DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis — Quiz II
Instructor: Rajesh Srivastava — November 17, 2013
Time duration: 1.5 hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. Let $\mathbb{P}(\mathbb{R})$ be the space of all polynomials with coefficients in $\mathbb{R}$. Does there exist a norm $\|.\|$ on $\mathbb{P}(\mathbb{R})$ such that $(\mathbb{P}(\mathbb{R}), \|.\|)$ is a Banach space? Justify your answer. $\boxed{\mathbf{1}}$

2. Can the quotient space l^∞ / c_o be separable? Justify your answer. $\boxed{\mathbf{1}}$

3. Show that l^∞ has no Schauder basis. $\boxed{\mathbf{2}}$

4. Defin a linear map $T : l^1 \to l^1$ by $T(x_1, x_2, \ldots) = \left(2x_1 + x_2, \frac{x_2 + x_3}{2}, \frac{x_3 + x_4}{2^2}, \ldots\right)$. Find the value of $\|T\|$. $\boxed{\mathbf{2}}$

5. Let(a_n) be a sequence such that the linear map $T : l^1 \to l^2$ defined by $T(x_1, x_2, \ldots) = (a_1 x_1, a_2 x_2, \ldots)$ is continuous. Show that (a_n) is bounded. $\boxed{\mathbf{2}}$

6. Let ϕ be a measurable measurable function on $\mathbb{R}$ such that for each $f \in L^\infty(\mathbb{R})$, $\phi f \in L^\infty(\mathbb{R})$. Define a linear map $T : L^\infty(\mathbb{R}) \to L^\infty(\mathbb{R})$ by $T(f) = \phi f$. Show that T is continuous. $\boxed{\mathbf{2}}$

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