

DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA547: Complex Analysis
Instructor: Rajesh Srivastava
Time duration: 1.5 hours

Quiz - I
February 12, 2025
Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

1. (a) Whether the series $\sum_{n=1}^{\infty} \frac{i^n}{n}$ is convergent? **1**
(b) Does the series $\sum_{n=0}^{\infty} \frac{z^n}{1+z^n}$ converges absolutely for $|z| > 1$? **1**
2. For z and w in \mathbb{C} , show that $|1 - z\bar{w}|^2 - |z - w|^2 = (1 - |z|^2)(1 - |w|^2)$. **2**
3. For each z in the open unit disk \mathbb{D} , show that there exists a sequence $\frac{k_1 + ik_2}{n}$ which converges to z , where k_1 and k_2 are integers smaller than n . **2**
4. For complex number z in \mathbb{C} , show that $\lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right)^n = e^z$. **2**
5. For given $\epsilon > 0$, show that there exists a natural number n such that $|e^{in} - 1| < \epsilon$. **2**

END