DEPARTMENT OF MATHEMATICS Indian Institute of Technology Guwahati

MA642: Real Analysis Instructor: Rajesh Srivastava Time duration: 02 hours Quiz I February 18, 2017 Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

- 1. (a) Let A and B be two closed subsets of \mathbb{C} . Is |A| + |B| a closed subset of \mathbb{R} ? 1
 - (b) Does there exist an open and dense subset of \mathbb{R} of arbitrarily small content? 1
 - (c) For $x, y \in (l^{\infty}, \|.\|_{\infty})$, write $d(x, y) = \limsup |x_n y_n|$. Does d a metric on l^{∞} ? 1
- 2. Show that $f_n(t) = e^{-\left(1 + \frac{1}{\sqrt{n}}\right)|t|}$ is a uniformly convergent sequence on \mathbb{R} . Does $\lim f_n$ a uniformly continuous function on \mathbb{R} ?
- 3. Let $C_o = \{f : \mathbb{R} \to \mathbb{R} : f \text{ is continuous and } \lim_{|x| \to \infty} f(x) = 0\}$. Find a norm on C_o that completes C_o .
- 4. Show that the quotient norm on l^{∞}/c_o satisfies $\|\tilde{x}\| = \limsup |x_n|$. Does l^{∞}/c_o a finite dimensional linear space? **2+1**

END