DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA650: Advanced Course on Hardy spaces — Quiz II
Instructor: Rajesh Srivastava — April 23, 2022
Time duration: two hour — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) For $p > 0$, let $f \in H^p(\mathbb{D})$ and $f \not\equiv 0$. Does it implies that $\log|f| \in L^1(\mathbb{T})$? **1**

   (b) Let $f \in \mathrm{Hol}(\mathbb{D})$. Does existence of non-tangential limits of f at a.e. $\xi \in \mathbb{T}$ imply the existence of point-wise radial limits of f at a.e. $\xi \in \mathbb{T}$? **1**

2. Let μ be finite Borel measure on $\mathbb{T}$, which is singular w.r.t. m. Define

$$f(z) = \exp\left(-\int_{\mathbb{T}} \frac{\xi + z}{\xi - z}\, d\mu(\xi)\right)$$

   for $z \in \mathbb{D}$. Show that $|f| = 1$ a.e. on $\mathbb{T}$. **2**

3. Let f be a holomorphic function on the open unit disc $\mathbb{D}$ and $f(0) > 0$. If

$$\lim_{r\to 1} \int_{\mathbb{T}} |\log|f_r||dm = 0,$$

   then show that f is a Blaschke product. **2**

4. Let $f \in \mathrm{Hol}\,(\mathbb{D})$. Show that there exists a function $g \in L^\infty(\mathbb{T})$ satisfies $\left|\frac{[g]}{f}\right| \leq 1$ a.e. on $\mathbb{T}$. **2**

5. Let f be in H^∞. Show that there exists a function $g \in \mathrm{Nev}(\mathbb{D})$ such that $Z(f) \cap \mathbb{D} = \{z \in \mathbb{D} : g(z) = 1\}$. **2**

END