DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA1501H (CSE): Multivarable Calculus **Quiz**
Instructor: Rajesh Srivastava **Date:** October 18, 2025
Duration: 1:30 hours **Maximum Marks:** 15

Note: Answers lacking rigorous justification will not be awarded marks.

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1. (a) Whether the set $\{(x, y, z) \in \mathbb{R}^3 : |x| + 2|y| + 3|z|^2 < 1\}$ is bounded in $\mathbb{R}^3$? $\boxed{1}$

   (b) Whether there exists an unbounded sequence (x_n) in $\mathbb{R}$ such that $((x_n, \sin x_n^2))$ has convergent subsequence? $\boxed{1}$

   (c) Does there exist a continuous function $f : \mathbb{R} \to \mathbb{R}^2$ such that $f(e^{-n^2}) = (n, \frac{1}{n})$ for each $n \in \mathbb{N}$? $\boxed{1}$

2. Show that the set $\{x \in \mathbb{R}^m : 2 \leq \|x\| < 3\}$ is neither open nor closed set in $\mathbb{R}^m$. $\boxed{2}$

3. If (x_n) is sequence in $\mathbb{R}^m$ such that the series $\sum_{n=1}^{\infty} n^3 \|x_n\|^2 < \infty$. Show that the series $\sum_{n=1}^{\infty} \|x_n\|^2$ is convergent. $\boxed{2}$

4. Let function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \dfrac{\sin^2(x - y)}{|x| + |y|} & \text{if } |x| + |y| \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

   Check the continuity of f at $(0, 0)$.

   $\boxed{3}$

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be such that $f \circ g$ is differentiable for every function $g : \mathbb{R} \to \mathbb{R}^2$ with $g(0) = (0, 0)$. Show that all the directional derivative of f exist $(0, 0)$. $\boxed{2}$

6. Show that the function f defined by $f(x, y) = \dfrac{1}{1 + x - y}$ is differentiable at $(0, 0)$. $\boxed{3}$

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