DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA224: Real Analysis — Quiz I
Instructor: Rajesh Srivastava — February 12, 2018
Time duration: 1.5 hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Does there exist a countable family of closed sets in $\mathbb{R}$ whose union is open? **1**

   (b) Let $A = \left\{\left(x, \cos\frac{1}{x}\right) : \ x \neq 0, \ x \in \mathbb{R}\right\}$. What is the interior of A? **1**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function given by $f(x,y) = \begin{cases} 1 & \text{if } y > 0 \text{ and } 0 < x < y^2, \\ 0 & \text{otherwise.} \end{cases}$

   Examine the continuity of f at $(0,0)$. **1**

3. Let $A : \mathbb{R}^2 \to \mathbb{R}$ be the linear transformation defined by $A(x,y) = 3x + 4y$. Show that $\|A\| = 5$. **2**

4. Let $g : [a,b] \to \mathbb{R}^2$ be defined by $g(t) = (at^3, 2b(1-t^2))$. Using appropriate mean value theorem (MTV), show that

$$\|g(b) - g(a)\|_2 \leq (b-a)\sqrt{9a^2b^4 + 16b^4}.$$

   2

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be function such that $f \circ g$ is differentiable at $t = 0$ for all functions $g : \mathbb{R} \to \mathbb{R}^2$ with $g(0) = (0,0)$. Show that directional derivatives $D_{\mathbf{v}} f(0,0)$ exist for all $\mathbf{v} \in \mathbb{R}^2$ with $\|\mathbf{v}\|_2 = 1$. **3**

END