DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA642: Real Analysis -1   Quiz - I
Instructor: Rajesh Srivastava   September 1, 2025
Time duration: 1.5 hours   Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

---

1. (a) Is it possible that every monotone continuous function on $\mathbb{R}$ is uniformly continuous? $\boxed{1}$

   (b) Does $\{f \in C[0,1] \text{ with } \|f\|_1 \leq 1\}$ a bounded subset of normed linea space $(C[0,1], \|\cdot\|_2)$? $\boxed{1}$

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a monotone decreasing and bounded function. Define $h(x) = \inf\{f(y) : x > y\}$. If f is continuous at a, then show that $f(a) = h(a)$. $\boxed{1}$

3. Let $f(x) = e^{-x^2}$. Show that for $n \in \mathbb{N}$, there exists $M > 0$ which is independent of x such that $|f(x)| \leq \dfrac{M}{(1 + x^{2n})^2}$. Whether M can be independent of n too? $\boxed{2}$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and $\lim\limits_{|x| \to \infty} f(x) = \infty$. Show that f is bounded below and attains its infimum. $\boxed{3}$

5. If $0 \neq x \in l^{p_o}$ for some $p_o \geq 1$, then show that $\lim\limits_{p \to \infty} \sup \|x\|_p \leq \|x\|_\infty$. $\boxed{2}$

END