DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA541: Real Analysis — Quiz II
Instructor: Rajesh Srivastava — November 8, 2017
Time duration: 02 hours — Maximum Marks: 12

N.B. Answer without proper justification will attract zero mark.

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1. (a) Does there exist a function $f : \mathbb{R}^2 \to \mathbb{R}$ which is differentiable at only $(0,0)$? **1**

   (b) Let $f(x,y) = \begin{cases} 1 & \text{if } x^2+y^2 = 1; \\ 0 & \text{otherwise.} \end{cases}$

   Find all those points in $\mathbb{R}^2$ such that f is discontinuous. **1**

   (c) Is it possible that a function on $\mathbb{R}^2$ has local minima at $(0,0)$ along all direction passing through $(0,0)$ but has no local minimia in any open neighbourhood of $(0,0)$? **1**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \begin{cases} 0 & \text{if } xy \neq 0; \\ 1 & \text{otherwise.} \end{cases}$

   Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not continuous at $(0,0)$. **2**

3. Find all possible directional derivative $D_v(0,0)$ for the function

   $f(x,y) = \begin{cases} \frac{\sin xy}{xy} & \text{if } xy \neq 0; \\ 0 & \text{otherwise.} \end{cases}$

   Whether f is differentiable at $(0,0)$? **2**

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function which is given by $f(x_1,x_2) = \cos(x_1+x_2)$. For $x, y \in \mathbb{R}^2$, show that

   $$|f(x) - f(y)| \leq \sqrt{2}\|x-y\|.$$

   2

5. For a continuous function $g : [-1,1] \to \mathbb{R}$, define $f(x,y) = \int_x^y g(t)dt$. Show that f is differentiable at $(0,0)$. Whether f is continuously differentiable at $(0,0)$? **2+1**

END