**DEPARTMENT OF MATHEMATICS**
**Indian Institute of Technology Guwahati**

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| MA731: Linear Algebra and Functional Analysis | Quiz II |
| Instructors: B. Bhattacharjya & R. Srivastava | April 2, 2013 |
| Time: 01:30 hours | Maximum Marks: 10 |

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1. Let $T : (C^1[0,1], \| \,.\, \|_\infty) \to (C[0,1], \| \,.\, \|_\infty)$ be a linear transformation defined by $Tf(s) = f'(s)$. Show that $T$ is not continuous. **1 Mark**

2. Let $M$ be a closed subspace of a normed linear space $X$. Show that the projection map $\pi : \ X \to X/M$ defined by $\pi(x) = x + M$ is an open map by showing that $\pi\left(B(0,1)\right)$ is an open subset of $X/M$. **1 Mark**

3. Let $X$ and $Y$ be two Banach spaces and let $T : X \to Y$ be a linear map. Define the linear map $T^* : Y^* \to X^*$ by $T^*(f) = f \circ T$, for all $f \in Y^*$. Show that $T$ is continuous. (**Hint:** Use closed graph theorem.) **2 Marks**

4. Let $1 \le p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. For $f \in L^p(\mathbb{R})$, prove that
$$\|f\|_p = \sup \left\{ \left| \int_{\mathbb{R}} f(x)g(x)dx \right| : g \in L^q(\mathbb{R}) \text{ and } \|g\|_q = 1 \right\}.$$
**2 Marks**

5. Let $c_o = \{x = (x_1, x_2, \ldots) : x_j \in \mathbb{C} \text{ and } \lim_{j\to\infty} x_j = 0\}$. Define a family of linear maps $f_n : c_o \to \mathbb{C}$ by
$$f_n(x) = \frac{1}{n} \sum_{j=1}^{n} x_j.$$
Show that $f_n(x) \to 0$ as $n \to \infty$ and $\|f_n\| = 1$. **2 Marks**

6. Let $\{e_1, e_2 \ldots, e_n\}$ be a linearly independent set in an infinite dimentional normed linear space $X$. For $(a_1, a_2, \ldots, a_n) \in \mathbb{C}^n$, prove that there exists $f \in X^*$ such that $f(e_j) = a_j$ for $j = 1, 2, \ldots, n$. (**Hint:** Use Hahn-Banach theorem.) **2 Marks**

**END**