DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA550: Measure Theory	Quiz I
Instructor: Rajesh Srivastava	September 6, 2014
Time duration: 1.5 hours	Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let E be a Lebesgue measurable subset of $\mathbb{R}$ and $F \subset \mathbb{R}$ be a countable set. Whether $E + F$ is a Lebesgue measurable set? **[1]**

   (b) If $A \subset \mathbb{R}$ with $m^*(A) > 0$. Does it imply $A \cap (\mathbb{R} \setminus \mathbb{Q}) \neq \emptyset$? **[1]**

2. Let S_X be a σ-algebra on a non-empty set X. For $x_1, x_2 \in X$, define $\delta_{x_i} : S_X \to [0, \infty]$ by $\delta_{x_i}(A) = \begin{cases} 0 & \text{if } x_i \notin A, \\ 1 & \text{if } x_i \in A. \end{cases}$
   Show that $\mu = \delta_{x_1} + \delta_{x_2}$ is a measure on X. Is $\nu = \delta_{x_1} - \delta_{x_2}$ a measure on X? **[2]**

3. For $E \subset (0, 1)$ write $E^2 = \{x^2 : x \in E\}$. Show that $m^*(E^2) \leq 2m^*(E)$. **[2]**

4. Let C be the Cantor ternary set. Show that for each pair of points $x, y \in C$ there exists $z \notin C$ such that $x < z < y$. **[2]**

5. Let $E \subset \mathbb{R}$ be Lebesgue measurable and $m(E) < \infty$. Show that for each $\epsilon > 0$, there exist compact set K and open set O with $K \subseteq E \subseteq O$ such that $m(O \setminus K) < \epsilon$. **[2]**

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