

DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA224: Real Analysis
Instructor: Rajesh Srivastava
Time duration: 1.5 hours

Quiz I
February 14, 2014
Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

1. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be function given by $f(x, y) = \begin{cases} \frac{3x^2y-y^3}{x^2+y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Show that all the first order partial derivatives of f exist at $(0, 0)$ but f is not differentiable at $(0, 0)$. **3**

2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be map $f(x, y) = \begin{cases} (x + y)\log(x^2 + y^2) & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Show that f is continuous at $(0, 0)$. Find all possible directions \mathbf{v} in which the directional derivative $D_{\mathbf{v}}f(0, 0)$ exist. **2**

3. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be map defined by $f(x, y) = \begin{cases} e^{-\frac{1}{x^2+y^2}} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

Whether f has Taylor's series expansion at $(0, 0)$. Justify your answer. **3**

4. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be function given by $f(x_1, x_2, \dots, x_n) = \sin(x_1 + x_2 + \dots + x_n)$. For $x, y \in \mathbb{R}^n$, show that

$$|f(x) - f(y)| \leq \sqrt{n}\|x - y\|.$$

2

END