MA15010H: Multi-variable Calculus

(Assignment 5: Riemann Integration, Fubini's Theorem)

September - November, 2025

(1) If $f : D = [a, b] \times [c, d] \to \mathbb{R}$ is continuous, then f is uniformly continuous.

(2) Let f be real valued continuous function on $[a, b]$. Show that the graph of f is a set of content zero.

(3) Let $D = \{(x, y) : a \leq x \leq b \text{ and } \varphi(x) \leq y \leq \psi(x)\}$, where φ and ψ are continuous functions on $[a, b]$. If f is a bounded continuous functions on D, then

$$\iint\limits_D f(x, y)dxdy = \int_a^b \left(\int_{\varphi(x)}^{\psi(x)} f(x, y)dy \right) dx.$$

(4) Evaluate the following integral applying Fubini's Theorem

(a) $\int\limits_0^1 \int\limits_0^{\sqrt{1-x^2}} \sqrt{1 - y^2}dydx$

(b) $\int\limits_0^\pi \int\limits_x^\pi \frac{\sin y}{y} dydx$

(c) $\int\limits_0^1 \int\limits_y^1 x^2 e^{xy} dxdy$

(5) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Show that $\int\limits_{y=0}^{x} \int\limits_{t=0}^{y} f(t)dtdy = \int\limits_{t=0}^{x} (x - t)f(t)dt$.

(6) Let f be a continuous function on the bounded domain D. If $\iint\limits_R f(x, y)dxdy = 0$ for all rectangle R in D, then $f = 0$ on D.

(7) Let $f : D = [a, b] \times [c, d] \to \mathbb{R}$ be a continuous function. If f_x, f_y, f_{xy} and f_{yx} are continuous then, by using Fubini's theorem, show that $f_{xy} = f_{yx}$.