

**DEPARTMENT OF MATHEMATICS**  
**Indian Institute of Technology Guwahati**

MA547: Complex Analysis  
Instructor: Rajesh Srivastava  
Time duration: 1.5 hours

Quiz - II  
April 6, 2025  
Maximum Marks: 12

**N.B.** Answer without proper justification will attract zero mark.

---

1. (a) Suppose  $\gamma$  is closed smooth path such that  $0 \notin \text{range}(\gamma)$ . Does it imply that  $\int_{\gamma} \frac{\sin z}{z^{2n+1}} dz = 0$  for each  $n \in \mathbb{Z}$ ? **1**  
(b) Suppose an entire function  $f$  does not meet the curve  $\text{Re}(z) = (\text{Im}(z))^2$ . Does it imply that  $f$  is constant? **1**
2. Examine if there exists an entire function  $f$  with  $f(x) = 1 + |x|^5$  for each  $x \in \mathbb{R}$ . **2**
3. If  $f$  is an entire function with  $f(0) = 0$  and  $\lim_{|z| \rightarrow \infty} \left| \frac{f(z)}{z} \right| = 1$ . Show that  $f(z) = cz$ , where  $|c| = 1$ . **2**
4. Let  $G = \{z \in \mathbb{C} : \text{Im}(z) < -2\}$ . Define  $F(z) = \int_0^1 \frac{e^{-itz}}{t^2 + z^2} dt$ . Show that  $F$  is an analytic function and satisfies  $|F(z)| \leq \frac{1}{3}$  for all  $z \in G$ . **3**
5. Let  $f$  be a analytic function on  $G = \{z \in \mathbb{C} : |z| > 1\}$ . Suppose there exists  $K > 0$  such that  $|f(z)| \leq K$  for  $z \in \partial G$ . If  $\lim_{|z| \rightarrow \infty} |f(z)| \leq K$ . Then show that  $|f(z)| \leq K$  for each  $z \in G$ . **3**

**END**