

DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA211(Minor): Real Analysis
Instructor: Rajesh Srivastava
Time duration: 1.5 hours

Quiz - II
November 16, 2022
Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

1. (a) If (x_n) is a Cauchy sequence in (\mathbb{R}, u) , does it imply that $\sin(x_n)$ is a Cauchy sequence in (\mathbb{R}, u) ? **1**
(b) Let $A_n = \{(x, y) \in \mathbb{R}^2 : 0 < \frac{1}{x} < y < \frac{1}{n}\}$. Whether the set $\bigcap_{n=1}^{\infty} A_n$ is open/closed? **1**
2. Examine for $d(x, y) = \frac{|x - y|}{1 + |xy|}$ defines a metric on \mathbb{R} . **2**
3. Define a metric d on $X = (0, 2]$ by $d(x, y) = \left| \frac{1}{x} - \frac{1}{y} \right|$. Show that (X, d) is a complete metric space. **2**
4. Find the point-wise limit of the sequence $f_n(t) = e^{-nt^2} \sin nt$. Examine for uniform convergence of f_n on \mathbb{R} . **2**
5. Let $f : [0, 1] \rightarrow [0, 1]$ be such that $|f(x) - f(y)| \leq \frac{1}{4}|x - y|$ and $g(x) = \frac{1}{8}x^2 - f(x)$ for all x and y in $[0, 1]$. Show that there exists a unique x_o in $[0, 1]$ such that $f(x_o) = -x_o + \frac{1}{8}x_o^2$. **2**

END