DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

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1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   Show that the function f is contnuous at $(0,0)$.   **2 Marks**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   Show that the function f is not differentiable at $(0,0)$.   **2 Marks**

3. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a function given by

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   Prove that the directional derivative of f exists in all directions at $(0,0)$ but f is not differentiable at $(0,0)$.   **3 Marks**

4. Use the method of Lagrange's multiplier to find the extremum values of the function $f(x,y) = xy$ on the ellipse $\dfrac{x^2}{2} + \dfrac{y^2}{8} = 1$.   **3 Marks**

END