DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA541: Real Analysis Quiz I
Instructor: Rajesh Srivastava August 26, 2017
Time duration: 02 hours Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $\lim\limits_{n\to\infty} f\left(\frac{1}{n}\right) = f(0)$. Does it imply that f is continuous at 0? $\boxed{1}$

   (b) Does there exist a monotone function $f : (0,\infty) \to \mathbb{R}$ such that each $c \in (0,\infty)$ satisfies $|f(c+) - f(c-)| = \frac{1}{c}$? $\boxed{1}$

2. Find the infimum and supremum of the set $\{x \in \mathbb{R} : |x-1| + |x+1| < 3\}$. $\boxed{2}$

3. For a sequence a_n of nonzero real numbers, define $A_n = \sup\limits_{k\geq n} a_k$ and $B_n = \inf\limits_{k\geq n} a_k$. If $A_n \to -\infty$ and $B_n \to \infty$, then show that $\lim\limits_{n\to\infty} \frac{1}{a_n} = 0$. $\boxed{2}$

4. Let a_n be a bounded sequence of non-negative real numbers. Show that $(1 + a_n)^{\frac{1}{n}}$ is convergent. $\boxed{2}$

5. Prove or disprove that there does not exist a monotone function $f : \mathbb{R} \to \mathbb{Q}$ which is onto. $\boxed{2}$

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