Indian Institute of Technology Guwahati

MA 102 Mathematics-II
Supplementary Tutorial Sheet

Differentiability

1. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a function. Then f is differentiable at $X_0 \in \mathbb{R}^n$ if and only if each component function f_i is differentiable.

2. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation. Then show that $T'(X_0) = [T]$, where $[T]$ is the matrix of the linear transformation T with respect to the standard basis.

3. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a differentiable function and let $T : \mathbb{R}^m \longrightarrow \mathbb{R}^k$ be a linear transformation. Show that $(T \circ f)'(X_0) = [T]f'(X_0)$.

4. If $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a differentiable function, then show that f is continuous.

5. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a differentiable function and $k \in \mathbb{R}$. Suppose that $f(rX) = r^k f(X)$ for all $r > 0$ and $X \in \mathbb{R}^n$, then show that $f'(X)X = kf(X)$.

6. Let D be a nonempty open subset of $\mathbb{R}^n$ and $g : D \longrightarrow \mathbb{R}^n$ be a cotinuous function. If $f : D \longrightarrow \mathbb{R}$ is such that $f(X) - f(X_0) = g(X) \cdot (X - X_0)$ for all $X \in D$, then show that f is differentiable at X_0.