DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA102 Summer-term: MATHEMATICS II — End Semester Exam
Instructors: S. N. Bora & R. K. Srivastava — July 5, 2013
Time: 03:00 hours — Maximum Marks: 40

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1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} (x^2+y^2)\cos\frac{1}{x^2+y^2} & \text{if } (x,y) \neq (0,0); \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Show that the function f is differentiable at $(0,0)$. Further show that none of the partial derivative f_x and f_y is continuous at $(0,0)$. **[4]**

2. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a function given by

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Let $\mathbf{v} = \left(\frac{3}{5}, \frac{4}{5}\right)$. Find the directional derivative $D_{\mathbf{v}} f(0,0)$ of f and show that $D_{\mathbf{v}} f(0,0) \neq (\nabla f.\mathbf{v})(0,0)$.

(b) Show that the function f is not differentiable at $(0,0)$.

[2+2]

3. Show that $(0,0)$ is a saddle point for the function $f(x,y) = (x-y)(x-y^2)$. **[3]**

4. Find the nearest point on the plane $x + 2y + 3z = 6$ from the point $(1,0,0)$. **[4]**

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Show that

$$\int_{y=0}^{x} \int_{t=0}^{y} f(t)\,dt\,dy = \int_{t=0}^{x} (x-t) f(t)\,dt.$$

[2]

6. Evaluate the double integral

$$\iint_D \sqrt{x+y}\,(y-2x)^2\,dy\,dx$$

over the domain D bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$. **[4]**

7. Let $\overrightarrow{N}$ the unit outward normal vector on the ellipse $x^2 + 2y^2 = 1$. Evaluate the line integral

$$\int_C \overrightarrow{N}.\overrightarrow{dg}$$

along the circle $C = \{(x, y) : \; x^2 + y^2 = 1\}$. **3**

8. Use fundamental theorem of calculus for line integral to show that

$$\int_C y \; dx + (x + z) \; dy + y \; dz$$

is independent of any path C joining the points $(2, 1, 4)$ and $(8, 3, -1)$. **3**

9. Find the surface integral

$$\int\int_S z dS,$$

where S it the part of the paraboloid $2z = x^2 + y^2$ which lies in the cylinder $x^2 + y^2 = 1$. **5**

10. Let C be the boundary of the cone $z = x^2 + y^2$ and $0 \leq z \leq 1$. Use Stoke's theorem to evaluate the line integral

$$\int_C \overrightarrow{F}.\overrightarrow{dg}$$

where $\overrightarrow{F} = (y, xz, 1)$. **4**

11. Let $\overrightarrow{F} = (xy, yz, zx)$ and S be the surface $z = 4 - x^2 - y^2$ with $2 \leq z \leq 4$. Use divergence theorem to find the surface integral

$$\int\int_S \overrightarrow{F}.\overrightarrow{n}\, dS.$$

4

END