DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA224: Real Analysis	EndSem Exam
Instructor: Rajesh Srivastava	May 2, 2014
Time duration: Three hour	Maximum Marks: 45

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1. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $F(x,y) = (x + e^y, y + e^{-x})$. Show that F is everywhere locally invertible. Compute $D(F^{-1})$. **3**

2. Let $u = 2x^2 - xy$ and $v = y - x$. Show that the system of equations can be solved for x, y everywhere except when $3x = y$. Compute $\frac{\partial x}{\partial u}(u_o, v_o)$, for $3x_o \neq y_o$. **3**

3. Find a sufficient condition under which the equation $x^2 + ye^x - \cos xy = 0$ can be solved for y. **3**

4. Let $f : [0,\infty) \to [0,\infty)$ be defined by $f(x) = \dfrac{x}{1+x}$. Show that f is uniformly continuous. **3**

5. Let A be a closed subset of a metric space X. Suppose diameter $\delta(A) \leq r$ and $A \cap B_r(x) \neq \emptyset$. Show that $A \subset B_{2r}(x)$. **3**

6. Let F be a subset of a metric space X such that $B_\epsilon(x) \cap F \neq \emptyset$, for each $\epsilon > 0$, implies $x \in F$. Show that F is closed. **3**

7. Let $T : \left(C\left[0, \frac{\pi}{2}\right], \|.\|_\infty\right) \to \left(C\left[0, \frac{\pi}{2}\right], \|.\|_\infty\right)$ be defined by $(Tf)(x) = \int\limits_{s=0}^{x} f(s)\sin s ds$. Show that T is **not** a contraction but T^2 is a contraction. **2+2**

8. Let $c = \left\{x = (x_1, x_2, \ldots) : \lim\limits_{n\to\infty} |x_n| < \infty\right\}$. Show that $(c, \|.\|_\infty)$ is a complete normed linear space. Whether $(c, \|.\|_\infty)$ is a proper subspace of $(l^\infty, \|.\|_\infty)$? **4**

9. Let X be a linear space. Suppose $\phi : X \to \mathbb{R}$ is a convex function satisfying $\phi(x) = 0$ if and only if $x = 0$ and $\phi(\alpha x) = |\alpha|\phi(x)$, for each $(\alpha, x) \in \mathbb{R} \times X$. Show that ϕ is a norm on X. **3**

10. Let $0 < p < 1$. Show that $(\mathbb{R}^n, \|.\|_p)$ $(n \geq 2)$ is **not** a normed linear space. **4**

11. Let $A \subset [0,1]$ such that $m^*(A) = 0$. Show that $m^*\{x^5 : x \in A\} = 0$. **2**

12. Let $\{E_k : k = 1, 2, \ldots, n\}$ be a collection of disjoint measurable sets in $\mathbb{R}$, having each of them has finite Lebesgue measure. Let $f = \sum_{k=1}^n \alpha_k \chi_{E_k}$, $\alpha_k \in \mathbb{C}$. For $1 < p < \infty$, evaluate the integral $\int\limits_{\mathbb{R}} |f|^p dm$. **3**

P.T.O.

13. Suppose $E \subset [\frac{-\pi}{4}, \frac{\pi}{4}]$ is a measurable set and satisfying $\int_E x^k \sec x \, dx = 0$, for all $k = 0, 1, 2, \ldots$. Show that $m(E) = 0$. **4**

14. Let $f_n : [0, 1] \to \mathbb{R}$ be a sequence of measurable functions given by $f_n(t) = \dfrac{n^2 t}{1 + n^4 t^3}$. Evaluate $\lim_{n\to\infty} \int_{[0,1]} f_n dm$. **3**

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