DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA211(Minor): Real Analysis — MidSem
Instructor: Rajesh Srivastava — September 25, 2022
Time duration: 02 hours — Maximum Marks: 30

N.B. Answer without proper justification will attract zero mark.

---

1. (a) Does there exist a function $f : \mathbb{R}^2 \to \mathbb{R}$, which is differentiable only at one point in $\mathbb{R}^2$? **[1]**

   (b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be such that $f_x(0,0)$ and $f_y(0,0)$ both exist. Does it imply that f has derivative at $(0,0)$ along $y = x$? **[1]**

   (c) Whether $\varphi(x) = \frac{1}{3}x \sin x$ is a contraction mapping on the interval $(0,1)$? **[1]**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \begin{cases} \frac{xy^2}{x^2-y} & \text{if } x^2 \neq y, \\ 0 & \text{otherwise.} \end{cases}$

   Determine all possible points in $\mathbb{R}^2$ where f is differentiable. **[3]**

3. Let $f : [1,2] \to [0,1]$ be defined by $f(x) = (2-x)\sin\frac{\pi}{2x}$. Find all possible fix points for f. **[2]**

4. For a continuous function $g : \mathbb{R} \to \mathbb{R}$, define $F(x,y) = \int_x^y g(t)dt$. Show that F is differentiable on $\mathbb{R}^2$ and find its derivative. Whether F is a bounded function always? **[3]**

5. Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $A(x,y) = (2x+y, x+2y)$. Find the norm of A. **[2]**

6. Let $g : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function and let $f(x,y) = x\, e^{g(y)}$. Show that f is not injective on $\mathbb{R}^2$. **[5]**

7. Find all possible points $(x,y) \in \mathbb{R}^2$ where $f(x,y) = (x^3 + x + \cos x, y^3)$ is locally injective. **[4]**

8. Let $f(x,y) = (2x^2 + y, x - 2y^2)$ and $g(x,y) = (2x + ye^x, 2y + xe^y)$. Show that $g^{-1} \circ f$ is differentiable at $(0,0)$ (with meaningful justification of g^{-1} exists) and find the value of $(g^{-1} \circ f)'(0,0)$. **[4]**

9. Show that equation $e^x + y - 2\cos(xyz) = 0$ can be solved for x in terms of y and z in some neighborhoods of $(0,1,1)$ in $\mathbb{R}^3$. Whether this equation is solvable for z in terms of x and y in a neighborhood of $(0,1,1)$ in $\mathbb{R}^3$? **[4]**

END