Indian Institute of Technology Guwahati

Quiz-1, Even Semester, 2014-15

Time: 50 Minutes | *MA 102 Mathematics-II* | Marks: 10

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1. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function defined by $f(x, y) = \begin{cases} \dfrac{x}{y} - \dfrac{y}{x} & \text{if } xy \neq 0; \\ 0 & \text{otherwise.} \end{cases}$

   (a) Prove or disprove: f is continuous at $(0, 0)$. [1]

   (b) Find all possible directions along which directional derivatives of f at $(0, 0)$ exist. [1]

2. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function defined by $f(x, y) = \begin{cases} xy \log(x^2 + y^2) & \text{if } x^2 + y^2 \neq 0; \\ 0 & \text{otherwise.} \end{cases}$

   (Here, log is the natural logarithm to the base e.)

   (a) Show that $f_x(0, 0) = f_y(0, 0) = 0$. [1]

   (b) Check the differentiability of f at $(0, 0)$. [1]

   (c) Prove that $(0, 0)$ is a saddle point of f. [1]

   (**Hint:** It is recommended to avoid second derivative test for Part (c))

3. Consider the surface $z = 3x^2 - y^2$. Write the equation of the tangent plane, if exists, to the surface at $(0, 0, 0)$. (Do not write your calculations here.) [1]

4. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a function. Suppose $f \circ g$ is differentiable at $t = 0$ for all functions $g : \mathbb{R} \longrightarrow \mathbb{R}^2$ with $g(0) = (0, 0)$. Show that directional derivatives of f at $(0, 0)$ exist in all directions. [2]

5. Let D be a closed subset of $\mathbb{R}^2$. Show that the complement of D in $\mathbb{R}^2$, i.e. $\mathbb{R}^2 \setminus D$, is an open subset of $\mathbb{R}^2$. [2]