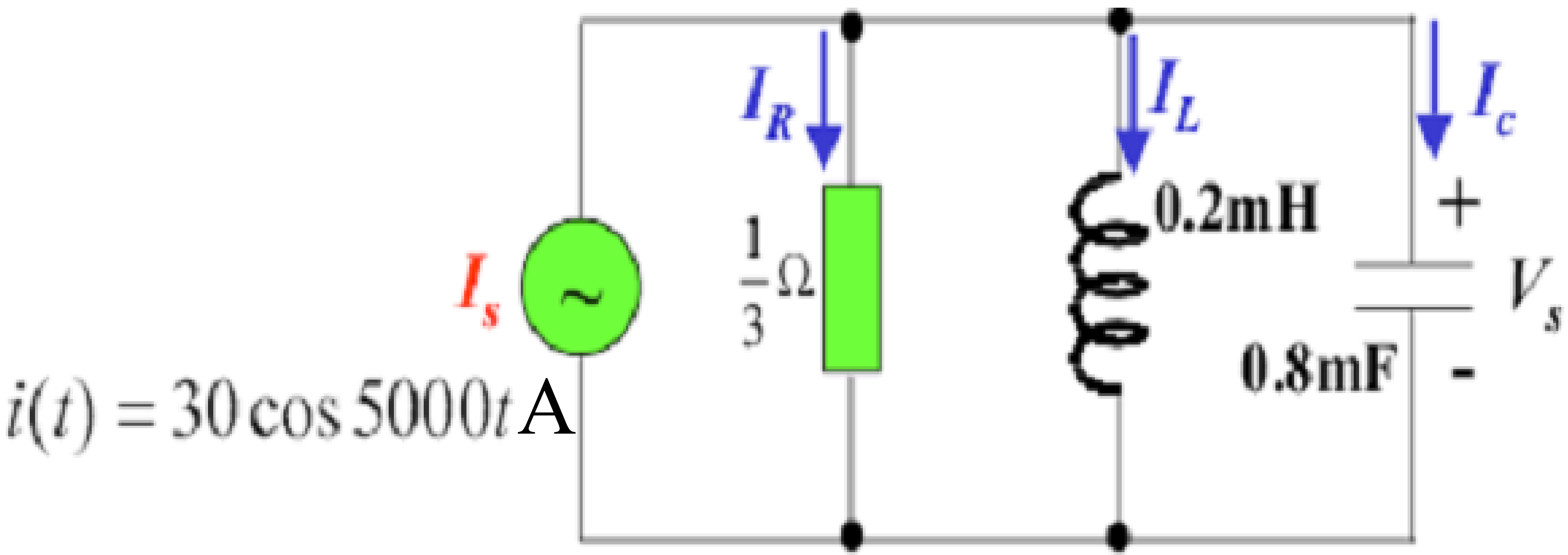
Lecture - 18

Phasor Diagram

Find I_R, I_L, I_C and I_S as functions of V_s.

Draw phasor diagram of the circuit.

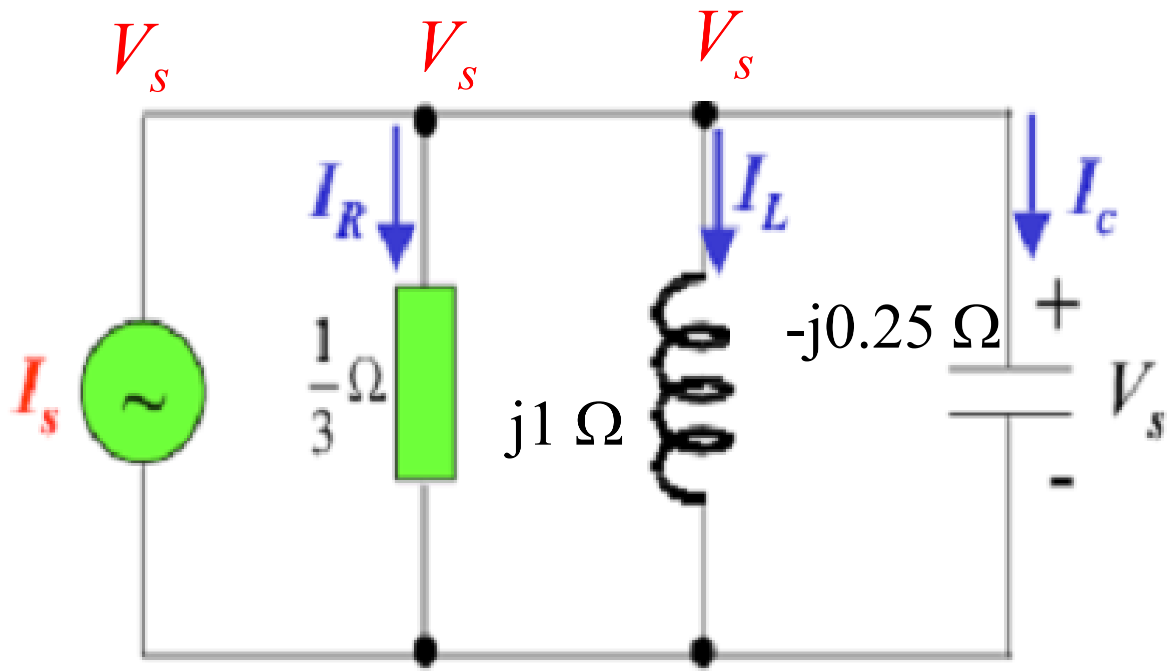
Determine v_s(t) and i_L(t).



Solution

The circuit in its equivalent PHASOR form can be redrawn as:

$I_s = 30 < 0^0$ A



Current through the resistor $I_R = 3V_s$

Answers

Current through the inductor $I_L = -jV_s$

Current through the capacitor $I_C = j4V_s$

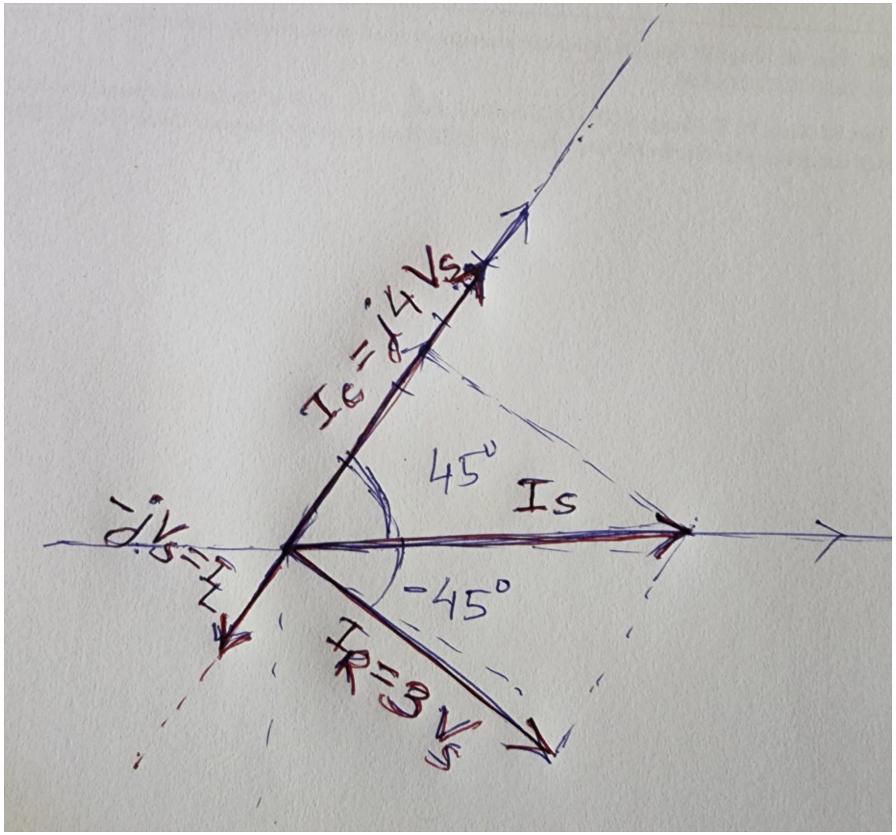
But, Current source $I_s = I_R + I_L + I_C = (3 + j3)V_s$

$$\Rightarrow V_s = I_s / (3+j3) = 30 / (3+j3)$$

$$\Rightarrow \; V_s = \frac{10}{\sqrt{2}} \angle -\frac{\pi}{4}$$

Answers

The PHASOR DIAGRAM can be drawn as:



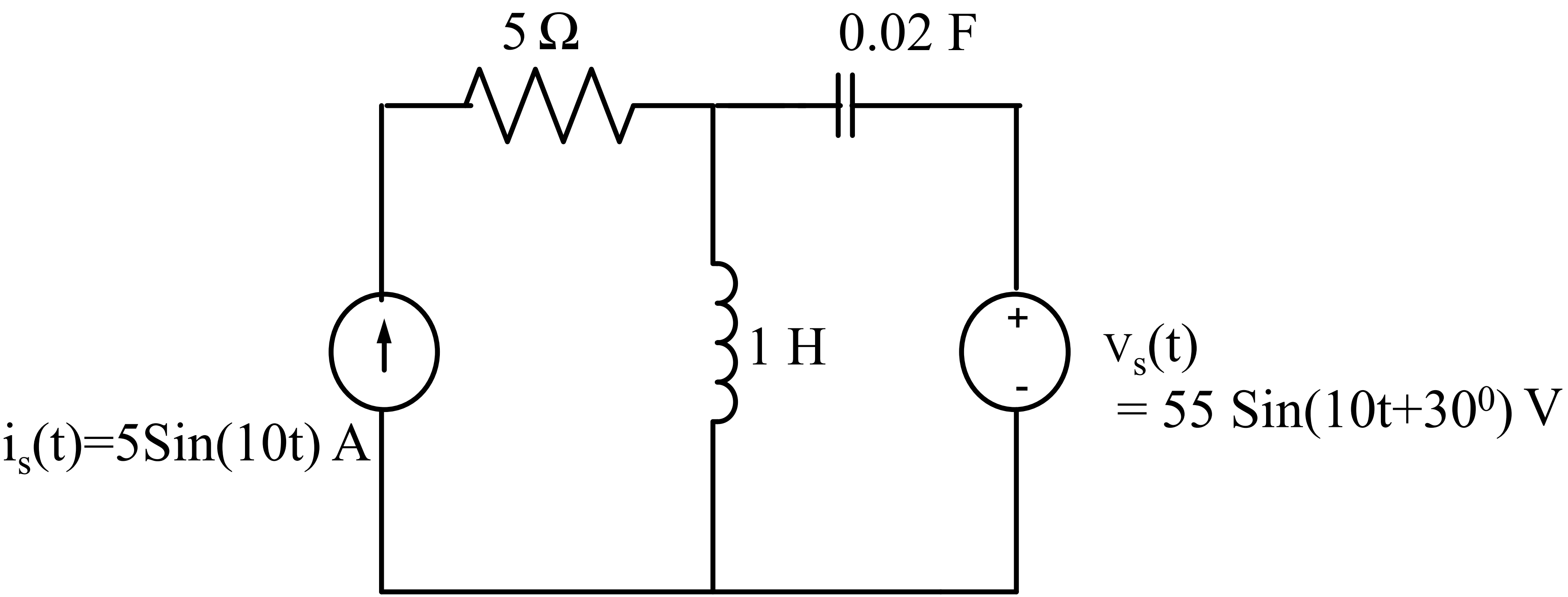
$$v_s(t) = \frac{10}{\sqrt{2}}\cos(5000t - \frac{\pi}{4}) \quad \text{V}$$

$$i_L(t) = \frac{10}{\sqrt{2}}\cos(5000t - \frac{3\pi}{4}) \quad \text{A}$$

Example: Benefits of Phasor Representation

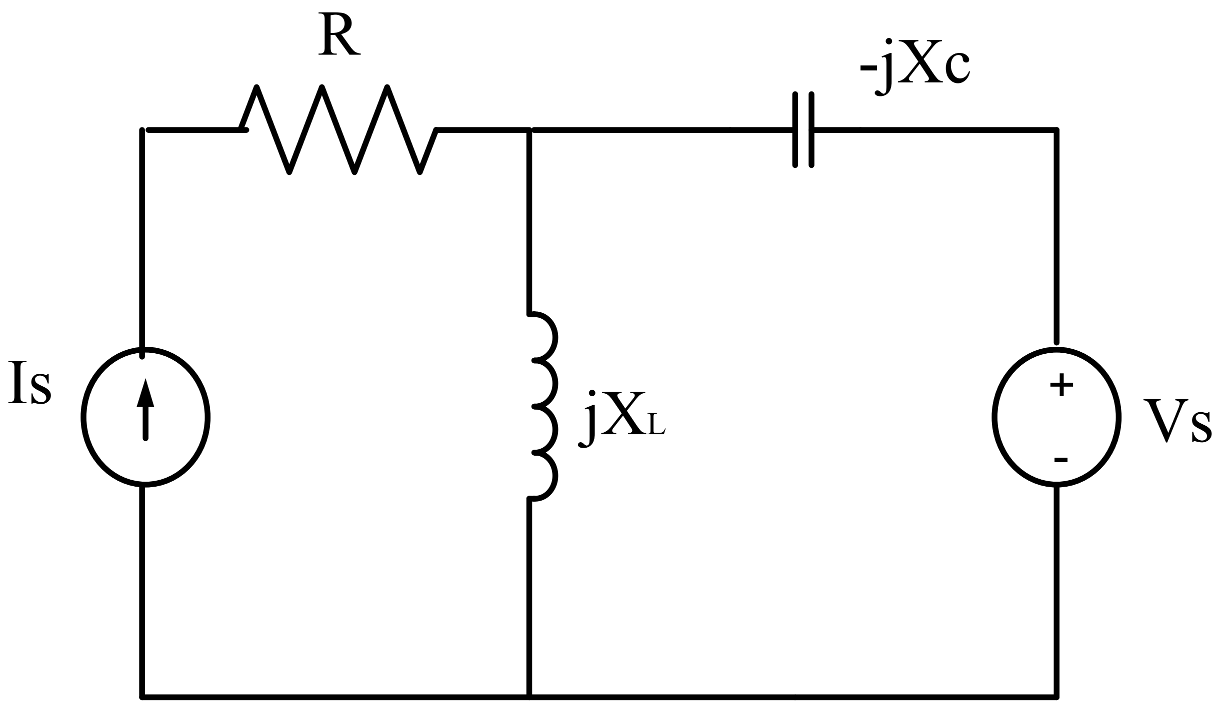
(a) Find voltage across the current source in phasor form.

(b) Find voltage across the inductor in phasor form.



Solution using Thevenin's Theorem

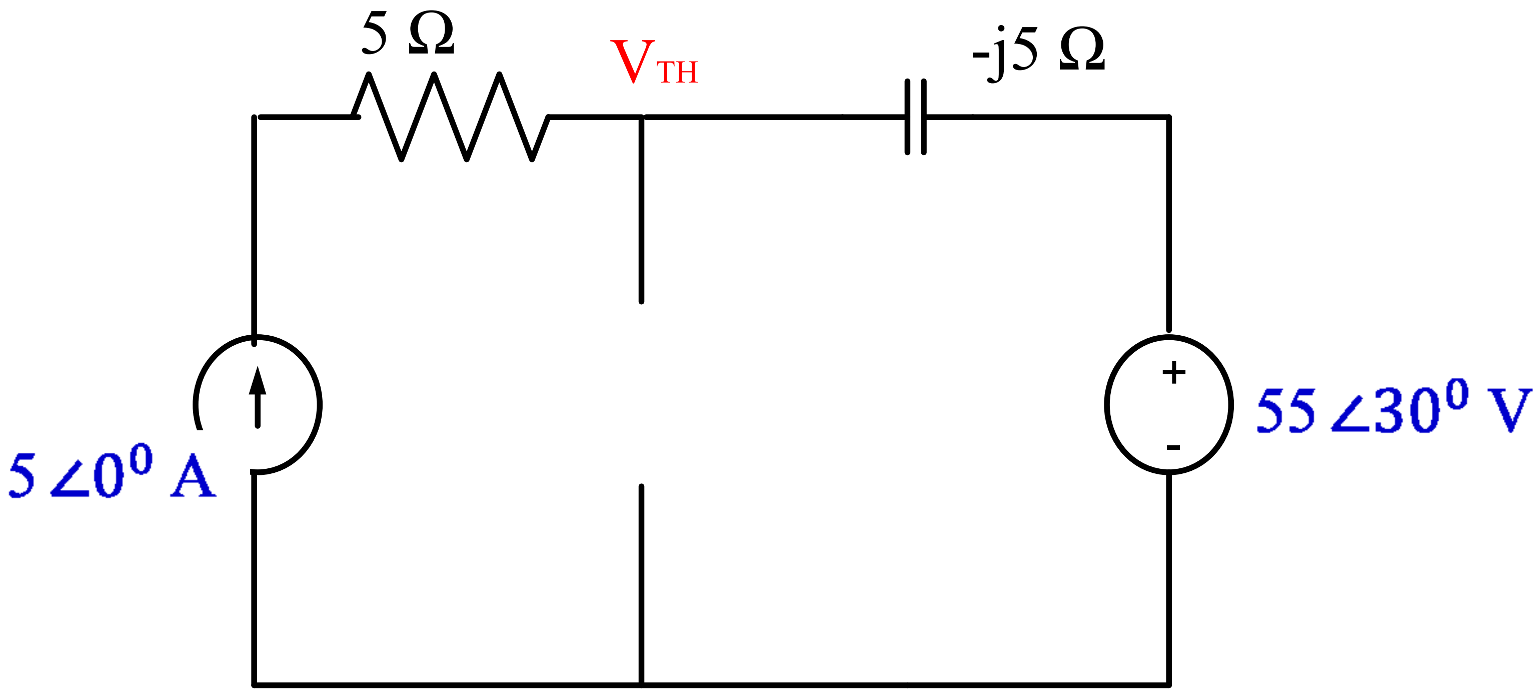
The given ac circuit in PHASOR form can be drawn as:



where Is = $5\angle 0^0$ A; Vs= $55\angle 30^0$ V; R = 5 Ω

$X_L = \omega L = 10\ \Omega$ and $X_c = \dfrac{1}{\omega C} = 5\ \Omega$

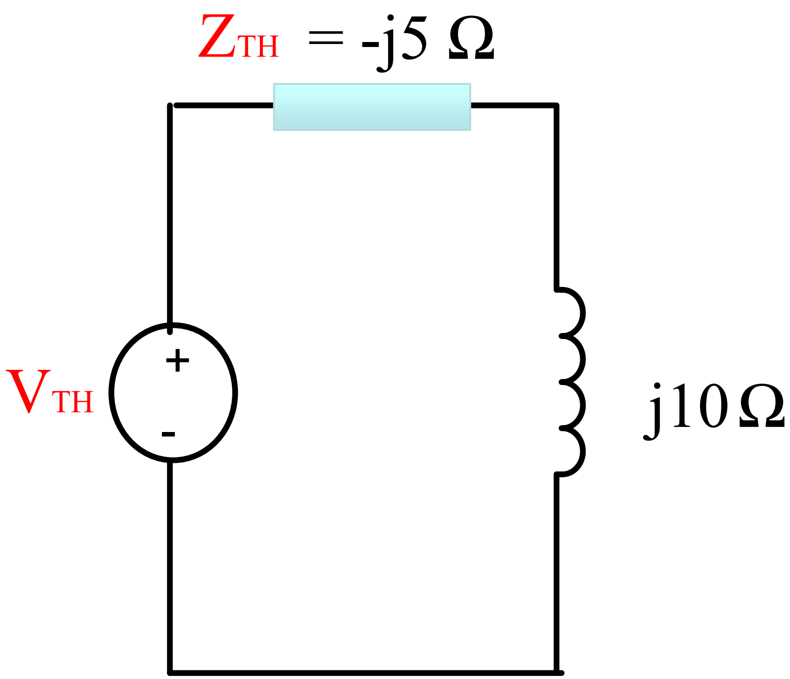
The given ac circuit in PHASOR form with the INDUCTOR detached to find Thevenin Voltage (V_{TH}) and Thevenin Impedance Z_{TH}:



Current supplied by the voltage source: $$\frac{55\angle 30^0 - V_{TH}}{-j5} = -5$$

$\Rightarrow V_{TH} = 55\angle 30^0 - j25$

After setting the INDEPENDENT SOURCES to ZERO, impedance seen from the open circuit nodes become: $Z_{TH} = -j5\Omega$.



Thevenin Equivalent

Voltage across the Inductor

$$= \frac{V_{TH}}{(-j5 + j10)} \text{ x } j10 = 2V_{TH}$$

$$= 110\angle 30^0 - j50$$

$$= 95.394\ \angle 3^0\ \text{V}$$ (Answer)

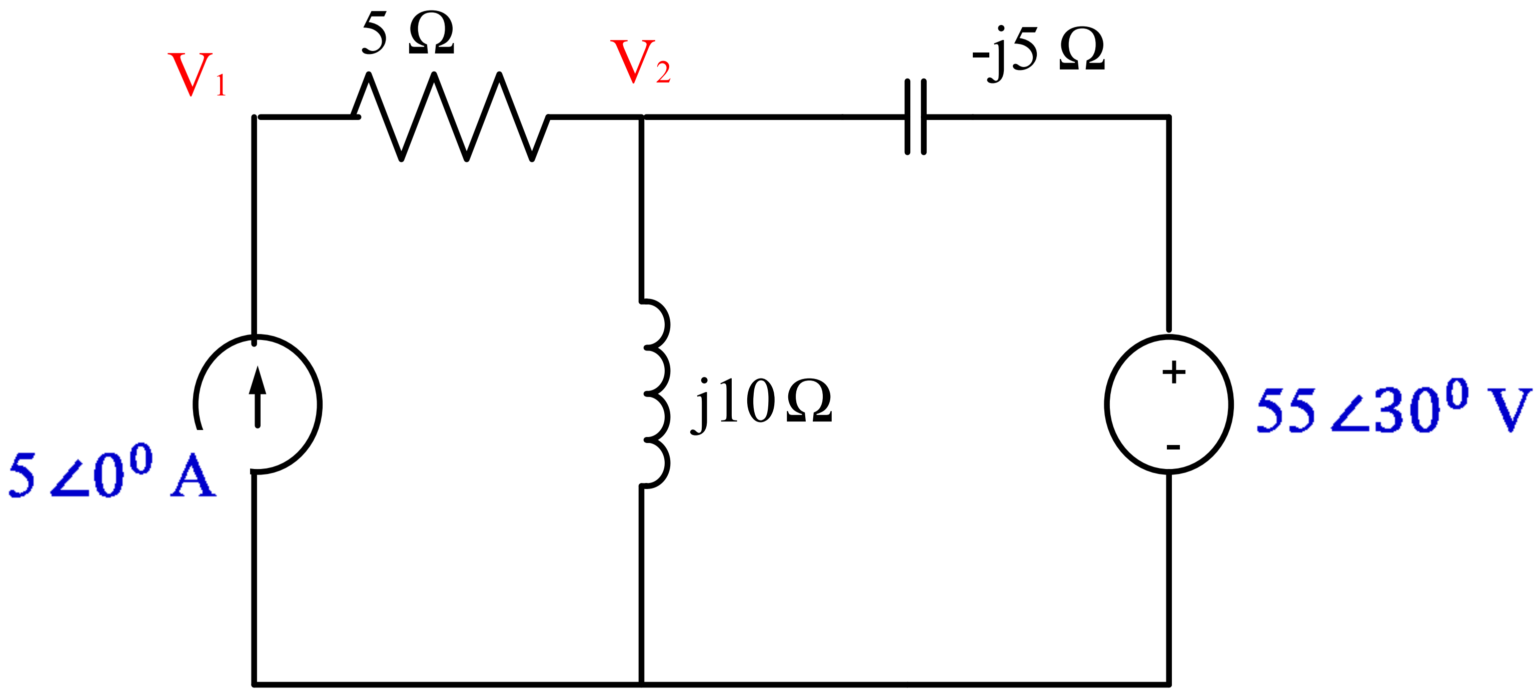
Voltage across the current source

$$= 25 + (95.394\ \angle 3^0)$$

$$= 120.367\ \angle 2.381^0\ \text{V}$$ (Answer)

Solution by Nodal Analysis Method

The given ac circuit in PHASOR form can be drawn as:



Using NODAL Analysis at Node V_2

$$5\angle 0^0 + \frac{55\angle 30^0 - V_2}{-j5} = \frac{V_2}{j10}$$

$$\Rightarrow j50 - 2(55\angle 30^0 - V_2) = V_2$$

$$\Rightarrow V_2 = 95.263 + j5$$

So, $V_2 = 95.394 < 3^0$ V (Answer)

Also, $V_1 = 5X5 + V_2 = 120.263 + j5$

$$= 120.367\angle 2.381^0 \text{ V}$$

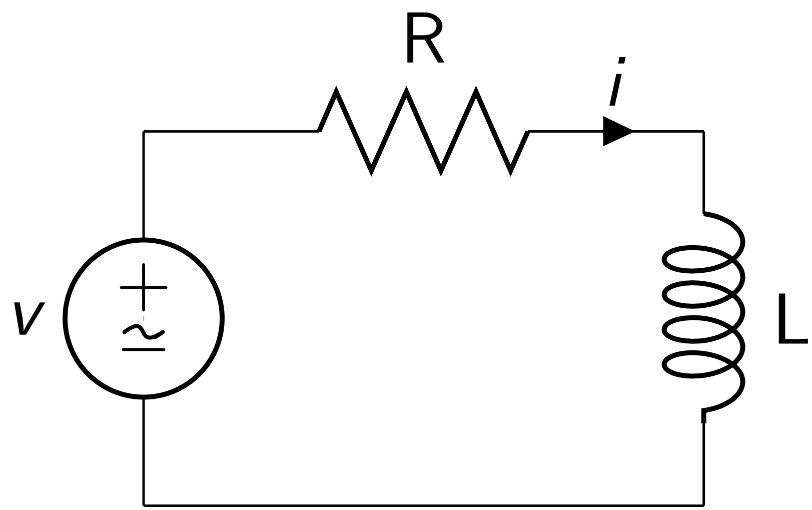
Instantaneous Power, Average Power

Complex Power,

Apparent Power

and Power Factor

Instantaneous Power



Let $v = V_m \sin(\omega t + \theta)$

Then $i = I_m \sin(\omega t + \phi)$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \text{ and } \theta - \phi = \tan^{-1}\frac{\omega L}{R}$$

Instantaneous Power

$$p(t) = v(t)i(t)$$

$$p(t) = V_m \sin(\omega t + \theta) I_m \sin(\omega t + \phi)$$

$$p(t) = \frac{V_m I_m}{2}(\cos(\theta - \phi) - \cos(2\omega t + \theta + \phi))$$

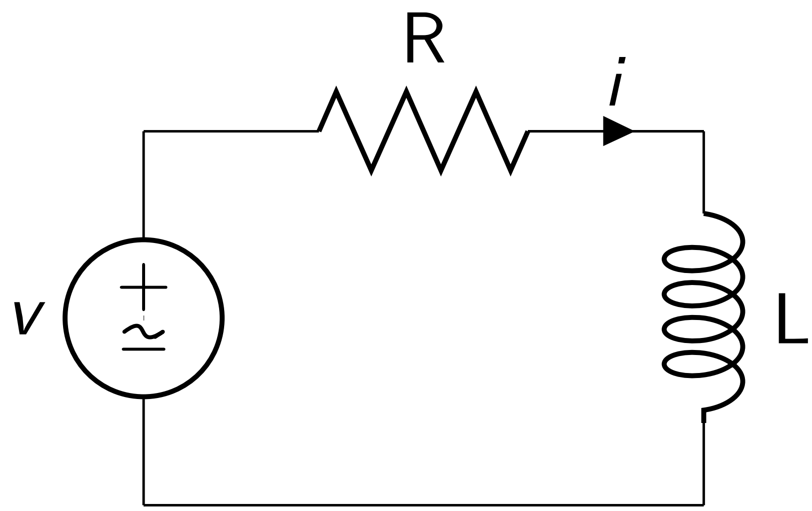
$$p(t) = \frac{V_m I_m}{2}\cos(\theta - \phi) - \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)$$

Average Power=Real Power

$$P = \frac{1}{T}\int_0^T p(t)dt$$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{rms} I_{rms} Cos(\theta - \emptyset)$$

EXAMPLE: Instantaneous Power



Let: R = 1 Ω
L = 1 H
v = $\sqrt{2}$ Sin (t) Volts

Then, $\omega = 1$ rad/s
$\theta = 0^0$

$V_m = \sqrt{2}$ V

$I_m = 1$ A

$\theta - \phi = \tan^{-1}(1) = 45^0$

Let $v = V_m \sin(\omega t + \theta)$

Then $i = I_m \sin(\omega t + \phi)$

$I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L)^2}}$ and $\theta - \phi = \tan^{-1} \dfrac{\omega L}{R}$

Instantaneous Power

$$p(t) = \frac{V_m I_m}{2}\cos(\theta - \emptyset) - \frac{V_m I_m}{2}\cos(2\omega t + \theta + \emptyset)$$

$$= 0.5 - 0.707\ \mathrm{Cos}(2t - 45^0)\ \ \mathrm{W}$$

Average Power or Real Power

(a) $P = \frac{1}{T}\int_0^T p(t)dt = 0.5$ W

(b) Also, $P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = 0.5\mathrm{x}\sqrt{2}\mathrm{x}1\mathrm{xCos}(0 - 45^0) = 0.5\ \ \mathrm{W}$

(c) But, $V_{rms} = V_m / \sqrt{2}$; $I_{rms} = I_m / \sqrt{2}$; So, P $= V_{rms} I_{rms}\ Cos(\theta - \emptyset)$

$$= 1\mathrm{x}\left(\frac{1}{\sqrt{2}}\right)\mathrm{x\ Cos}(0 - 45^0) = 0.5\ \ \mathrm{W}$$

(d) Also, real power is drawn by the RESISTOR only. Then, average or real power delivered is :

$$(I_{rms})^2 R = (1/2) \times 1 = 0.5\ \mathrm{W}$$

END