DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA746: Fourier Analysis **Quiz I**
Instructor: Rajesh Srivastava September 4, 2025
Time Allowed: 1.5 hours Maximum Marks: 10

Note: Answers must be accompanied by rigorous justification; unsupported answers will not be credited.

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1. (a) Does there exist a series that is not summable in the usual sense, but which is Abel summable and not Cesàro summable? **1**

   (b) Consider the series $\sum_{n=-\infty}^{\infty} \frac{(-1)^n i e^{-int}}{n}$. Is this series uniformly convergent on the interval $[-\pi, \pi]$? **1**

2. Determine the Fourier series expansion of the function $f(t) = |t - \pi|$ defined on the interval $0 \leq t \leq 2\pi$. Does the Fourier series of f converge uniformly almost everywhere on $[0, 2\pi]$? **2**

3. Let $G_n : [-\pi, \pi] \to \mathbb{C}$ be defined by

$$G_n(x) = \frac{1}{n}\left(\frac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\right)^2 \quad \text{for } x \neq 0, \quad \text{and} \quad G_n(0) = 1.$$

   For any $\delta > 0$ such that $0 < \delta < \pi$, prove that

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_{-\delta}^{\delta} G_n(x)\, dx = 1.$$

   Is this result valid for every family of good kernels $K_n \in L^1(S^1)$? **2**

4. Given a continuous function $f : \mathbb{C} \to \mathbb{C}$ and parameter $r > 0$, define $f_r(z) = f(rz)$. Determine the Fourier series of f_r on the unit circle S^1. If f is analytic, does the Fourier series of f_r coincide with the power series expansion in a neighborhood of $[-\pi, \pi]$? **2**

5. Let μ be a finite measure on $[-\pi, \pi]$ and $n \in \mathbb{Z}$. Define the Fourier coefficient of μ by

$$\hat{\mu}(n) = \int_{-\pi}^{\pi} e^{-int}\, d\mu(t).$$

   Also, for $f \in L^1([-\pi, \pi])$, define the convolution

$$f * \mu(x) = \int_{-\pi}^{\pi} f(x - y)\, d\mu(y).$$

   Show that the Dirac delta measure δ_0 at 0 satisfies $f * \delta_0(x) = f(x)$. Further, find $\hat{\delta_0}(n)$. **2**

END