## DEPARTMENT OF MATHEMATICS
## Indian Institute of Technology Guwahati

MA224: Real Analysis         Quiz I

Instructor: Rajesh Srivastava        April 9, 2014

Time duration: One hour        Maximum Marks: 10

**N.B.** Answer without proper justification will attract zero mark.

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1. Let $(X, d)$ be metric space. Show that any two distinct points in $X$ are separated by disjoint open sets. $\boxed{2}$

2. Let $(f_n)$ be sequence in $C[0, 1]$ given by $f_n(t) = \begin{cases} nt & \text{if } 0 \leq t \leq \frac{1}{n}, \\ 1 & \text{if } \frac{1}{n} < t \leq 1. \end{cases}$

   Show that $(f_n)$ is not a Cauchy sequence in $(C[0, 1], d_\infty)$. $\boxed{2}$

3. Let $(X, d)$ be metric space. Let $f_n : X \to \mathbb{R}$ be sequence of bounded functions. Suppose $f_n$ converges uniformly to $f$. Then show that $f$ is also bounded. $\boxed{2}$

4. Consider the IVP $y' = x + y$ and $y(0) = 0$. Using fixed point theorem, find the best possible $\delta > 0$ such that the above IVP has unique solution in $C[0, \delta]$. $\boxed{2}$

5. Show that the function $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ given by $f(x) = \dfrac{x}{|x|}$ can not be extended continuously to $\mathbb{R}$. $\boxed{2}$

**END**