DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA543: Functional Analysis	Quiz I
Instructor: Rajesh Srivastava	September 8, 2013
Time duration: 1.5 hours	Maximum Marks: 10

N.B. Each question carries **2 marks**. You can attempt at most **FIVE** question out of **SEVEN**.

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1. Find the linear map $T : \mathbb{C}^2 \to \mathbb{C}^2$ whose range space is span $\{(1,1),(1,i)\}$.

2. Let $0 < p < 1$. Let $X = \{x = (x_1, x_2, \ldots) : \ x_j \in \mathbb{C}\}$. Define $\|x\|_p = \left(\sum_{j=1}^{\infty} |x_j|^p\right)^{\frac{1}{p}}$. Show that $(X, \| \ . \ \|_p)$ is **not** a normed linear space.

3. Show that the space $c_o(\mathbb{N})$ is a closed and proper subspace of $l^\infty(\mathbb{N})$.

4. Let M be a proper closed subspace of a normed linear space X. Define a map $f : X \to \mathbb{R}$ by $f(x) = \inf\limits_{m \in M} \|x + m\|$. Show that f is an uniformly continuous function on X.

5. Let $X = C[0,1]$. For $f \in X$, define $\|f\|_2 = \left(\int_0^1 |f(t)|^2 dt\right)^{\frac{1}{2}}$. Show that $(X, \ \| \ . \ \|_2)$ is **not** a Banach space.

6. Let $\{E_k : \ k = 1, 2, \ldots, n\}$ be a collection of disjoint measurable sets in $\mathbb{R}$, having each of them has finite measure. Let $f = \sum_{k=1}^{n} \alpha_k \chi_{E_k}, \ \alpha_k \in \mathbb{C}$. For $0 < p < \infty$, evaluate the integral $\int\limits_{\mathbb{R}} |f|^p dm$.

7. Let $f_n : [0,1] \to \mathbb{R}$ be a sequence of measurable functions given by $f_n(t) = \dfrac{n \sin t}{1 + n^2\sqrt{t}}$. Evaluate $\lim\limits_{n \to \infty} \int\limits_{[0,1]} f_n dm$.

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