DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA550: Measure Theory | Quiz II
Instructor: Rajesh Srivastava | November 11, 2021
Time duration: One hour | Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) For $x \in \mathbb{R}$, define $f(x) = \frac{\sin x}{x}$ if $x \neq 0$ and $f(0) = 1$. What is the Lebesgue measure of the set $\cup_{n=1}^{\infty}\{x \in \mathbb{R} : f(x) = \frac{1}{n}\}$? **[1]**

   (b) Define a sequence of function on $\mathbb{R}$ by $f_n = e^{-x}\chi_{(\frac{1}{n},n)}$, where $n \in \mathbb{N}$. Whether f_n increases uniformly to a Lebesgue integrable function on $\mathbb{R}$? **[1]**

2. Let $f : \mathbb{R} \to \mathbb{R}$ be such that f is Lebesgue measurable on $(n, n+1)$ for every $n \in \mathbb{Z}$. Show that f is Lebesgue measurable on $\mathbb{R}$. **[2]**

3. Let (X, S) be a measure space. Let $f : X \to \mathbb{R}$ be a S-measurable function and $g : \mathbb{R} \to \mathbb{R}$ be differentiable. Show that $g' \circ f$ is S-measurable function. **[2]**

4. Let $f \in L^1(\mathbb{R}, M, m)$. Evaluative $\lim\limits_{n\to\infty} \int\limits_{\mathbb{R}} e^{-nx^2} f(x)dm(x)$. **[2]**

5. Let (X, S, μ) be a finite measure space on the finite set X. Show that $L^1(X, S, \mu)$ is a finite dimensional linear space. **[2]**

END