DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA1501H (CSE): Multivariable Calculus **EndSem**
Instructor: Rajesh Srivastava November 18, 2025
Duration: 03:00 hours **Maximum Marks:** 45

Note: Answers lacking rigorous justification will not be awarded marks.

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1. (a) Whether the interior of $\{(x, \sin\frac{\pi}{x}) : x \neq 0 \text{ and } x \in \mathbb{R}\}$ is non-empty in $\mathbb{R}^2$? **1**

   (b) Whether the set $\{(x, \frac{1}{x}) : x \neq 0 \text{ and } x \in \mathbb{R}\}$ is closed in $\mathbb{R}^2$? **1**

   (c) Whether $(0,0)$ is a saddle point of the function $f(x,y) = (x-y)(x-y^2)$? **1**

2. Show that the content of the set $\{\frac{1}{n} : n \in \mathbb{N}\}$ in $[0,1]$ is zero. **2**

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 - y^2} & \text{if } x^2 \neq y^2, \\ 0 & \text{otherwise.} \end{cases}$$

   Find all possible directions along which f has directional derivatives at $(0,0)$. **3**

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} x^2 y^2 \dfrac{x-y}{x^2+y^2} & \text{if } x^2 + y^2 \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

   Examine whether $f_{xy}(0,0) = f_{yx}(0,0)$. **4**

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2} \sin\left(\frac{1}{x^2+y^2}\right) & \text{if } x^2 + y^2 \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

   Show that f is continuous at $(0,0)$. **3**

6. Let $A = \int_0^1 e^{-x^2} dx$. Show that $\int_0^1 \int_0^x e^{-t^2} dt dx = A + \frac{1}{2}\left(\frac{1}{e} - 1\right)$. **4**

7. Let $f(x,y,z) = xyz$ and $S = \{(x,y,z) : x^2 + y^2 + z^2 = 6\}$. Use Lagrange multiplier method to find maximum and minimum values of f on S. **5**

P.T.O.

8. Evaluate the double integral $\iint_D \sqrt{x+y}(y-2x)^2 dydx$ over the domain D bounded by $x = 0$, $y = 0$, and $x + y = 1$. **5**

9. Let C be the intersection of the cylinder $x^2 + y^2 = 1$ with plane $x + 2y + 3z = 3$ which is parameterised by R. Let F be a vector field with $curlF = i + 2j - \alpha k$ for some $\alpha \in \mathbb{R}$. If $\oint_C F \cdot dR = -\frac{3\pi}{2}$, then find α. **5**

10. Evaluate the line integral $\oint_C 2xyzdx + x^2zdy + x^2ydz$, where C is parametrised by $R(t) = \cos t\, i + \frac{t}{2\pi}\, j + \sin t\, k$, $0 < t < \frac{\pi}{2}$. **3**

11. Find the area of the surface $z = 2xy$ inside the cylinder $x^2 + y^2 = 2$. **4**

12. Evaluate the line integral $\oint_C xydx + 2x^2dy$, where C is the line joining $(-2, 0)$ and $(2, 0)$ and the upper half of the circle $x^2 + y^2 = 4$. **4**

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