MA15010H: Multi-variable Calculus

(Assignment 6: Change of variables, triple integral)

September - November, 2025

1. Using double integral, find the area enclosed by the curve $r = \sin 3\theta$ given in polar coordinates.
2. Evaluate the double integral $\iint\limits_D \sqrt{x+y}\,(y-2x)^2 dydx$ over the domain D bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$.
3. Evaluate the integral $\iint\limits_D e^{(x-2y)} dxdy$ over the domain D bounded by the lines $x - 2y = 0$, $2x - y = 0$ and $x + y = 1$.
4. Compute $\lim\limits_{a\to\infty} \iint\limits_{D(a)} e^{-(x^2+y^2)} dxdy$, where

   (a) $D(a) = \{(x, y) : x^2 + y^2 \leq a^2\}$ and (b) $D(a) = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq a\}$

   Hence prove that (c) $\displaystyle\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ (d) $\displaystyle\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$
5. Let D denote the solid bounded by the surfaces $y = x$, $y = x^2$, $z = x$ and $z = 0$. Evaluate $\iiint\limits_D y dxdydz$.
6. Let D denote the solid bounded above by the plane $z = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$. Evaluate $\iiint\limits_D \sqrt{x^2 + y^2 + z^2} dxdydz$.
7. Find the surface integral $\iint\limits_S z d\sigma$, where S it the part of the paraboloid $2z = x^2 + y^2$ which lies in the cylinder $x^2 + y^2 = 1$.
8. What is the integral of the function $x^2 z$ taken over the entire surface of a right circular cylinder of height h which stands on the circle $x^2 + y^2 = a^2$.