DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA650: Advanced Course on Hardy spaces — Quiz I
Instructor: Rajesh Srivastava — February 20, 2022
Time duration: two hour — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Whether $H^2(\mathbb{T}, m) \cap L^\infty(\mathbb{T}, m)$ is dense in $L^2(\mathbb{T}, m)$? **[1]**

   (b) Let μ be finite Borel measure on $\mathbb{T}$. If $\bar{z}^2 \in H^2(\mu)$, does it imply that $H^2(\mu) = z^2 H^2(\mu)$? **[1]**

   (c) Let $f = \chi_{[0, \frac{\pi}{2}]}$. Does it imply that $\overline{\text{span}}\{z^n f : n \geq 0\}$ is a non-reducing subspace of $H^2(\mathbb{T}, m)$? **[1]**

   (d) Let $0 \leq \mu \ll m$. Is it possible that $H^2(\mu)$ is a proper reducing subspace of $L^2(\mu)$? **[1]**

2. Let μ be finite Borel measure on $\mathbb{T}$ and $H^2_0(\mu) = \overline{\text{span}}\{z^n : n \geq 1\}$ in $L^2(\mu)$. Let $f \in L^2(\mu)$. Evaluate $\text{dist}(f, H^2_0(\mu))$. **[2]**

3. Let $f \in H^1(\mathbb{T}, m) \cap L^\infty(\mathbb{T}, m)$. Show that there exist $f_j \in L^2(\mathbb{T}, m) : \ j = 1, 2$ such that $E_{f^2} = f_1 E_{f_2}$, where E_g stands for $\overline{\text{span}}\{z^n\, g : n \geq 0\}$. **[2]**

4. Let $f(z) = e^z$ and $g \in H^2(\mathbb{T}, m)$ be such that $f * g = 1$. Show that g must be constant. **[2]**

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