DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA642: Real Analysis — Quiz - II
Instructor: Rajesh Srivastava — April 16, 2017
Time duration: 02 hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Does there exist a non-empty open and connected set $A \subset \mathbb{R}^n$ such that every real valued function on A is continuous? **1**

   (b) Does there exist a discontinuous function $f : \mathbb{R} \to \mathbb{R}$ such that the graph G_f is connected in $\mathbb{R}^2$ but $\text{int}(\overline{G}_f)$ is non-empty? **1**

   (c) Let $f_n \in C^1[0,1]$. Does it imply that the set $\{f_n' : \ n = 1, 2, \ldots\}$ is equicontinuous in $C[0,1]$? **1**

   (d) Let $f_n : [0,1] \to \mathbb{R}$ be defined by $f_n = \chi_{[0,1/n]}$ and f_n converges point-wise to f. Does it imply that $\{f, f_n : \ n = 1, 2, \ldots\}$ is compact in $L^\infty[0,1]$? **1**

2. Show that $GL_n(\mathbb{C})$ is path connected by using the fact that every polynomial on $\mathbb{C}$ has finitely many zeros. Does the set $GL_n(\mathbb{C})$ is open in $M_n(\mathbb{C})$? **2+1**

3. Let $f_n : (0,1) \to \mathbb{R}$ be a sequence of uniformly continuous functions having uniform limit f. Show that $\{f, f_n : \ n = 1, 2, \ldots\}$ is a compact subset of $C[0,1]$. **2+1**

END