**DEPARTMENT OF MATHEMATICS**
**Indian Institute of Technology Guwahati**

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| MA224: Real Analysis | Quiz II |
| Instructor: Rajesh Srivastava | April 16, 2018 |
| Time duration: 1.5 hours | Maximum Marks: 11 |

**N.B.** Answer without proper justification will attract zero mark.

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1. (a) Does there exist a set $A \subset (\mathbb{R}, u)$ such that $\delta\left(A^o \cup \{0\}\right) = 0$ but $\delta\left((\bar{A})^o\right) = 1$? Where $\delta$ stands for diameter. **[1]**

   (b) Whether the function $f$ defined by $f(x) = e^{x^2}$ is uniformly continuous on $\mathbb{R}$? **[1]**

   (c) Let $l^\infty$ be the space of all bounded sequences in $\mathbb{R}$. For $x, y \in l^\infty$, does $d(x, y) = \min\{1, \limsup |x_n - y_n|\}$ define a metric on $l^\infty$? **[1]**

2. Let $c_o$ be the space of all sequences in $\mathbb{R}$ which converges to zero. For $x \in c_o$, define $f_n(x) = \frac{e^{-n}}{n+1} \sum\limits_{j=1}^{n} x_j$. Show that $\lim\limits_{n\to\infty} f_n(x) = 0$. **[2]**

3. Suppose $f$ is a continuous function on $\mathbb{R}$ that satisfies $|f(x) - f(y)| \geq 2|x - y|$ for all $x, y \in \mathbb{R}$. Show that $f(\mathbb{R})$ is complete in $(\mathbb{R}, u)$. **[2]**

4. Let $f : \mathbb{R} \to \mathbb{R}$ be such that its derivative $f'$ is bounded. Define $f_n(t) = f\left(t + \frac{1}{n}\right)$. Show that the sequence $f_n$ converges uniformly to $f$. **[2]**

5. Show that $A = \{f \in C[0,1] : \|f\|_1 < 1\}$ is an unbounded subset of the normed linear space $\left(C[0,1], \|\cdot\|_\infty\right)$. **[2]**

**END**