ME 101: Engineering Mechanics

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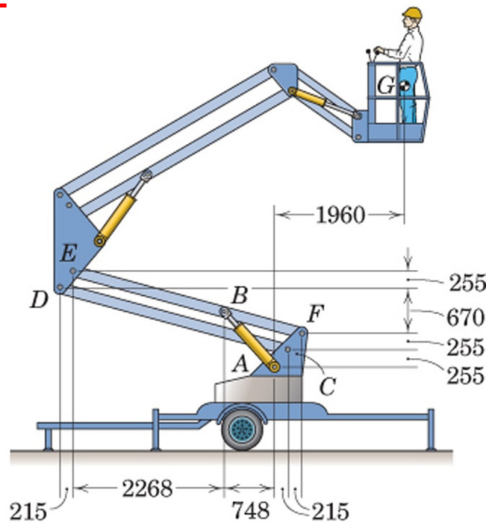
Frames and Machines

A structure is called a Frame or Machine if at least one of its individual members is a <u>multi-force member</u>

- member with 3 or more forces acting, or
- member with 2 or more forces and 1 or more couple acting



Frames: generally stationary and are used to support loads

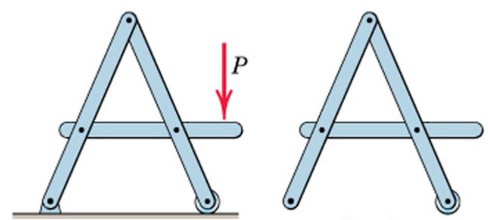
Machines: contain moving parts and are designed to transmit and alter the effect of forces acting

Multi-force members: the forces in these members in general will not be along the directions of the members

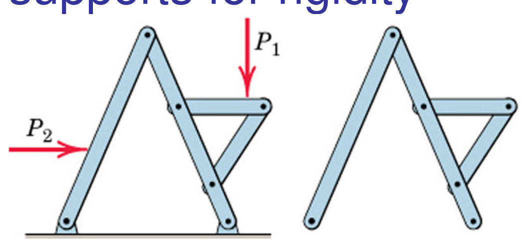
→ methods used in simple truss analysis cannot be used

Frames and Machines

Interconnected Rigid Bodies with Multi-force Members



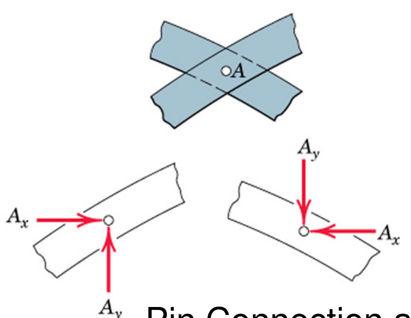
- Rigid Non-collapsible
  - –structure constitutes a rigid unit by itself when removed from its supports
  - –first find all forces external to the structure treated as a single rigid body
  - –then dismember the structure & consider equilibrium of each part
- Non-rigid Collapsible
  - –structure is not a rigid unit by itself but depends on its external supports for rigidity
  - –calculation of external support reactions cannot be completed until the structure is dismembered and individual parts are analysed.



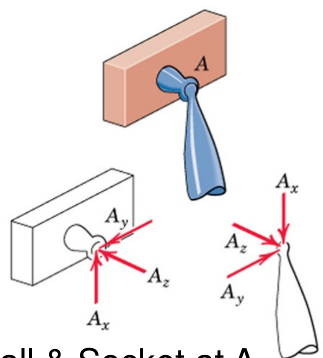
Frames and Machines

Free Body Diagrams: Forces of Interactions

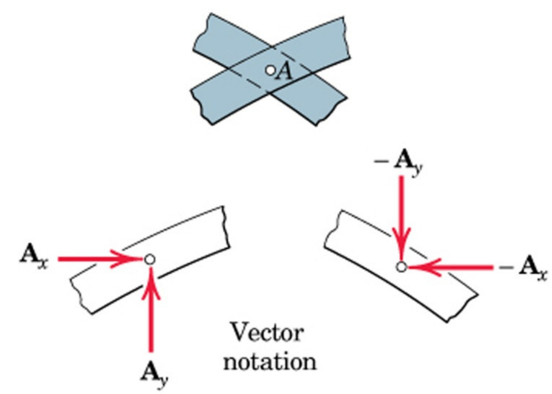
- force components must be consistently represented in opposite directions on the separate FBDs (Ex: Pin at A).
- apply action-and-reaction principle (Ex: Ball & Socket at A).
- Vector notation: use plus sign for an action and a minus sign for the corresponding reaction



Pin Connection at A



Ball & Socket at A



Frames and Machines

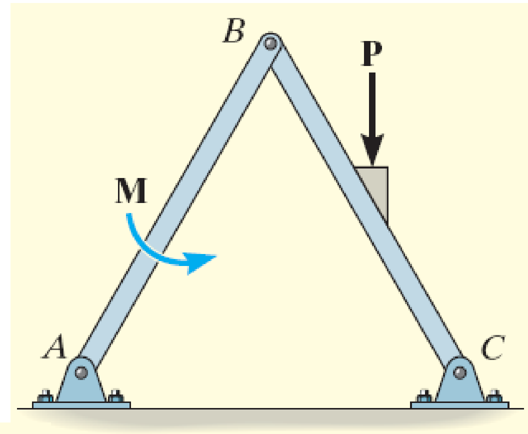
Example: Free Body Diagrams

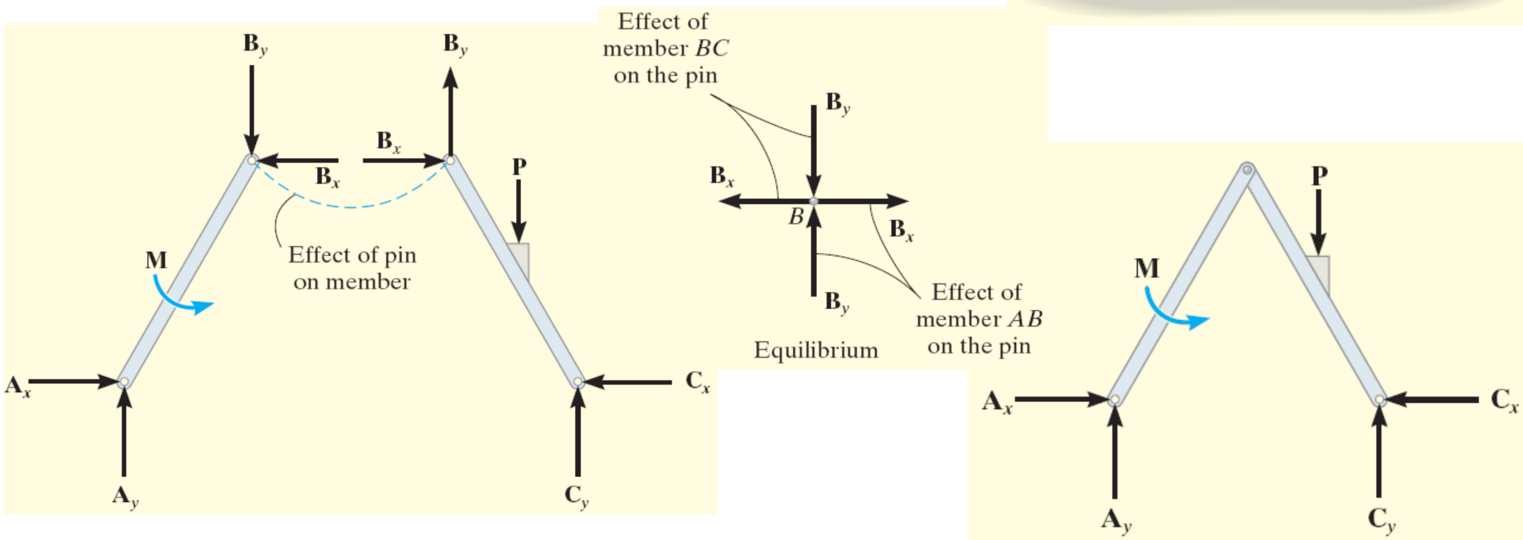
Draw FBD of

(a) Each member

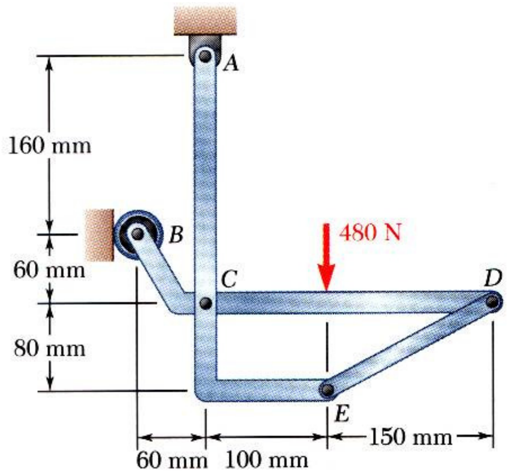
(b) Pin at B, and

(c) Whole system





Example

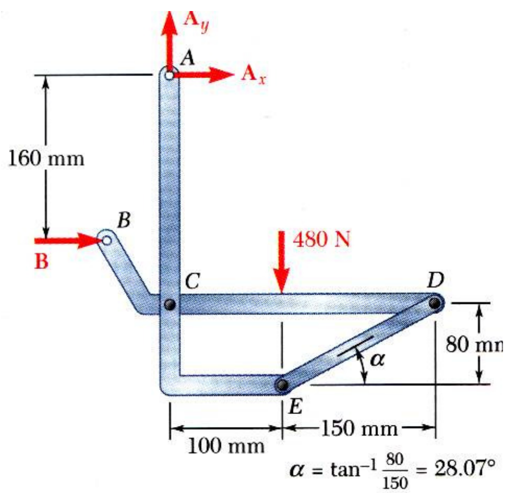


Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.
- With the force on the link *DE* known, the sum of forces in the *x* and *y* directions may be used to find the force components at *C*.
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

Example



<u>SOLUTION</u>:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\text{ N}$$

$$\boxed{A_y = 480\text{ N} \uparrow}$$

$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

$$\boxed{B = 300\text{ N} \rightarrow}$$

$$\sum F_x = 0 = B + A_x$$

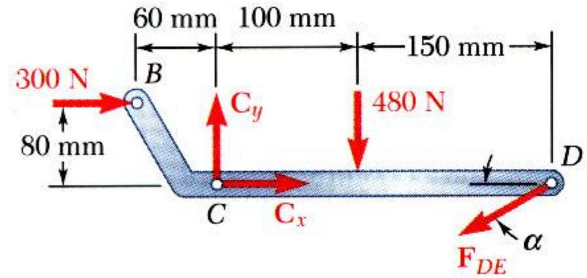
$$\boxed{A_x = -300\text{ N} \leftarrow}$$

Note:

$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

Example

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$\sum M_C = 0 = (F_{DE} \sin\alpha)(250 \text{ mm}) + (300 \text{ N})(60 \text{ mm}) + (480 \text{ N})(100 \text{ mm})$$

$$F_{DE} = -561 \text{ N} \qquad \boxed{F_{DE} = 561 \text{ N} \;\; C}$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

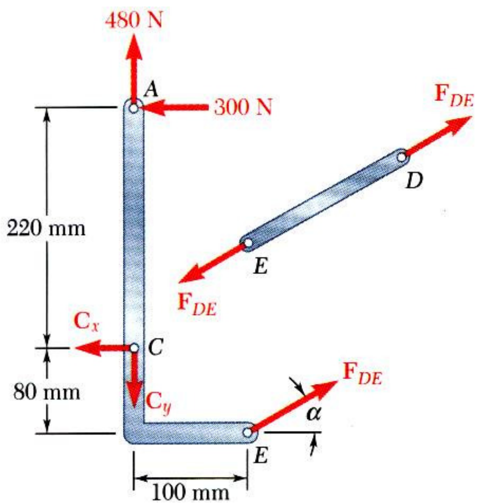
$$\sum F_x = 0 = C_x - F_{DE} \cos\alpha + 300 \text{ N}$$

$$0 = C_x - (-561 \text{ N})\cos\alpha + 300 \text{ N} \qquad \boxed{C_x = -795 \text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE} \sin\alpha - 480 \text{ N}$$

$$0 = C_y - (-561 \text{ N})\sin\alpha - 480 \text{ N} \qquad \boxed{C_y = 216 \text{ N}}$$

Example



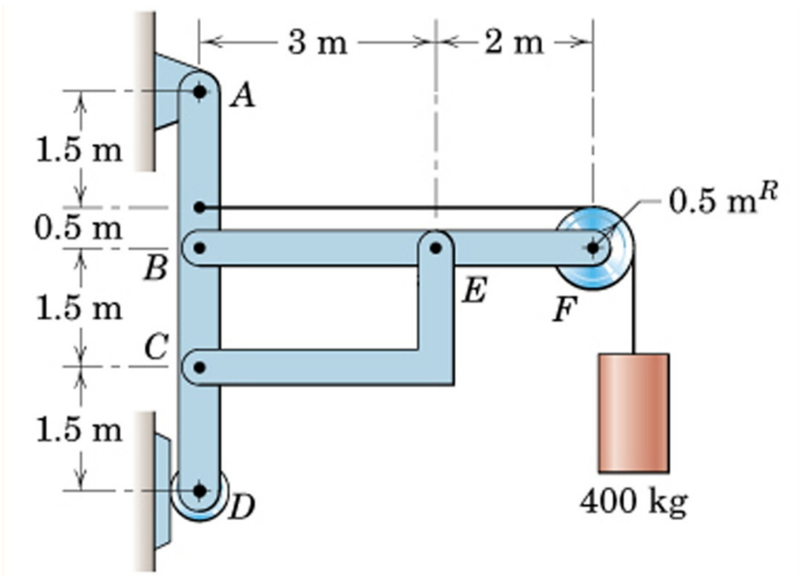
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

(checks)

Frames and Machines

Example: Compute the horizontal and vertical components of all forces acting on each of the members (neglect self weight)



Frames and Machines

Example Solution:

3 supporting members form a rigid non-collapsible assembly

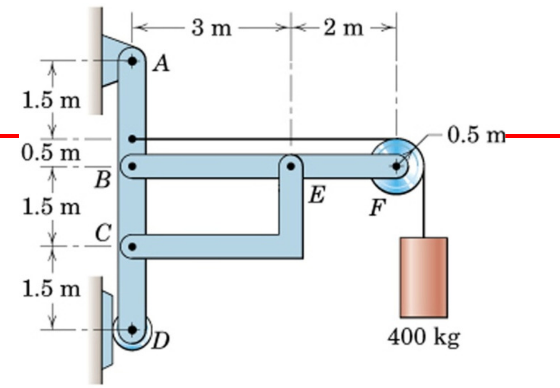
Frame Statically Determinate Externally

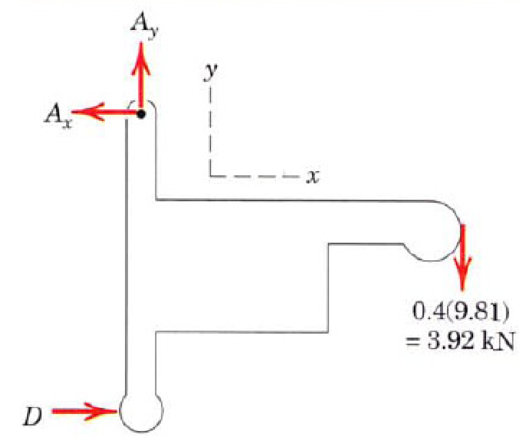
Draw FBD of the entire frame

3 Equilibrium equations are available

Pay attention to sense of Reactions

Reactions can be found out





$$[\Sigma M_A = 0] \quad 5.5(0.4)(9.81) - 5D = 0 \quad D = 4.32 \text{ kN}$$

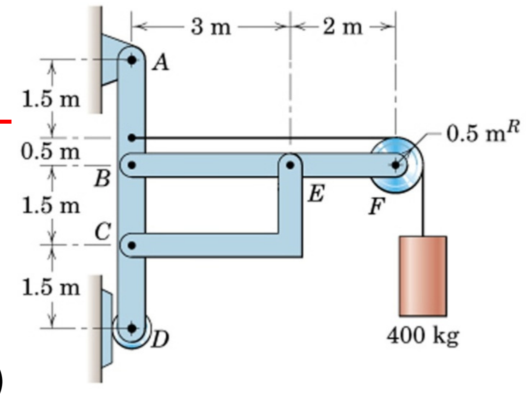
$$[\Sigma F_x = 0] \quad A_x - 4.32 = 0 \quad A_x = 4.32 \text{ kN}$$

$$[\Sigma F_y = 0] \quad A_y - 3.92 = 0 \quad A_y = 3.92 \text{ kN}$$

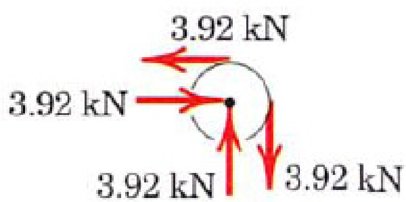
Frames and Machines

Example Solution: Dismember the frame and draw separate FBDs of each member

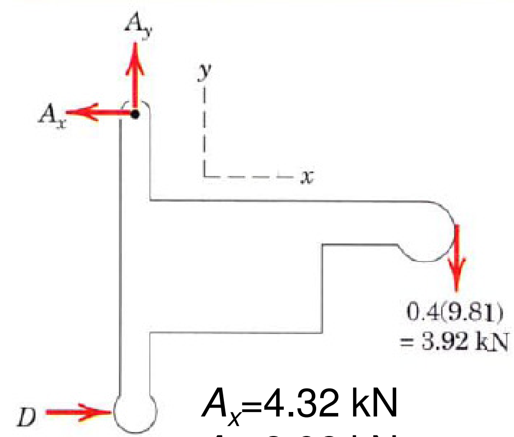
- show loads and reactions on each member due to connecting members (interaction forces)



Begin with FBD of Pulley



Then draw FBD of Members BF, CE, and AD



A_x=4.32 kN

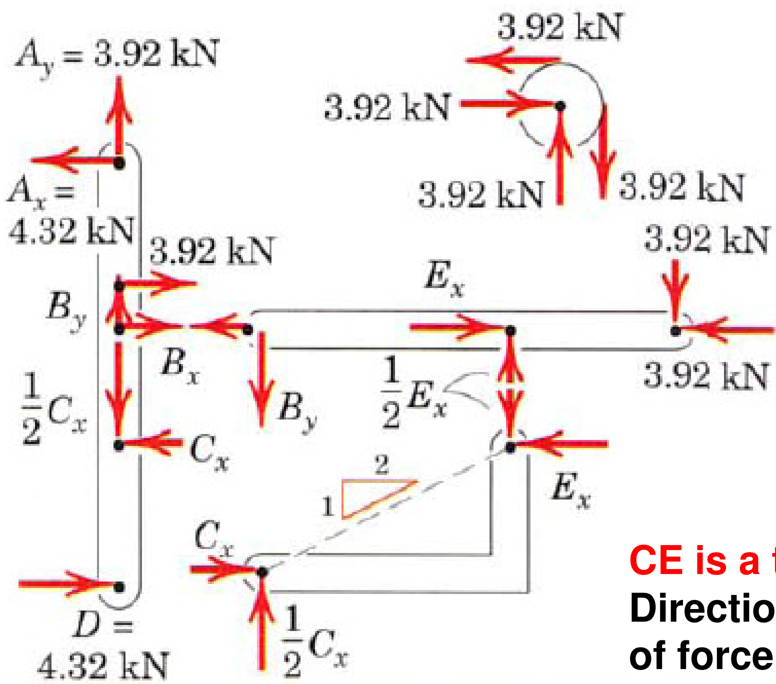
A_y=3.92 kN

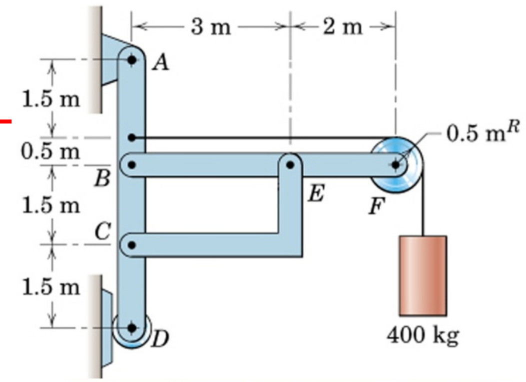
D=4.32 kN

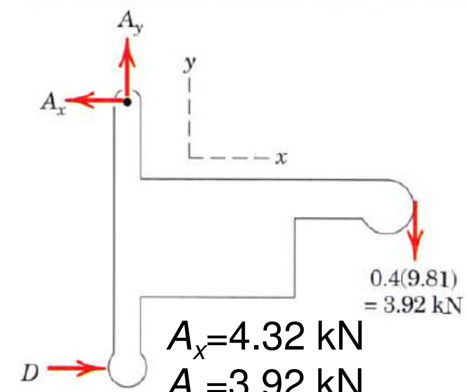
Frames and Machines

Example Solution:

FBDs







A_x=4.32 kN
A_y=3.92 kN
D=4.32 kN

CE is a two-force member

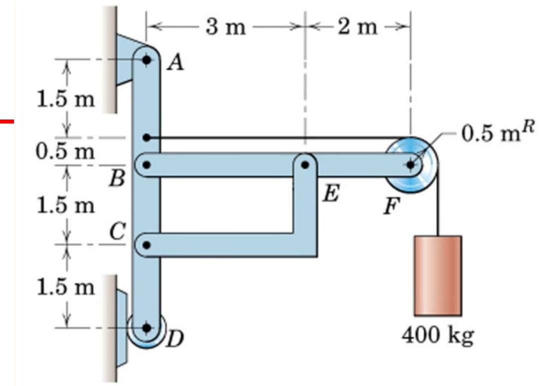
Direction of the line joining the two points of force application determines the direction of the forces acting on a two-force member.

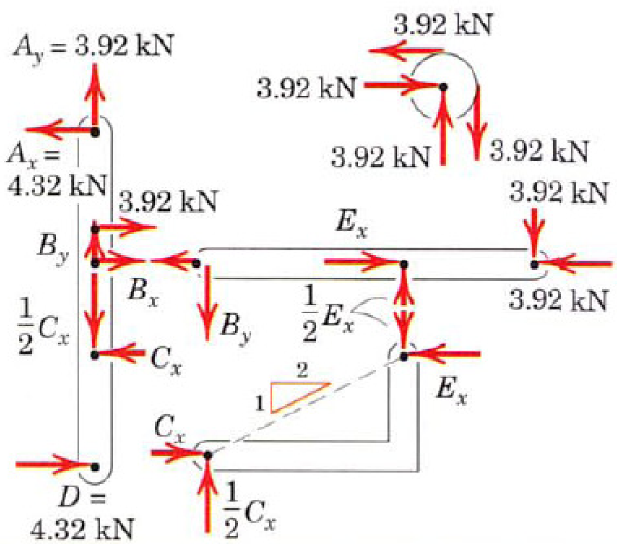
Shape of the member is not important.

Frames and Machines

Example Solution:

Find unknown forces from equilibrium





Member BF

$[\Sigma M_B = 0]$ $3.92(5) - \frac{1}{2}E_x(3) = 0$ $E_x = 13.08$ kN

$[\Sigma F_y = 0]$ $B_y + 3.92 - 13.08/2 = 0$ $B_y = 2.62$ kN

$[\Sigma F_x = 0]$ $B_x + 3.92 - 13.08 = 0$ $B_x = 9.15$ kN

Member CE

[∑Fx = 0] $C_x = E_x = 13.08$ kN

Checks:

$[\Sigma M_C = 0]$ $4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$

$[\Sigma F_x = 0]$ $4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$

$[\Sigma F_y = 0]$ $-13.08/2 + 2.62 + 3.92 = 0$

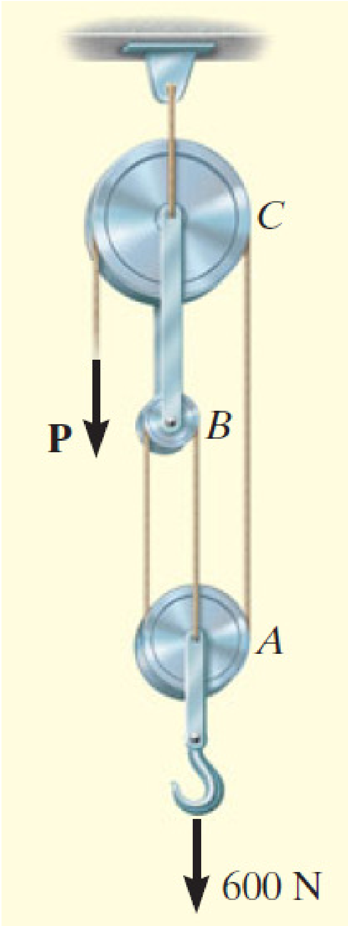
Frames and Machines

Example:

Find the tension in the cables and the force P required to support the 600 N force using the frictionless pulley system (neglect self weight)

Solution:

Draw the FBD



Frames and Machines

Example Solution:

Draw FBD and apply equilibrium equations

Pulley A

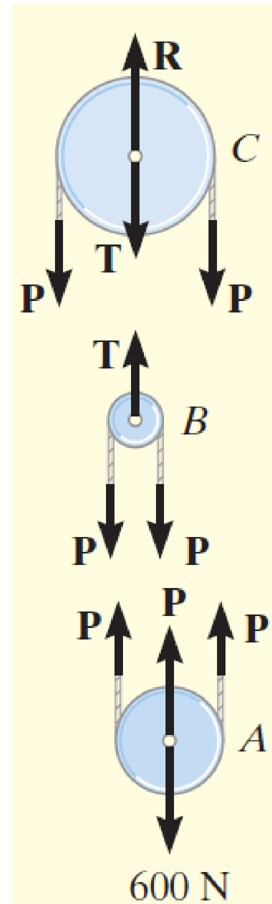
$+\uparrow \Sigma F_y = 0; \qquad 3P - 600\text{ N} = 0 \qquad P = 200\text{ N}$

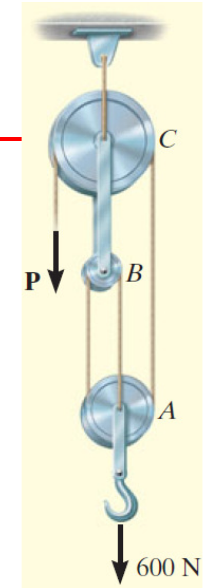
Pulley B

$+\uparrow \Sigma F_y = 0; \qquad T - 2P = 0 \qquad T = 400\text{ N}$

Pulley C

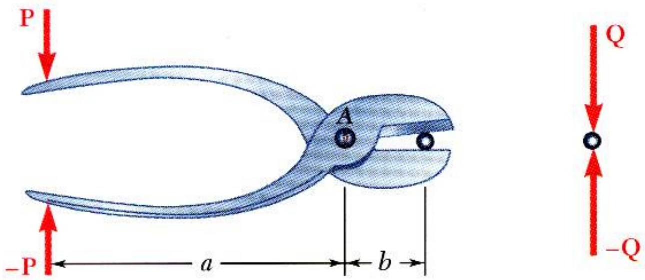
$+\uparrow \Sigma F_y = 0; \qquad R - 2P - T = 0 \qquad R = 800\text{ N}$



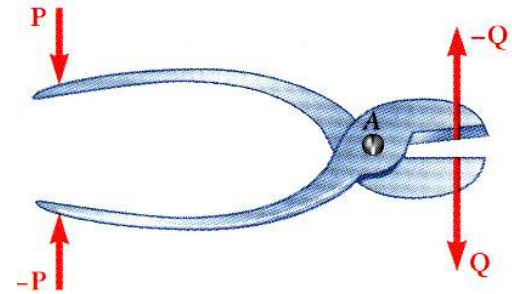


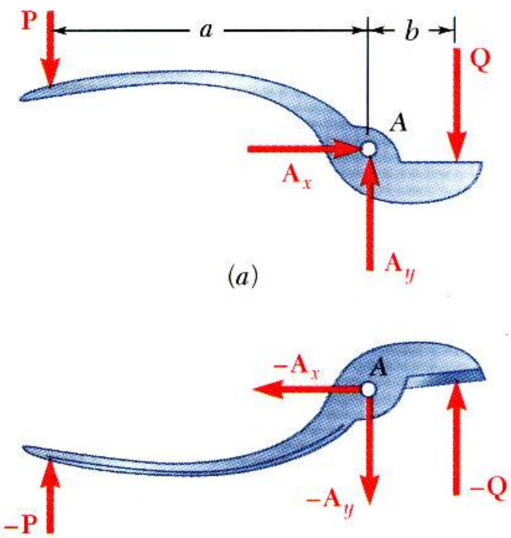
Frames and Machines

Example: Pliers: Given the magnitude of P, determine the magnitude of Q



FBD of Whole Pliers





(a)

FBD of individual parts

Taking moment about pin A

$Q = Pa/b$

Also pin reaction $A_x = 0$

$A_y = P(1 + a/b)$ OR $A_y = P + Q$

Example

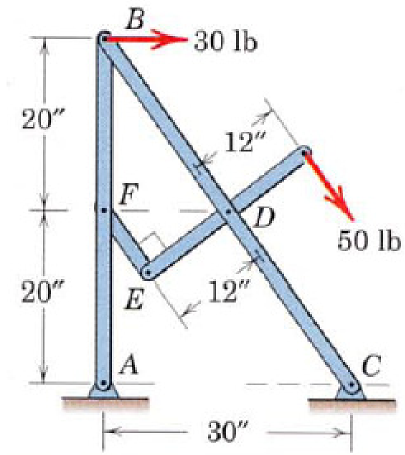
Q. Neglect the weight of the frame and compute the forces acting on all of its members.

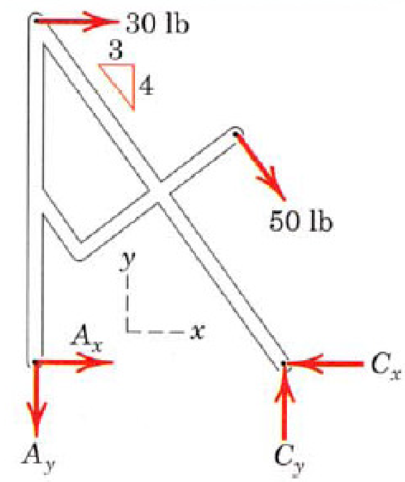
Step 1: Draw the FBD and calculate the reactions.

Is it a rigid frame?

The frame is not rigid, hence all the reaction can not the determined using the equilibrium equations.

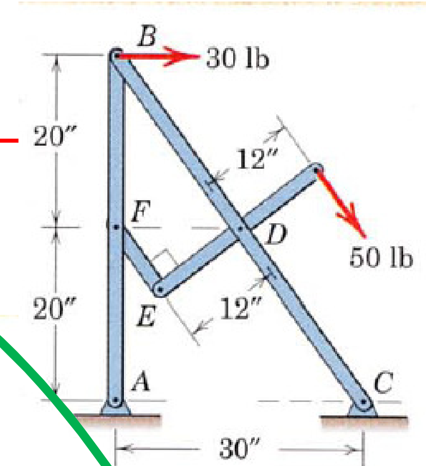
Calculate the reactions which are possible to calculate using the equilibrium equations





$[\Sigma M_C = 0]$	$50(12) + 30(40) - 30A_y = 0$	$A_y = 60$ lb
$[\Sigma F_y = 0]$	$C_y - 50(4/5) - 60 = 0$	$C_y = 100$ lb

Example



Member ED. The two unknowns are easily obtained by

$[\Sigma M_D = 0]$ $\quad 50(12) - 12E = 0 \quad E = 50$ lb

$[\Sigma F = 0]$ $\quad D - 50 - 50 = 0 \quad D = 100$ lb

Member EF. Clearly F is equal and opposite to E with the magnitude of 50 lb.

Member AB. Since F is now known, we solve for B_x, A_x, and B_y from

$[\Sigma M_A = 0]$ $\quad 50(3/5)(20) - B_x(40) = 0 \quad B_x = 15$ lb

$[\Sigma F_x = 0]$ $\quad A_x + 15 - 50(3/5) = 0 \quad A_x = 15$ lb

$[\Sigma F_y = 0]$ $\quad 50(4/5) - 60 - B_y = 0 \quad B_y = -20$ lb

The minus sign shows that we assigned B_y in the wrong direction.

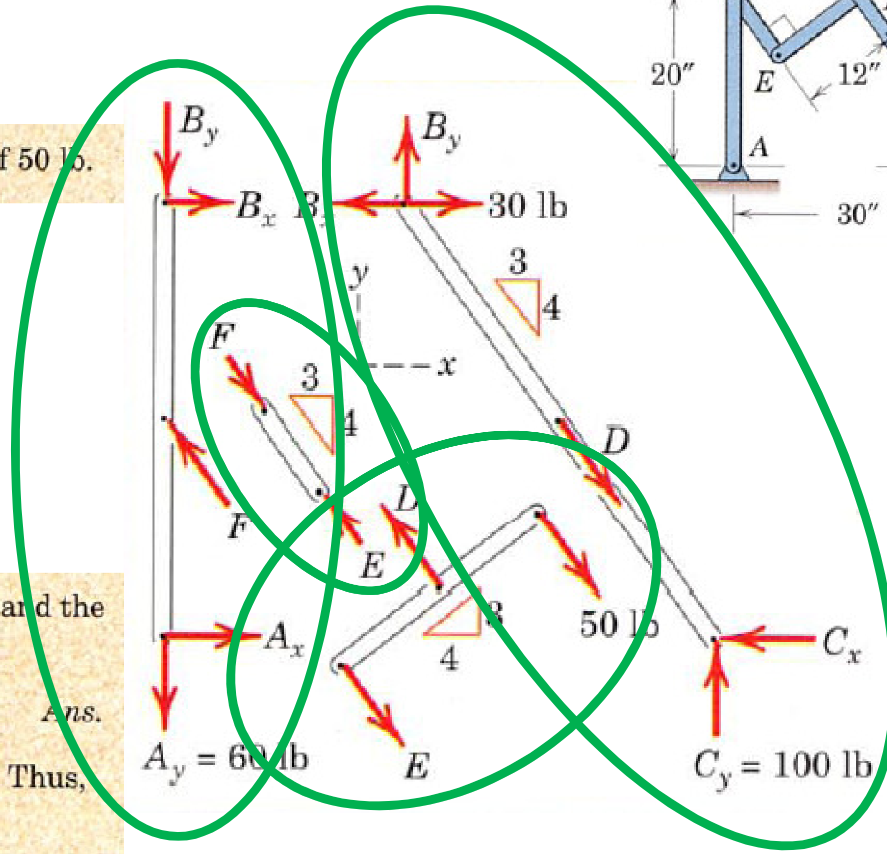
Member BC. The results for B_x, B_y, and D are now transferred to BC, and the remaining unknown C_x is found from

$[\Sigma F_x = 0]$ $\quad 30 + 100(3/5) - 15 - C_x = 0 \quad C_x = 75$ lb *Ans.*

We may apply the remaining two equilibrium equations as a check. Thus,

$[\Sigma F_y = 0]$ $\quad 100 + (-20) - 100(4/5) = 0$

$[\Sigma M_C = 0]$ $\quad (30 - 15)(40) + (-20)(30) = 0$



Frames and Machines

Definitions

- **Effort**: Force required to overcome the resistance to get the work done by the machine.
- **Mechanical Advantage**: Ratio of load lifted (W) to effort applied (P).

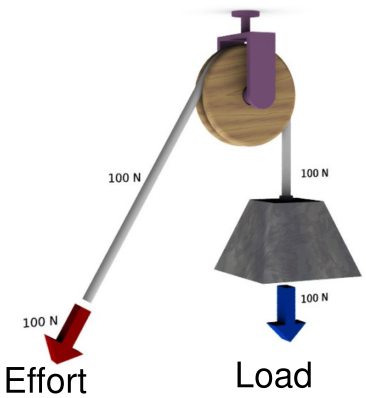
  Mechanical Advantage = W/P
- **Velocity Ratio**: Ratio of the distance moved by the effort (D) to the distance moved by the load (d) in the same interval of time.

  Velocity Ratio = D/d
- **Input**: Work done by the effort → Input = PD
- **Output**: Useful work got out of the machine, i.e. the work done by the load → Output = Wd
- **Efficiency**: Ratio of output to the input.

  Efficiency of an ideal machine is 1. In that case, $Wd = PD \rightarrow W/P = D/d$.

  For an ideal machine, mechanical advantage is equal to velocity ratio.

Frames and Machines: Pulley System



Fixed Pulley

Effort = Load
→ Mechanical Advantage = 1

Distance moved by effort is equal to the distance moved by the load.
→ Velocity Ratio = 1

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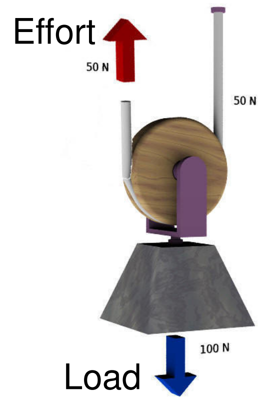
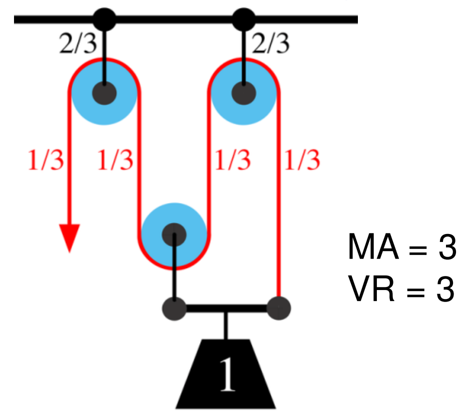


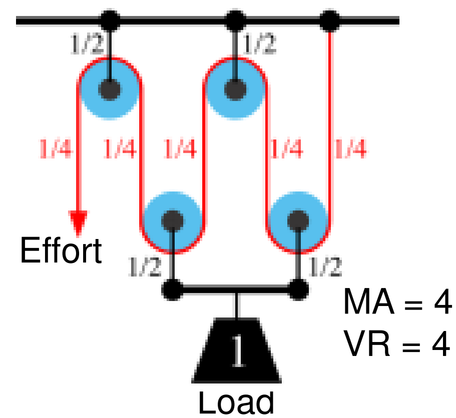
Movable Pulley

Effort = Load/2
→Mechanical Advantage = 2

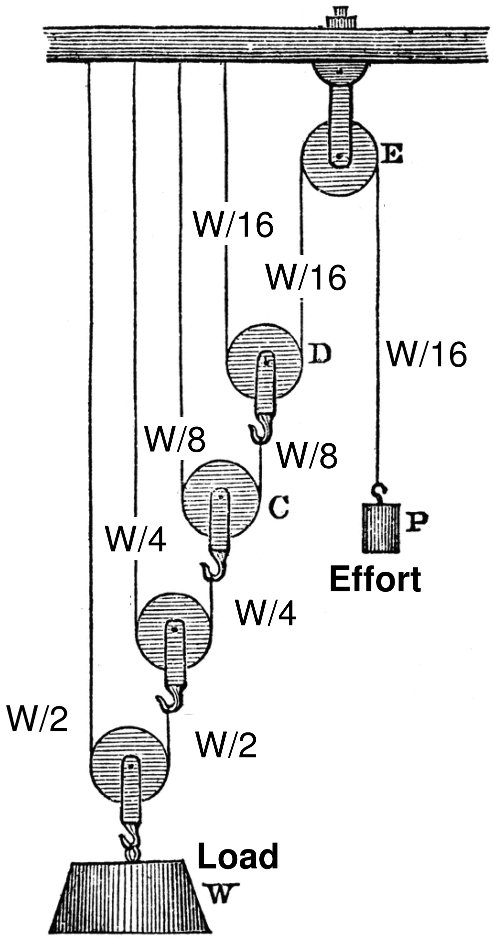
Distance moved by effort is twice the distance moved by the load (both rope should also accommodate the same displacement by which the load is moved).
→ Velocity Ratio = 2

Compound Pulley





Frames and Machines: Pulley System



Compound Pulley

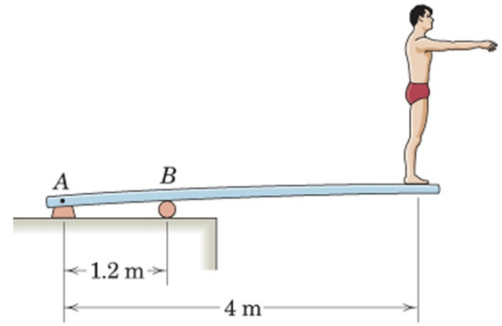
Effort required is 1/16th of the load.

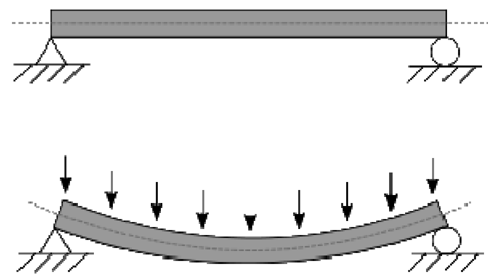
Mechanical Advantage = 16.
(neglecting frictional forces).

Velocity ratio is 16, which means in order to raise a load to 1 unit height; effort has to be moved by a distance of 16 units.

http://etc.usf.edu/

Beams

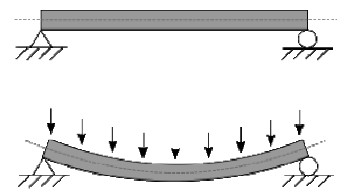




Beams are structural members that offer resistance to bending due to applied load

Beams

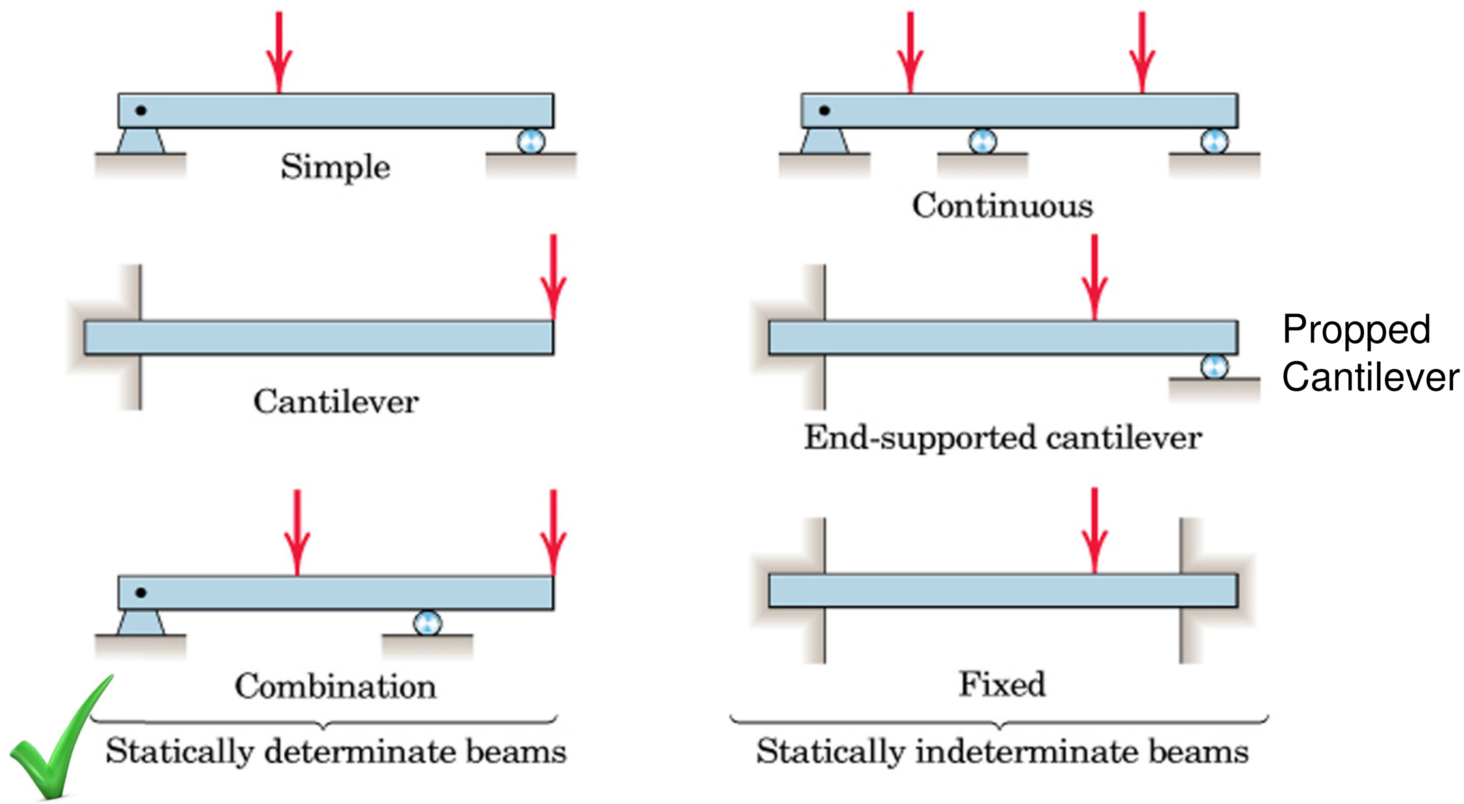
Beams



- mostly long prismatic bars
  - Prismatic: many sided, same section throughout
- non-prismatic beams are also useful
- cross-section of beams much smaller than beam length
- loads usually applied normal to the axis of the bar
- Determination of Load Carrying Capacity of Beams
- Statically Determinate Beams
  - Beams supported such that their external support reactions can be calculated by the methods of statics
- Statically Indeterminate Beams
  - Beams having more supports than needed to provide equilibrium

Types of Beams

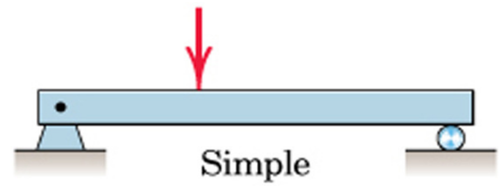
- Based on support conditions



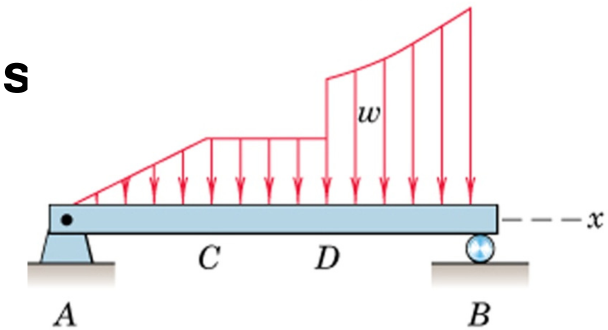
Types of Beams

- Based on type of external loading

Beams supporting Concentrated Loads



Beams supporting Distributed Loads

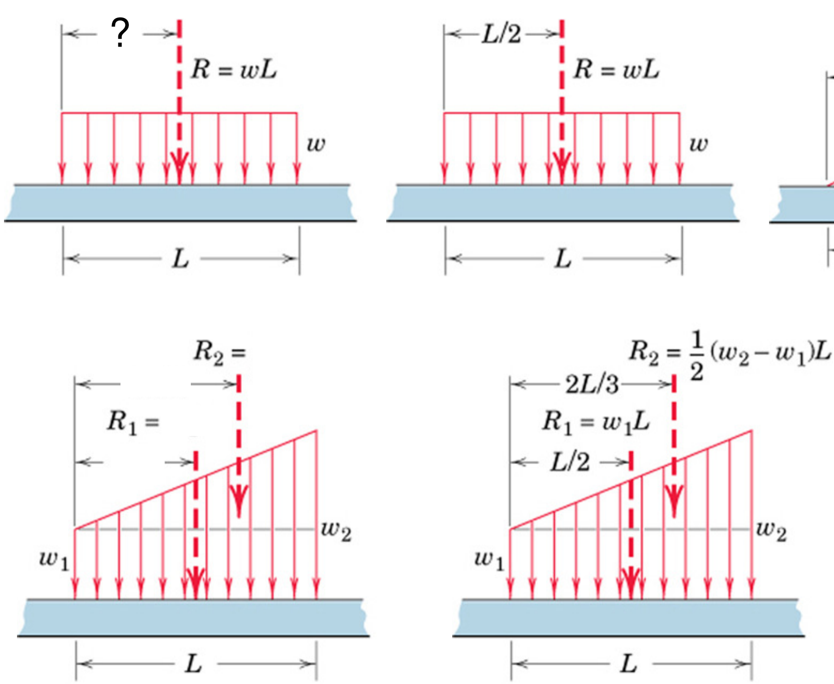


- Intensity of distributed load = *w*
- *w* is expressed as force per unit length of beam (N/m)
- intensity of loading may be constant or variable, continuous or discontinuous
  - discontinuity in intensity at D (abrupt change)
  - At C, intensity is not discontinuous, but rate of change of intensity (*dw*/*dx*) is discontinuous

Beams

Distributed Loads on beams

- Determination of Resultant Force (*R*) on beam is important



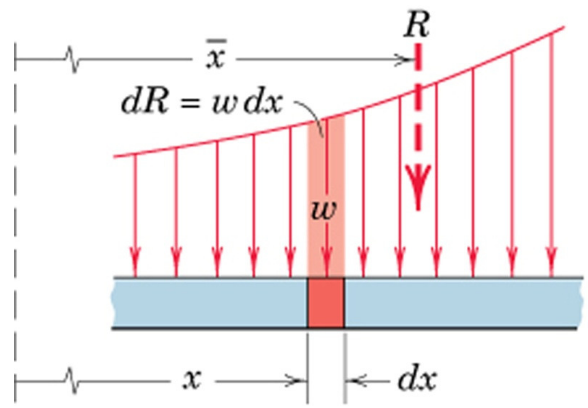
R = area formed by *w* and length *L* over which the load is distributed

R passes through centroid of this area

Beams

Distributed Loads on beams

- General Load Distribution



Differential increment of force is

$dR = w\,dx$

Total load R is sum of all the differential forces

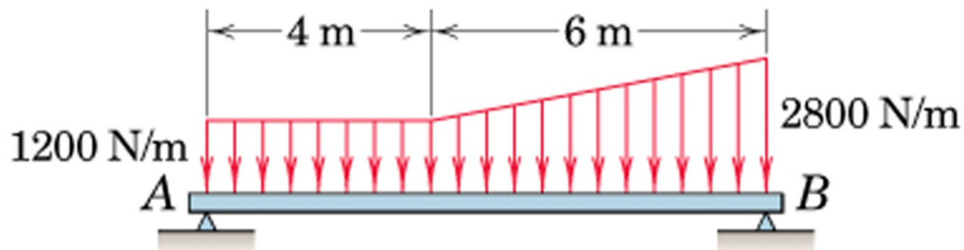
$R = \int w\,dx$ acting at centroid of the area under consideration

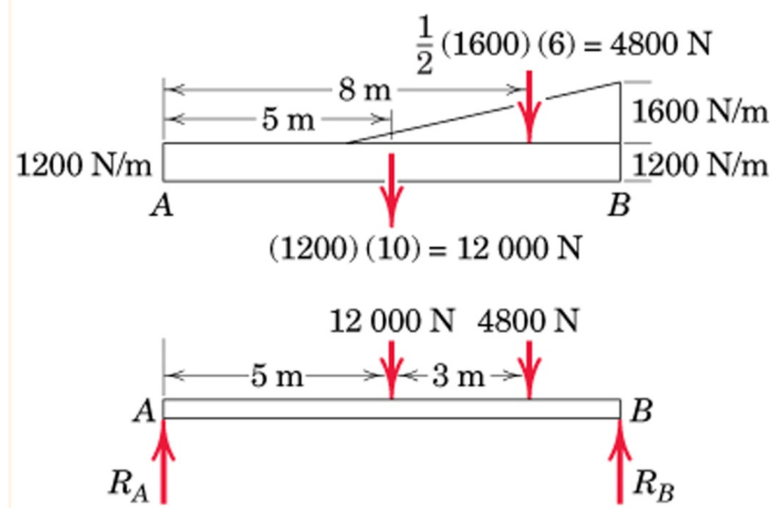
$$\bar{x} = \frac{\int xw\,dx}{R}$$

Once R is known reactions can be found out from Statics

Beams: Example

Determine the external reactions for the beam

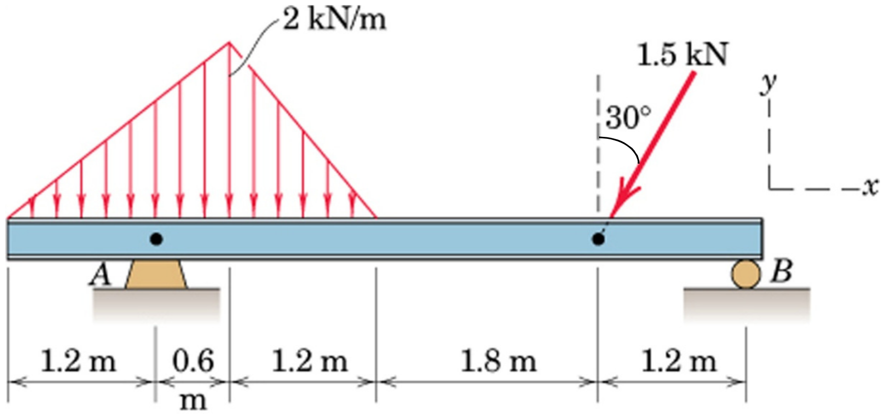




R_A = 6.96 kN, R_B = 9.84 kN

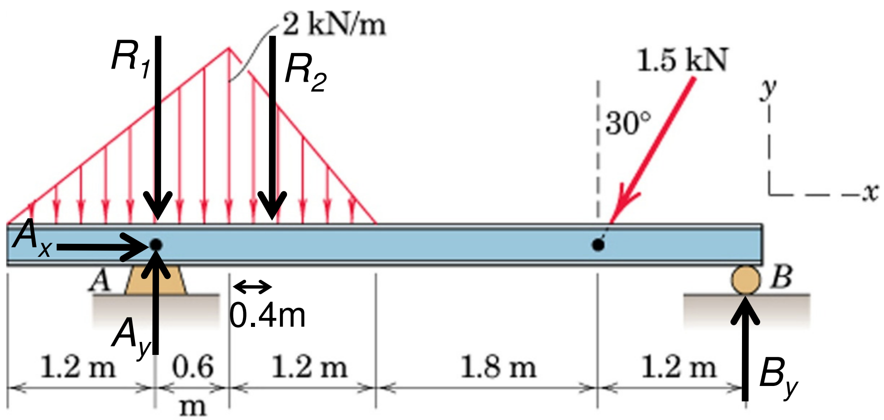
Beams: Example

Determine the external reactions for the beam



Dividing the un-symmetric triangular load into two parts with resultants R_1 and R_2 acting at point A and 1m away from point A, respectively.

R_1 = 0.5x1.8x2 = 1.8 kN
R_2 = 0.5x1.2x2 = 1.2 kN



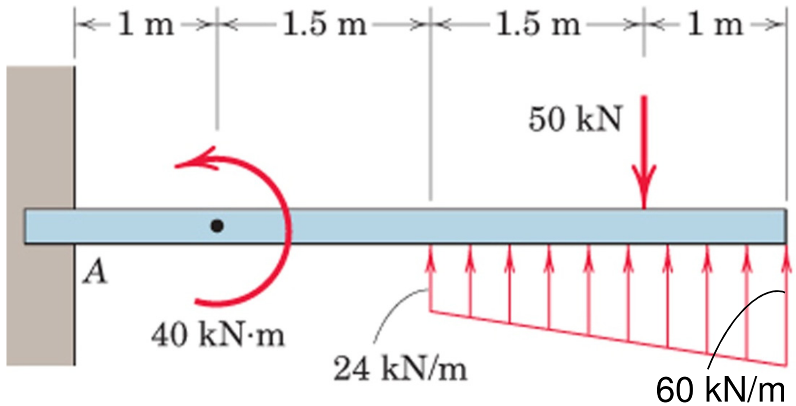
$\sum F_x=0 \rightarrow$ $\boldsymbol{A_x}$ = 1.5sin30 = **0.75 kN**

$\sum M_A=0 \rightarrow$
4.8xB_y = 1.5cos30x3.6 + 1.2x1.0
$\boldsymbol{B_y}$ **= 1.224 kN**

$\sum F_y=0 \rightarrow$
A_y = 1.8+1.2+1.5cos30-1.224
$\boldsymbol{A_y}$ **= 3.075 kN**

Beams: Example

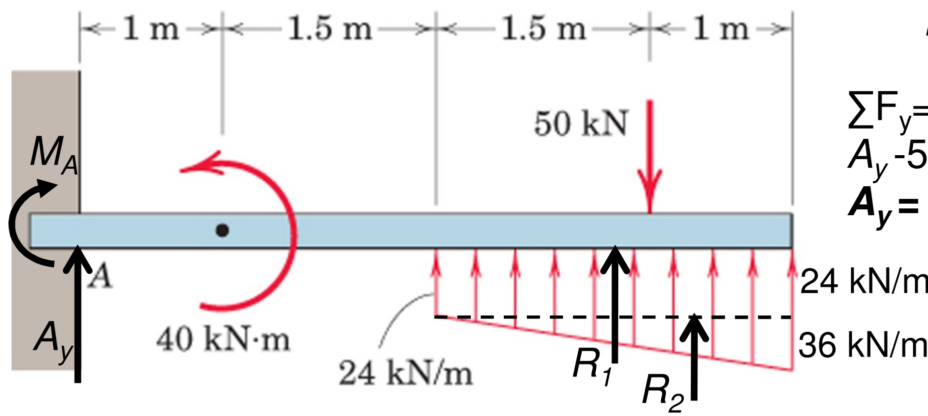
Determine the external reactions for the beam



Dividing the trapezoidal load into two parts with resultants R_1 and R_2

R_1 = 24x2.5 = 60 kN @ 3.75m → A
R_2 = 0.5x2.5x36=45 kN@4.17m → A
(distances from A)

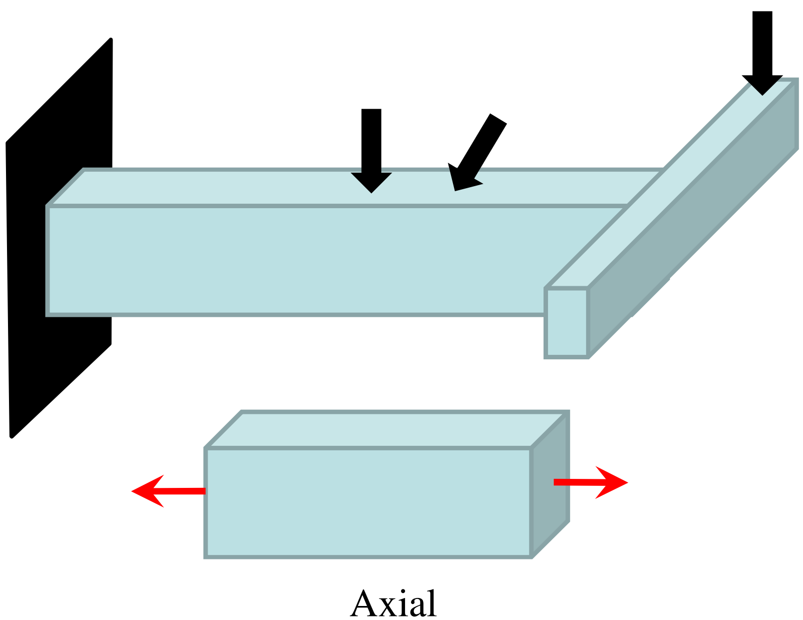
$\sum M_A=0$ →
M_A-40+50x4.0-60x3.75-45x4.17=0
M_A = 253 kNm

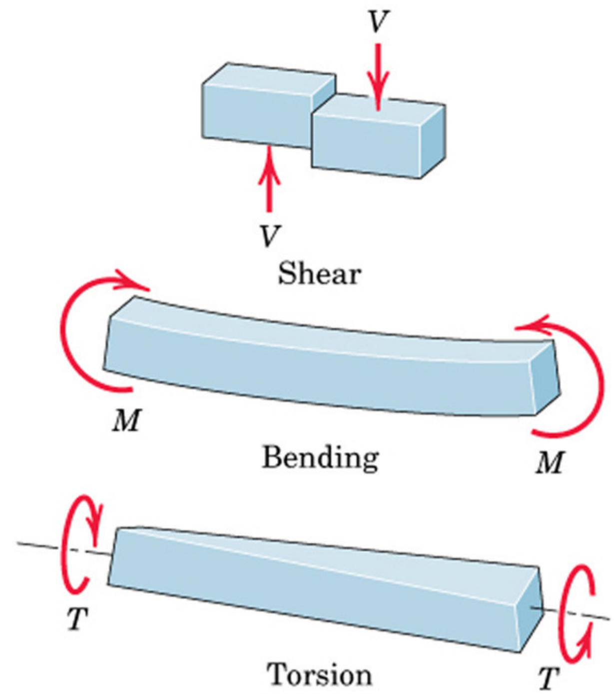


$\sum F_y=0$ →
A_y -50+60+45 = 0
A_y = -55 kN → Downwards

Beams – Internal Effects

- Internal Force Resultants
- Axial Force (*N*), Shear Force (*V*), Bending Moment (*M*), Torsional Moment (*T*) in Beam
  - Method of Sections is used

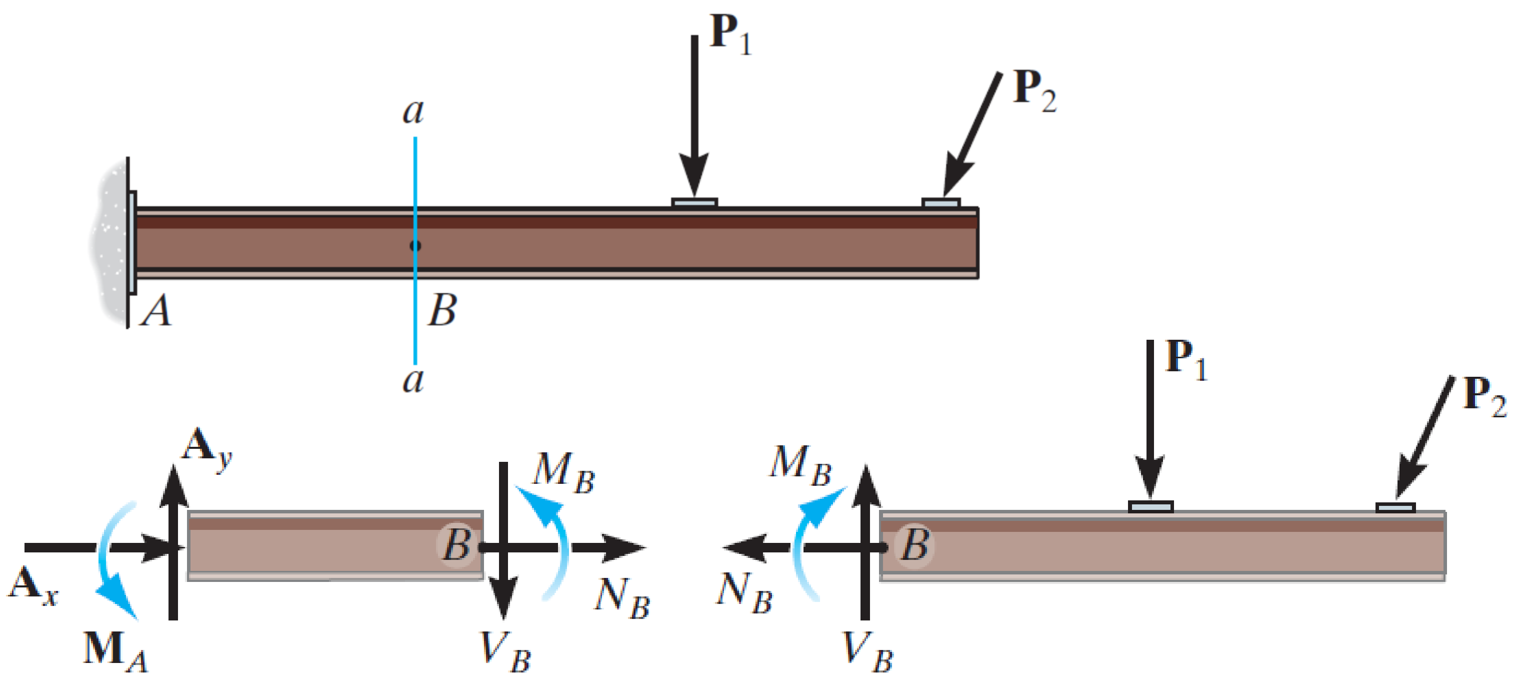




Beams – Internal Effects

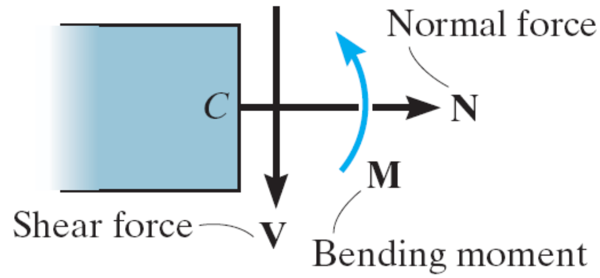
- Method of Section:

  Internal Force Resultants at B → Section a-a at B and use equilibrium equations in both cut parts

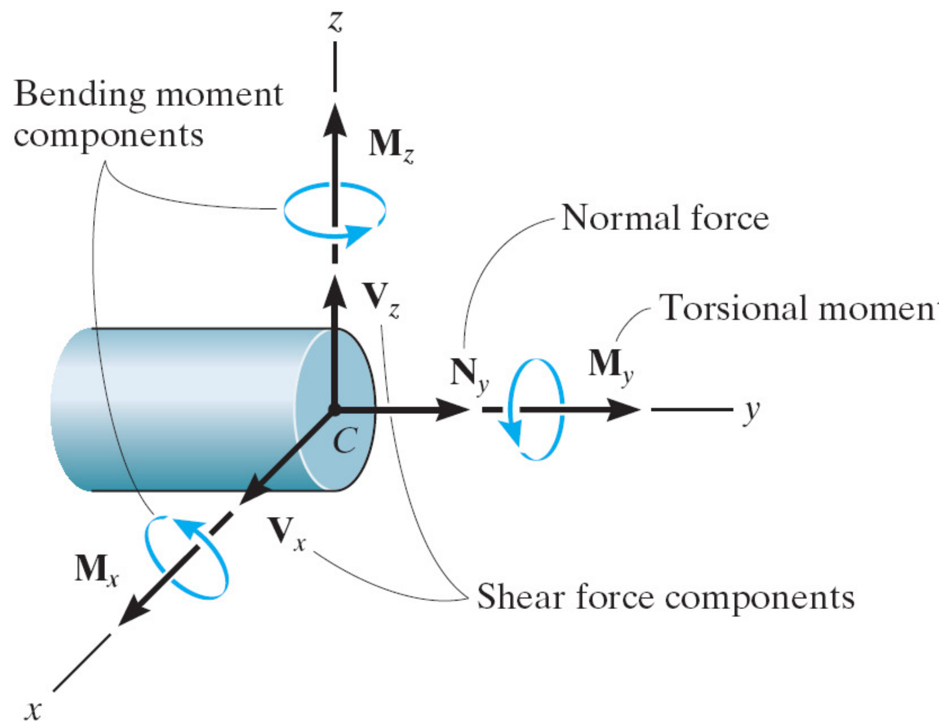


Beams – Internal Effects

- 2D Beam



- 3D Beam

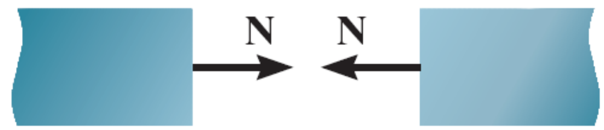


The Force Resultants act at centroid of the section's Cross-sectional area

Beams – Internal Effects

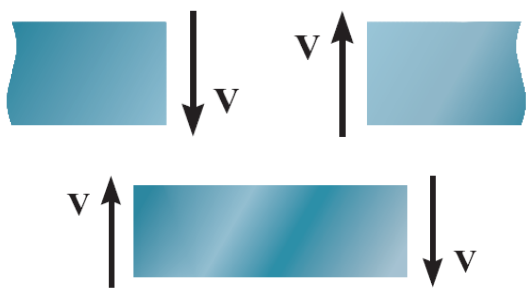
Sign Convention

Positive Axial Force creates Tension



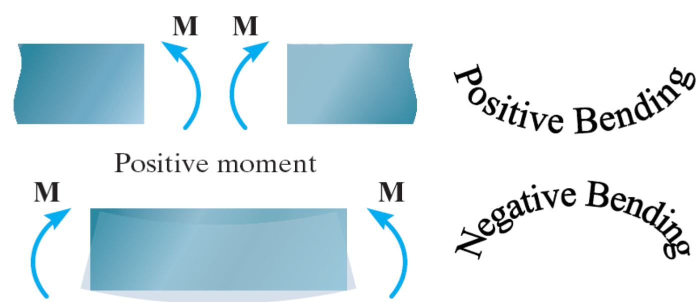
Positive normal force

Positive shear force will cause the Beam segment on which it acts to rotate clockwise



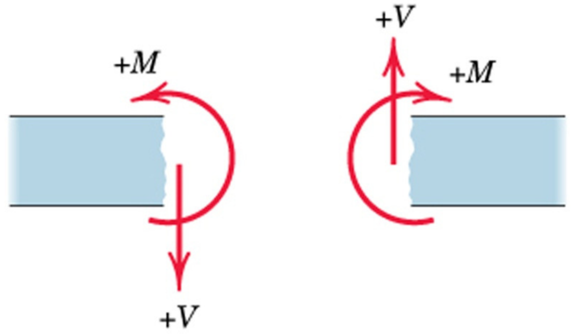
Positive shear

Positive bending moment will tend to bend the segment on which it acts in a concave upward manner (compression on top of section).

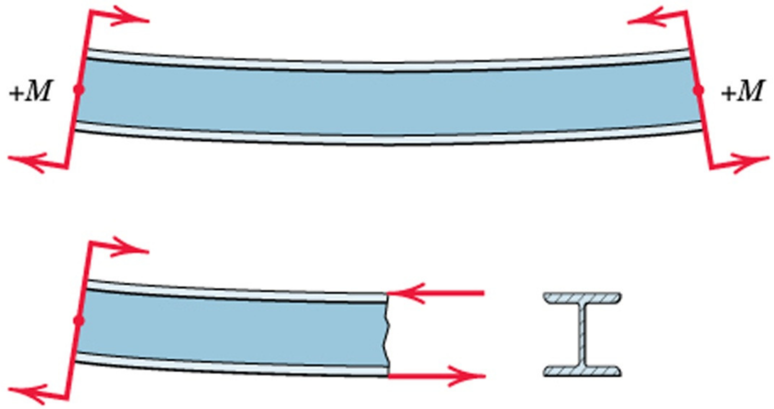


Beams – Internal Effects

Sign convention in a single plane



Interpretation of Bending Couple



H-section Beam bent by two equal and opposite positive moments applied at ends

Neglecting resistance offered by web

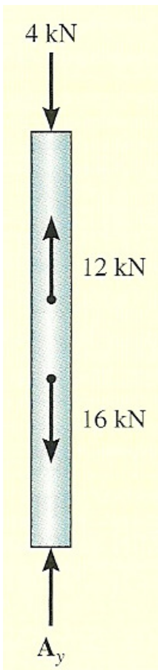
Compression at top; Tension at bottom

Resultant of these two forces (one tensile and other compressive) acting on any section is a Couple and has the value of the Bending Moment acting on the section.

Beams – Internal Effects

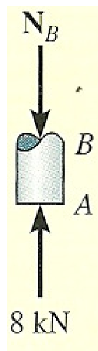
Example: Find the axial force in the fixed bar at points B and C

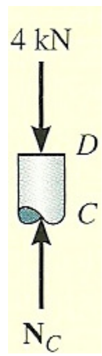
Solution: Draw the FBD of the entire bar



$$+\uparrow \Sigma F_y = 0; \quad A_y - 16\text{ kN} + 12\text{ kN} - 4\text{ kN} = 0 \quad A_y = 8\text{ kN}$$

Draw sections at B and C to get AF in the bar at B and C



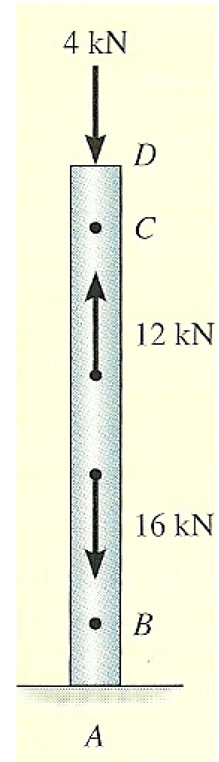


Segment AB

$$+\uparrow \Sigma F_y = 0; \quad 8\text{ kN} - N_B = 0 \quad N_B = 8\text{ kN}$$

Segment DC

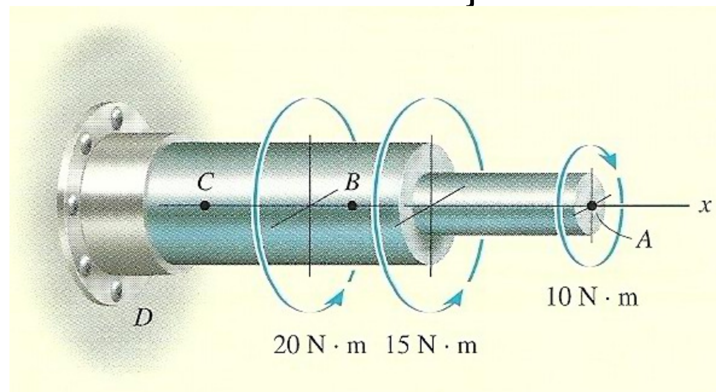
$$+\uparrow \Sigma F_y = 0; \quad N_C - 4\text{ kN} = 0 \quad N_C = 4\text{ kN}$$



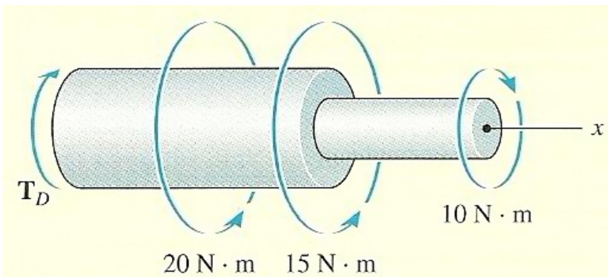
Alternatively, take a section at C and consider only CD portion of the bar
Then take a section at B and consider only BD portion of the bar
→ no need to calculate reactions

Beams – Internal Effects

Example: Find the internal torques at points B and C of the circular shaft subjected to three concentrated torques

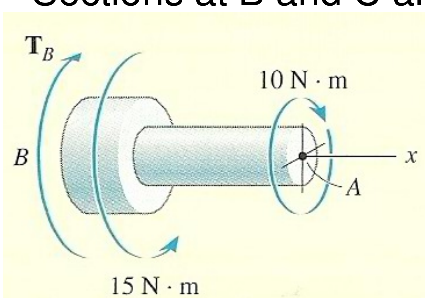


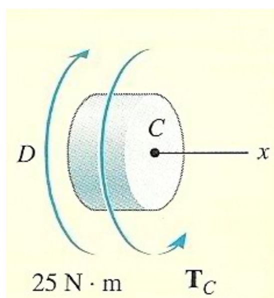
Solution: FBD of entire shaft



$$\Sigma M_x = 0; \qquad -10\ \mathrm{N\cdot m} + 15\ \mathrm{N\cdot m} + 20\ \mathrm{N\cdot m} - T_D = 0$$
$$T_D = 25\ \mathrm{N\cdot m}$$

Sections at B and C and FBDs of shaft segments AB and CD





Segment AB

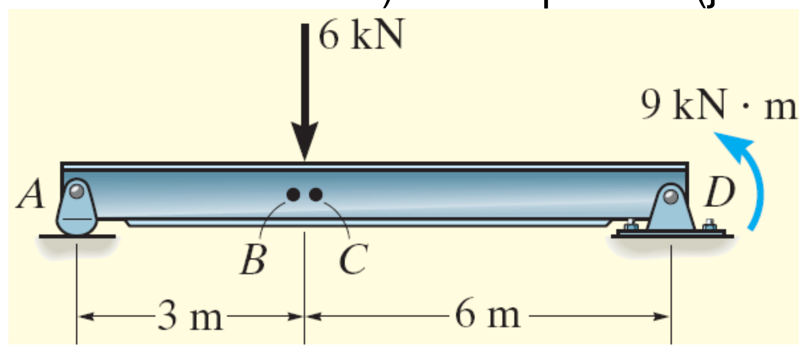
$$\Sigma M_x = 0; \qquad -10\ \mathrm{N\cdot m} + 15\ \mathrm{N\cdot m} - T_B = 0 \qquad T_B = 5\ \mathrm{N\cdot m}$$

Segment CD

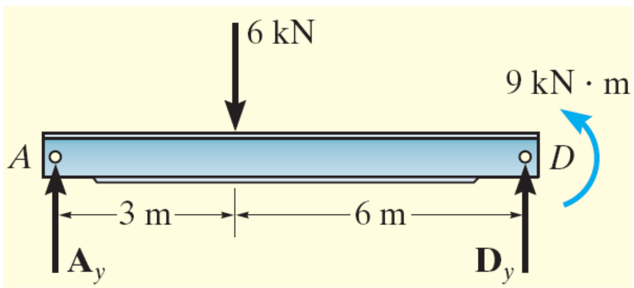
$$\Sigma M_x = 0; \qquad T_C - 25\ \mathrm{N\cdot m} = 0 \qquad T_C = 25\ \mathrm{N\cdot m}$$

Beams – Internal Effects

Example: Find the AF, SF, and BM at point B (just to the left of 6 kN) and at point C (just to the right of 6 kN)



Solution: Draw FBD of entire beam

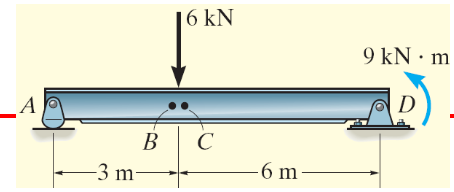


$$\circlearrowleft + \Sigma M_D = 0;\ \ 9\ \text{kN}\cdot\text{m} + (6\ \text{kN})(6\ \text{m}) - A_y(9\ \text{m}) = 0$$

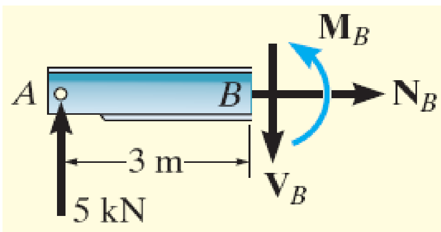
$$A_y = 5\ \text{kN}$$

D_y need not be determined if only left part of the beam is analysed

Beams – Internal Effects



Example Solution: Draw FBD of segments AB and AC and use equilibrium equations

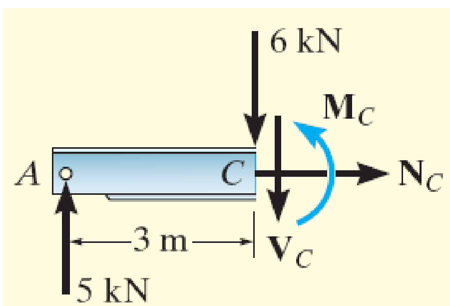


Segment AB

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N_B = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad 5\text{ kN} - V_B = 0 \qquad V_B = 5\text{ kN}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -(5\text{ kN})(3\text{ m}) + M_B = 0 \qquad M_B = 15\text{ kN}\cdot\text{m}$$



Segment AC

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N_C = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad 5\text{ kN} - 6\text{ kN} - V_C = 0 \qquad V_C = -1\text{ kN}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -(5\text{ kN})(3\text{ m}) + M_C = 0 \qquad M_C = 15\text{ kN}\cdot\text{m}$$

Beams – SFD and BMD

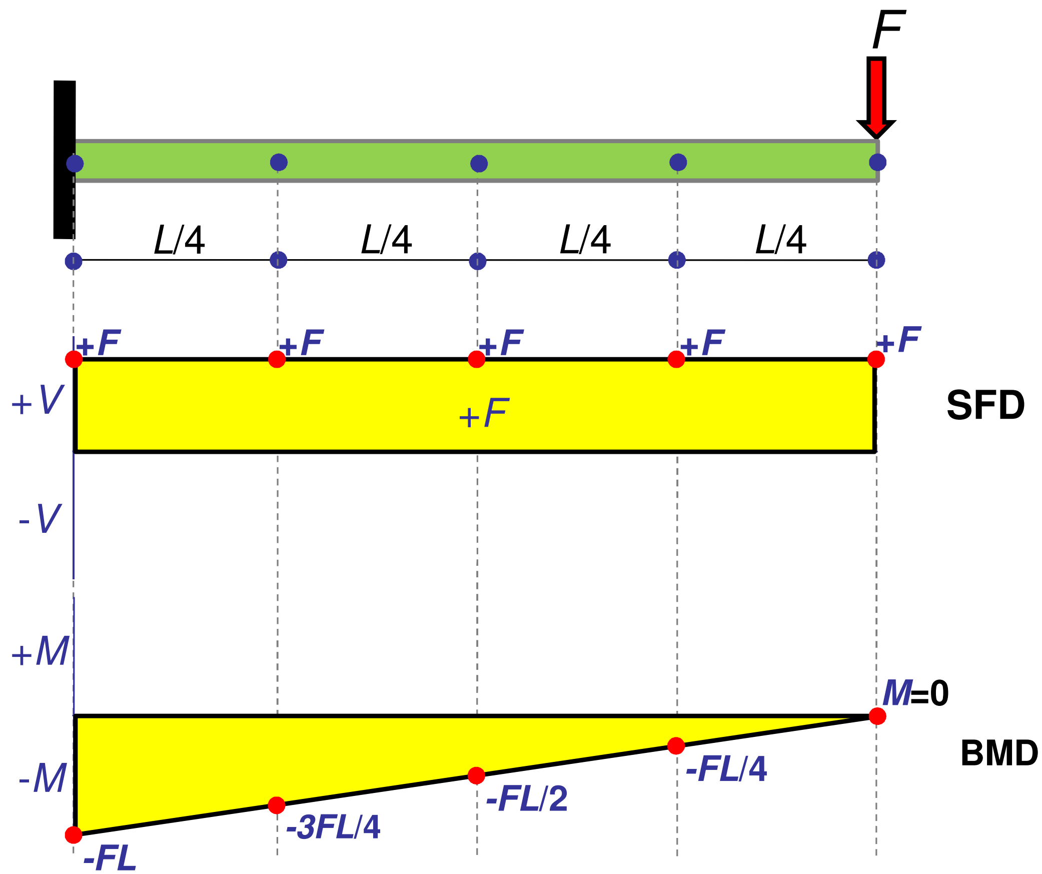
Shear Force Diagram and bending Moment Diagram

- Variation of SF and BM over the length of the beam → SFD and BMD
- Maximum magnitude of BM and SF and their locations is prime consideration in beam design
- SFDs and BMDs are plotted using method of section
  - Equilibrium of FBD of entire Beam
    → External Reactions
  - Equilibrium of a cut part of beam
    → Expressions for SF and BM at the cut section

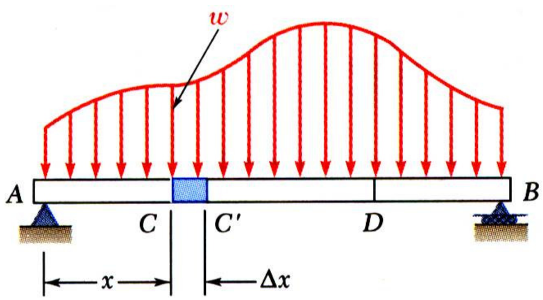
  Use the positive sign convention consistently

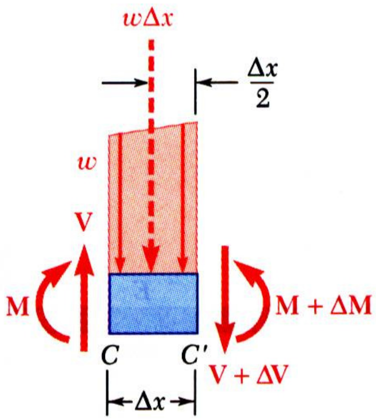
Beams – SFD and BMD

Draw SFD and BMD for a cantilever beam supporting a point load at the free end



Relations Among Load, Shear, and Bending Moment





- Relations between load and shear:

$$V - (V + \Delta V) - w\Delta x = 0$$

$$\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -w$$

$$\boxed{w = -\frac{dV}{dx}}$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\, dx = -(\text{area under load curve})$$

- Relations between shear and bending moment:

$$(M + \Delta M) - M - V\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\frac{dM}{dx} = \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \lim_{\Delta x \to 0} \left(V - \tfrac{1}{2} w\Delta x\right) = V$$

$$\boxed{V = \frac{dM}{dx}}$$

$$M_D - M_C = \int_{x_C}^{x_D} V\, dx = (\text{area under shear curve})$$

Beams – SFD and BMD

Shear and Moment Relationships

$$w = -\frac{dV}{dx}$$

Slope of the shear diagram = - Value of applied loading

$$V = \frac{dM}{dx}$$

Slope of the moment curve = Shear Force

Both equations not applicable at the point of loading because of discontinuity produced by the abrupt change in shear.

Beams – SFD and BMD

Shear and Moment Relationships

Expressing V in terms of w by integrating

$$w = -\frac{dV}{dx}$$

$$\int_{V_0}^{V} dV = -\int_{x_0}^{x} w\,dx$$

$V = V_0$ + (the negative of the area under the loading curve from x_0 to x)

V_0 is the shear force at x_0 and V is the shear force at x

Expressing M in terms of V by integrating

$$V = \frac{dM}{dx}$$

$$\int_{M_0}^{M} dM = \int_{x_0}^{x} V\,dx$$

$M = M_0$ + (area under the shear diagram from x_0 to x)

M_0 is the BM at x_0 and M is the BM at x

Beams – SFD and BMD

$w = -\frac{dV}{dx}$ Degree of *V* in *x* is one higher than that of *w*

$V = \frac{dM}{dx}$ Degree of *M* in *x* is one higher than that of *V*

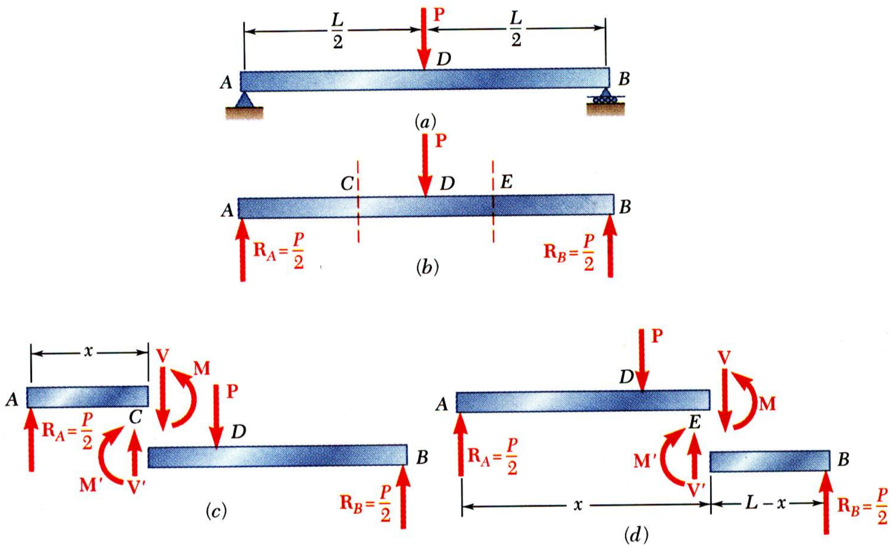
→ Degree of *M* in *x* is two higher than that of *w*

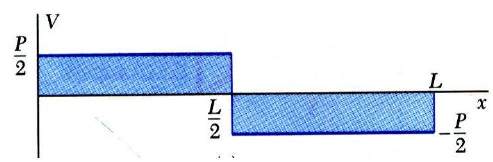
Combining the two equations → $$\boxed{\frac{d^2M}{dx^2} = -w}$$

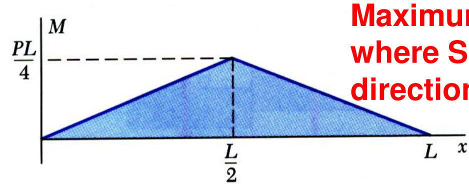
→ If *w* is a known function of *x*, BM can be obtained by integrating this equation twice with proper limits of integration.

- → Method is usable only if *w* is a continuous function of *x* (other cases not part of this course)

Beams – SFD and BMD: Example







Maximum BM occurs where Shear changes the direction

- Draw the SFD and BMD.

- Determine reactions at supports.

- Cut beam at *C* and consider member *AC*,

$$V = +P/2 \quad M = +Px/2$$

- Cut beam at *E* and consider member *EB*,

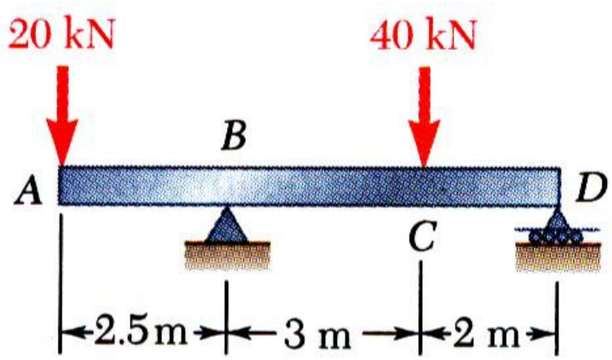
$$V = -P/2 \quad M = +P(L-x)/2$$

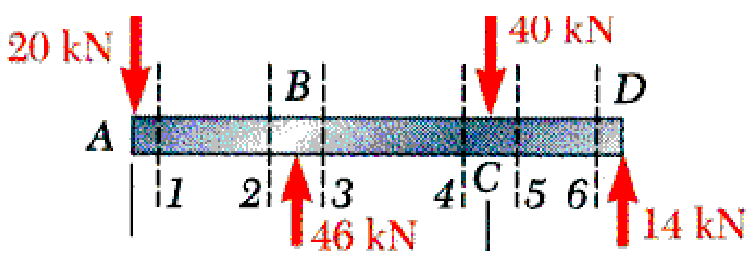
- For a beam subjected to <u>concentrated loads</u>, shear is constant between loading points and moment varies linearly.

Beams – SFD and BMD: Example

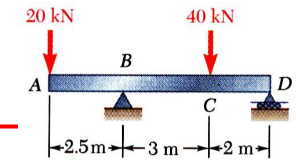
Draw the shear and bending moment diagrams for the beam and loading shown.

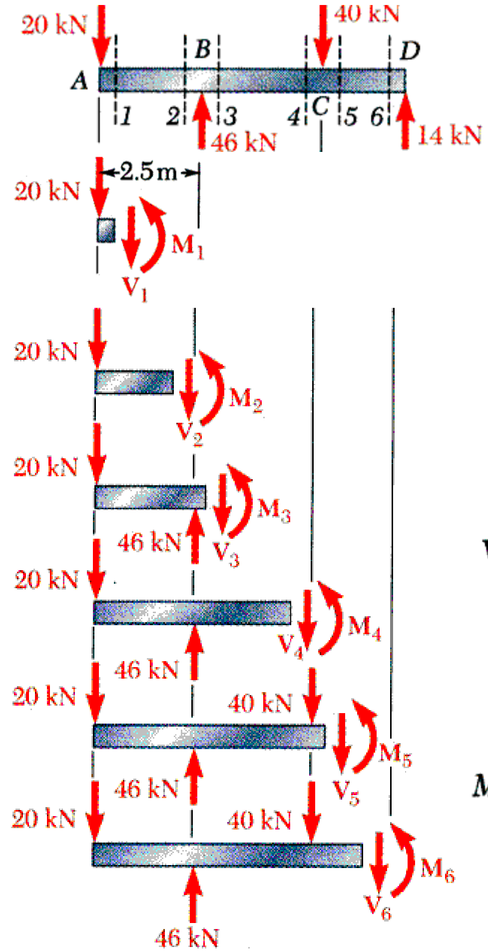
Solution: Draw FBD and find out the support reactions using equilibrium equations





Beams – SFD and BMD: Example





Use equilibrium conditions at all sections to get the unknown SF and BM

$$\sum F_y = 0: \quad -20\,\mathrm{kN} - V_1 = 0 \quad \boxed{V_1 = -20\,\mathrm{kN}}$$

$$\sum M_1 = 0: \quad (20\,\mathrm{kN})(0\,\mathrm{m}) + M_1 = 0 \quad \boxed{M_1 = 0}$$

V_2 = -20 kN; M_2 = -20x kNm

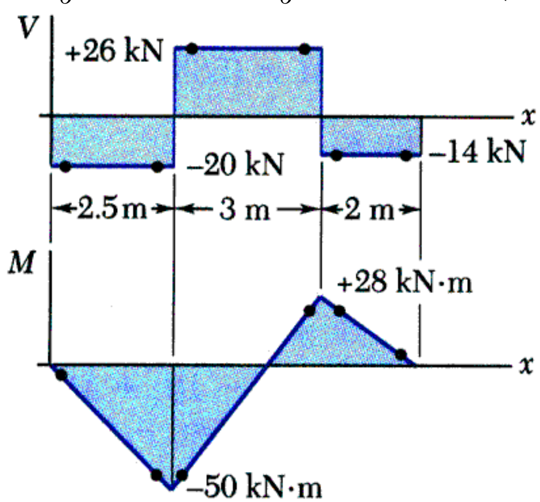
V_3 = +26 kN; M_3 = -20x+46×0 = -20x → M_B= -50 kNm

V_4 = +26 kN; M_4 = -20x+46×(x-2.5) → M_C= +28 kNm

V_5 = -14 kN; M_5 = -20x+46×(x-2.5)-40×0 → M_C= +28 kNm

V_6 = -14 kN; M_6 = -20x+46×(x-2.5)-40×(x-5.5)

→ M_D= 0 kNm



Alternatively, from RHS

V_5 = V_6 = -14 kN

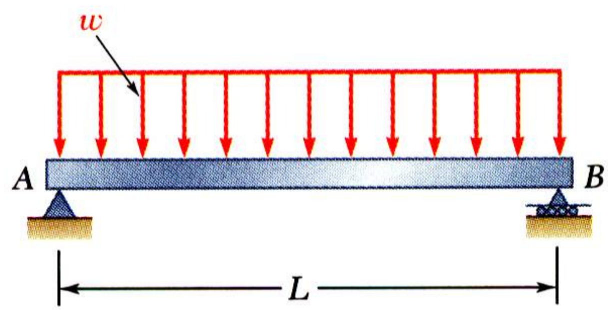
M_5 = +14x → M_C= +28 kNm

M_6 = +14×0 → M_D= 0 kNm

Important: BM≠0 at Hinged Support B. Why?

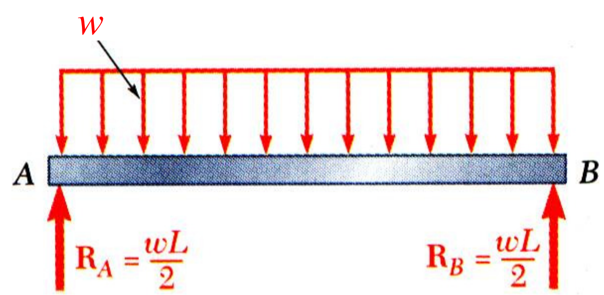
Beams – SFD and BMD: Example

Draw the SFD and BMD for the beam



Solution:

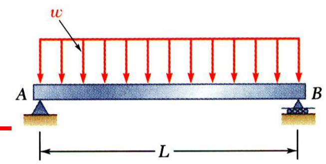
Draw FBD of the entire beam and calculate support reactions using equilibrium equations



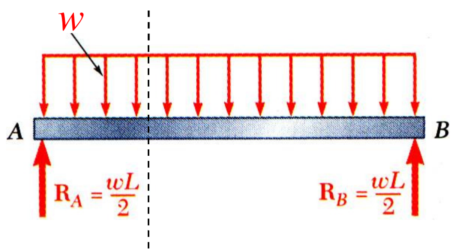
Reactions at supports: $R_A = R_B = \dfrac{wL}{2}$

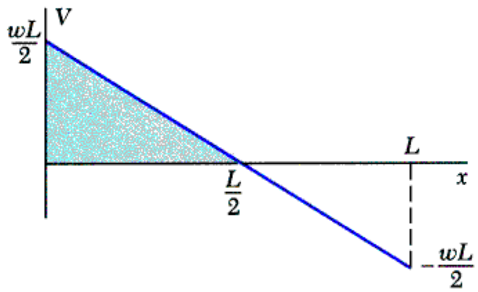
Develop the relations between loading, shear force, and bending moment and plot the SFD and BMD

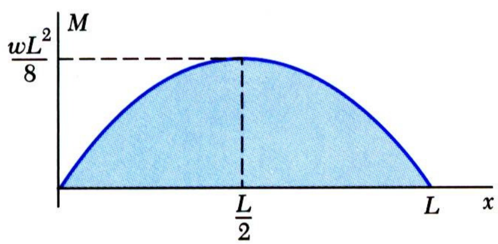
Beams – SFD and BMD: Example



Shear Force at any section:







$$V = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

Alternatively, $V - V_A = -\int_0^x w\,dx = -wx$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

BM at any section: $M = \frac{wL}{2}x - wx\frac{x}{2} = \frac{w}{2}\left(Lx - x^2\right)$

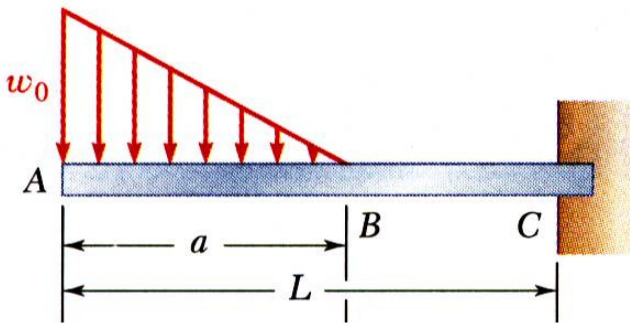
Alternatively, $M - M_A = \int_0^x V dx$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \because \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Beams – SFD and BMD: Example

Draw the SFD and BMD for the Beam

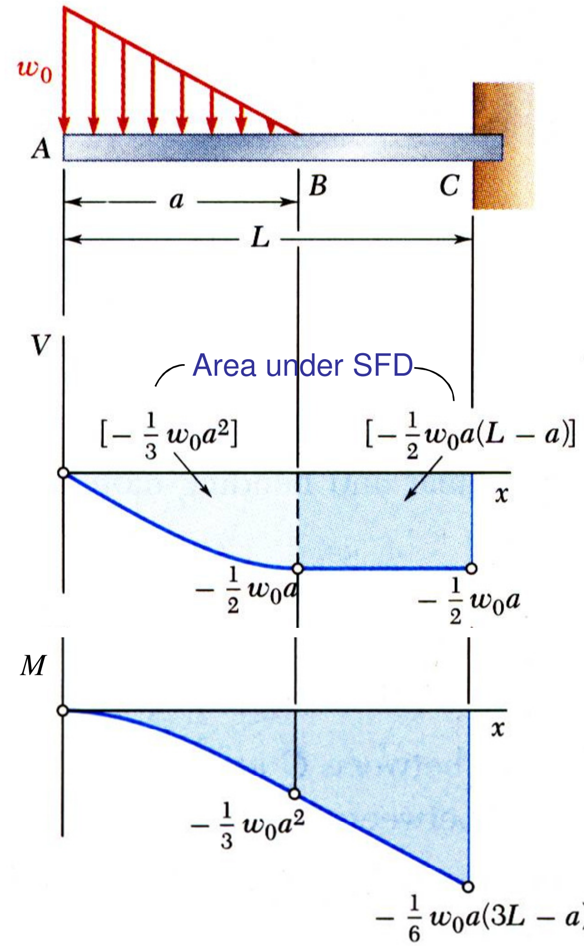


Solution:

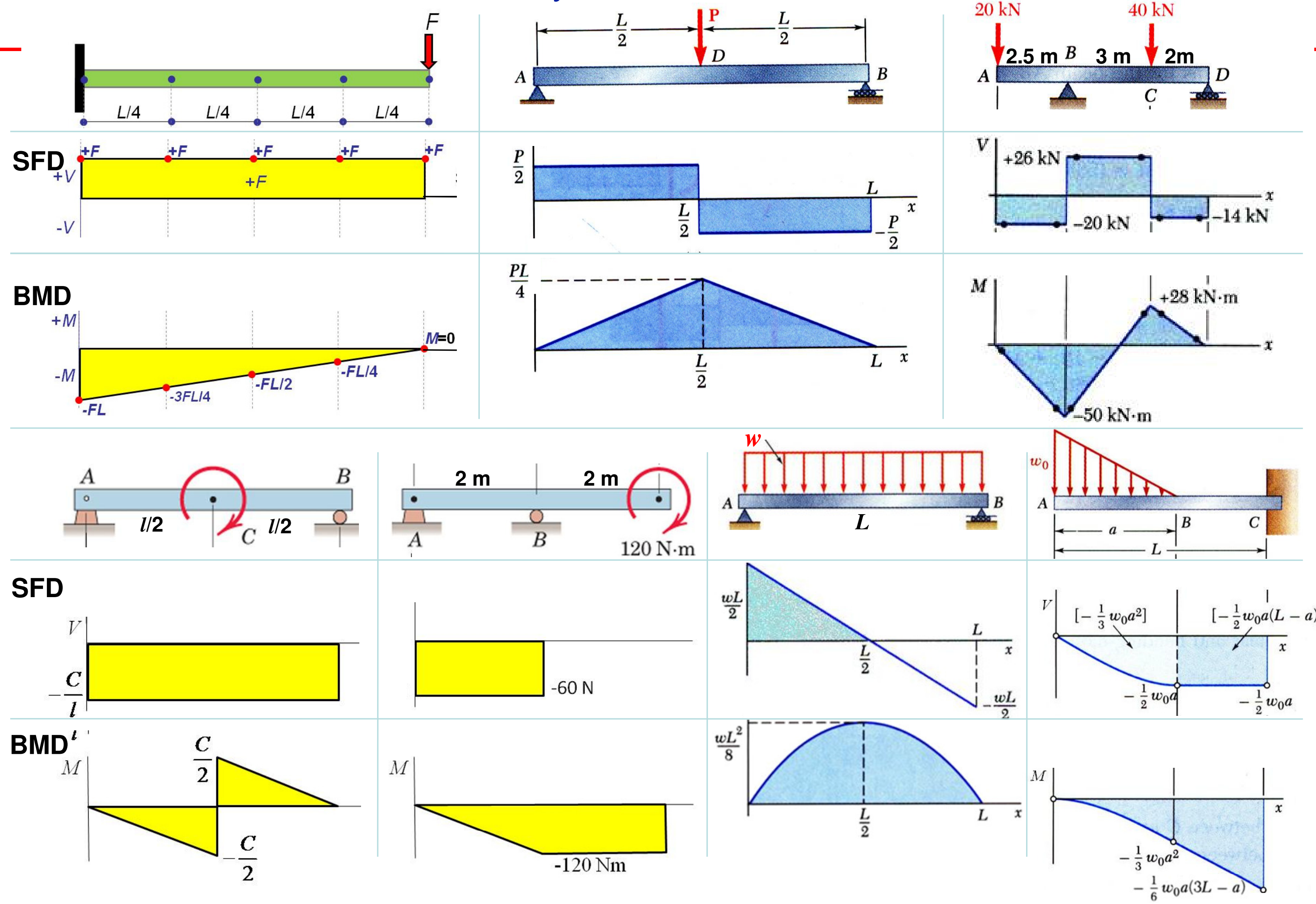
SFD and BMD can be plotted without determining support reactions since it is a cantilever beam.

However, values of SF and BM can be verified at the support if support reactions are known.

$$R_C = \frac{w_0 a}{2}\uparrow; \quad M_C = \frac{w_0 a}{2}\left(L - \frac{a}{3}\right) = \frac{w_0 a}{6}(3L - a)$$



Beams – SFD and BMD: Summary



Cables

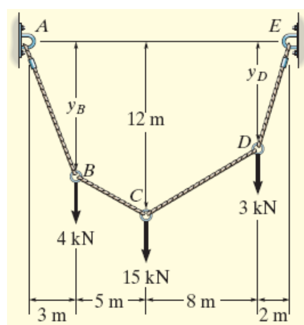
Flexible and Inextensible Cables

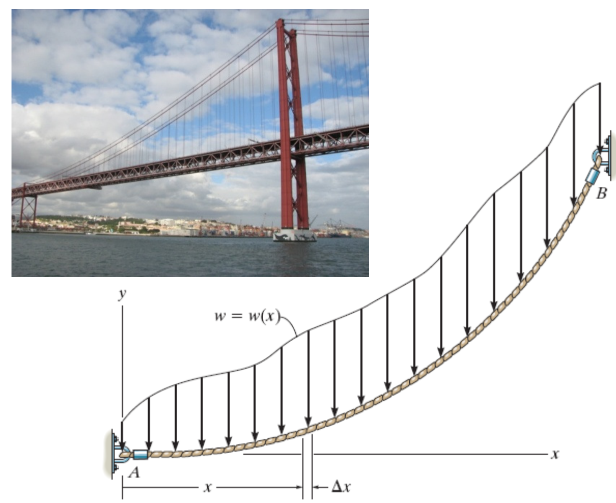
Important Design Parameters

Tension
Span
Sag
Length

Cables

- Relations involving Tension, Span, Sag, and Length are reqd
  - Obtained by examining the cable as a body in equilibrium
- It is assumed that any resistance offered to bending is negligible → Force in cable is always along the direction of the cable.
- Flexible cables may be subjected to concentrated loads or distributed loads





Cables

- In some cases, weight of the cable is negligible compared with the loads it supports.
- In other cases, weight of the cable may be significant or may be the only load acting → weight cannot be neglected.

Three primary cases of analysis: Cables subjected to

1. concentrated load, **2.** distributed load, **3.** self weight

→ Requirements for equilibrium are formulated in identical way **provided Loading is coplanar with the cable**

Cables

Primary Assumption in Analysis:

The cable is perfectly Flexible and Inextensible

Flexible

→ cable offers no resistance to bending

→ tensile force acting in the cable is always tangent to the cable at points along its length

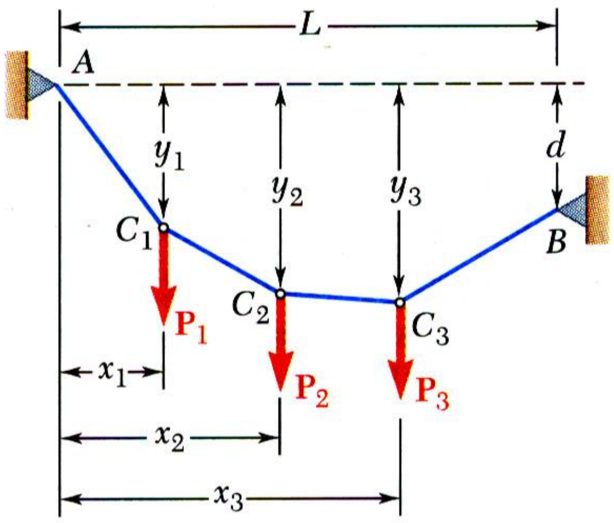
Inextensible

→ cable has a constant length both before and after the load is applied

→ once the load is applied, geometry of the cable remains fixed

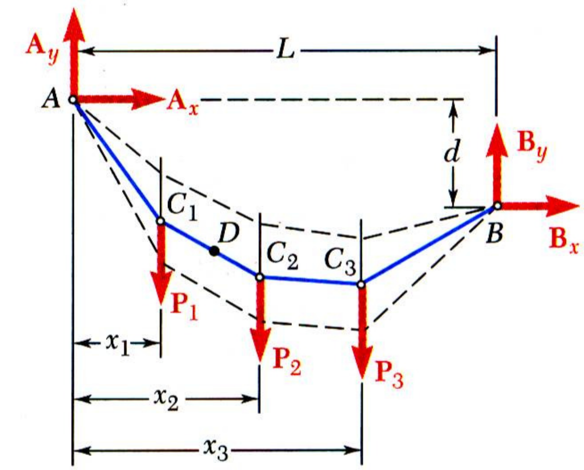
→ cable or a segment of it can be treated as a rigid body

Cables With Concentrated Loads

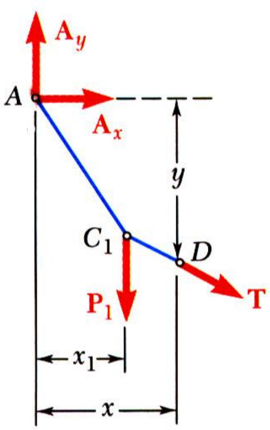


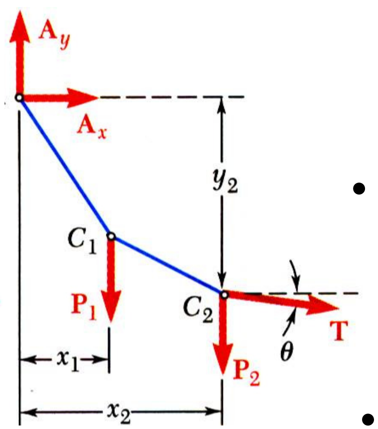
- Cables are applied as structural elements in suspension bridges, transmission lines, aerial tramways, guy wires for high towers, etc.
- For analysis, assume:
  a) concentrated vertical loads on given vertical lines,
  b) weight of cable is negligible,
  c) cable is flexible, i.e., resistance to bending is small,
  d) portions of cable between successive loads may be treated as two force members
- Wish to determine shape of cable, i.e., vertical distance from support *A* to each load point.

Cables With Concentrated Loads



- Consider entire cable as free-body. Slopes of cable at A and B are not known - two reaction components required at each support.
- Four unknowns are involved and three equations of equilibrium are not sufficient to determine the reactions.



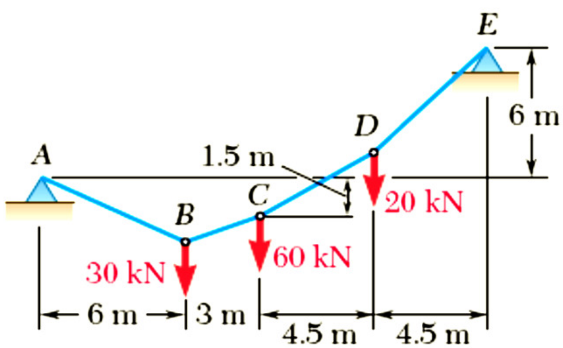


- Additional equation is obtained by considering equilibrium of portion of cable AD and assuming that coordinates of point D on the cable are known. The additional equation is $\sum M_D = 0$.
- For other points on cable,

  $\sum M_{C_2} = 0 \quad$ yields y_2

  $\sum F_x = 0, \sum F_y = 0$ yield T_x, T_y
- $T_x = T\cos\theta = A_x = \text{constant}$

Example



The cable *AE* supports three vertical loads from the points indicated. If point *C* is 1.5 m below the left support, determine *(a)* the elevation of points *B* and *D*, and *(b)* the maximum slope and maximum tension in the cable.

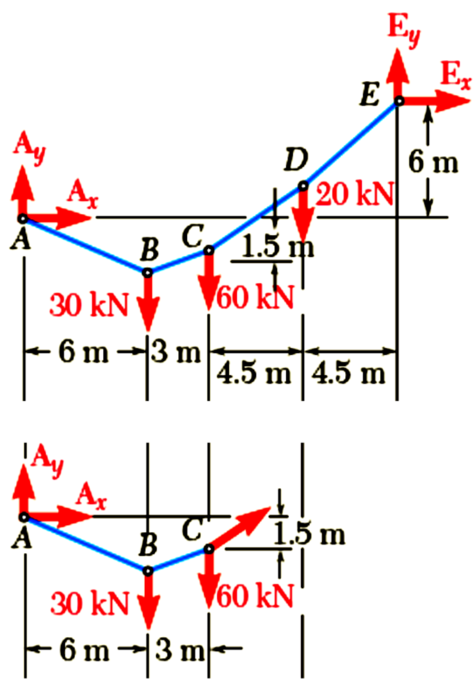
SOLUTION:

- Determine reaction force components at *A* from solution of two equations formed from taking entire cable as free-body and summing moments about *E*, and from taking cable portion *ABC* as a free-body and summing moments about *C*.

- Calculate elevation of *B* by considering *AB* as a free-body and summing moments *B*. Similarly, calculate elevation of *D* using *ABCD* as a free-body.

- Evaluate maximum slope and maximum tension which occur in *DE*.

Example



SOLUTION:

- Determine two reaction force components at *A* from solution of two equations formed from taking entire cable as a free-body and summing moments about *E*,

$$\sum M_E = 0:$$

$$6A_x - 18A_y + 12(30) + 9(60) + 4.5(20) = 0$$

$$6A_x - 18A_y + 990 = 0$$

and from taking cable portion *ABC* as a free-body and summing moments about *C*.

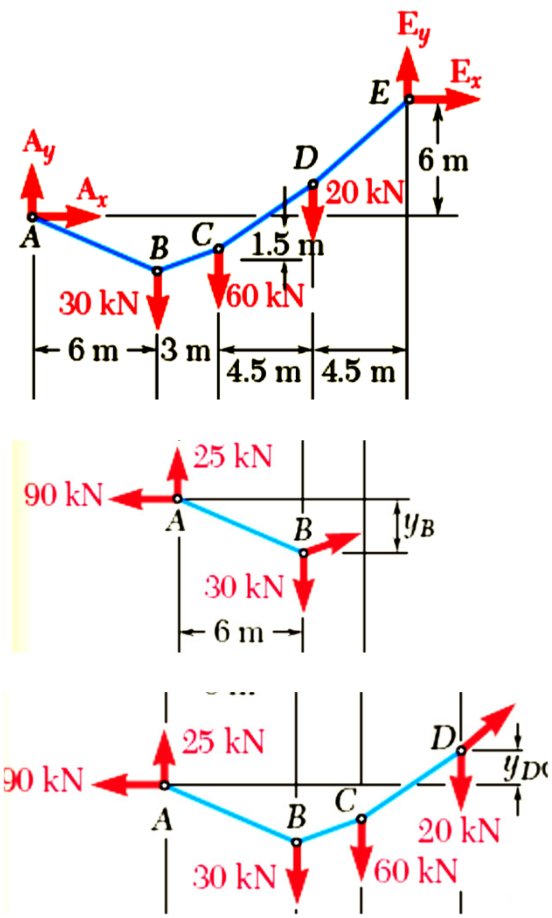
$$\sum M_C = 0:$$

$$-1.5A_x - 9A_y + 3(30) = 0$$

Solving simultaneously,

$$A_x = -90\,\text{kN} \qquad A_y = +25\,\text{kN}$$

Example



- Calculate elevation of *B* by considering *AB* as a free-body and summing moments *B*.

$$\sum M_B = 0: \qquad y_B(90) - 25(6) = 0$$

$$\boxed{y_B = -1.67\ \text{m}}$$

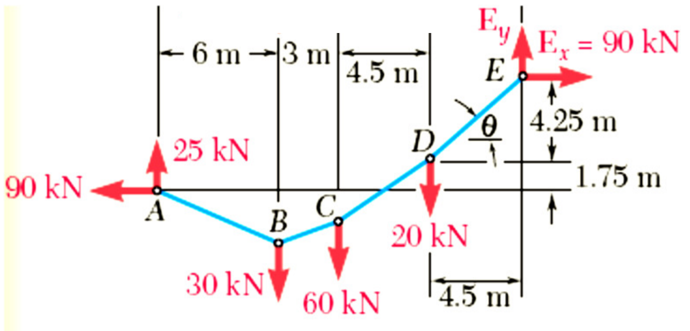
Similarly, calculate elevation of *D* using *ABCD* as a free-body.

$$\sum M = 0:$$

$$-y_D(90) - 13.5(25) + 7.5(30) + 4.5(60) = 0$$

$$\boxed{y_D = 1.75\ \text{m}}$$

Example

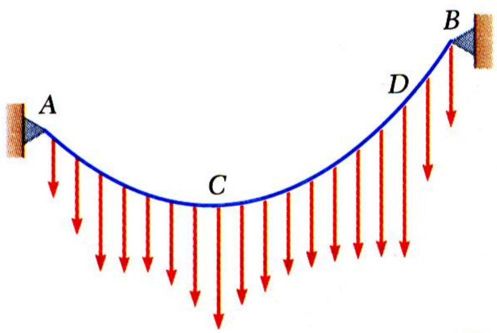


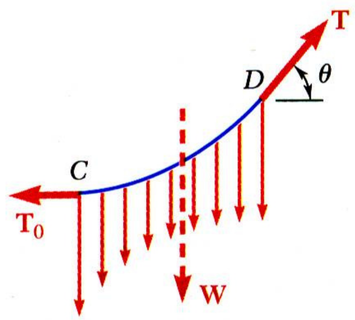
- Evaluate maximum slope and maximum tension which occur in *DE*.

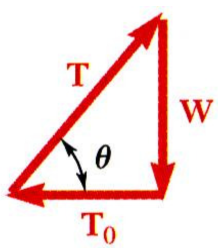
$$\tan\theta = \frac{4.25}{4.5} \qquad \boxed{\theta = 43.4°}$$

$$T_{\max} = \frac{90\,\text{kN}}{\cos\theta} \qquad \boxed{T_{\max} = 123.9\,\text{kN}}$$

Cables With Distributed Loads







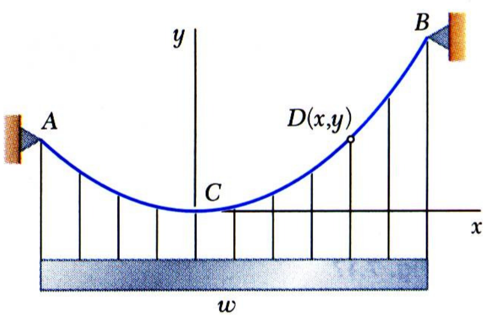
- For cable carrying a distributed load:
  a) cable hangs in shape of a curve
  b) internal force is a tension force directed along tangent to curve.
- Consider free-body for portion of cable extending from lowest point *C* to given point *D*. Forces are horizontal force $\boldsymbol{T_0}$ at C and tangential force $\boldsymbol{T}$ at *D*.
- From force triangle:

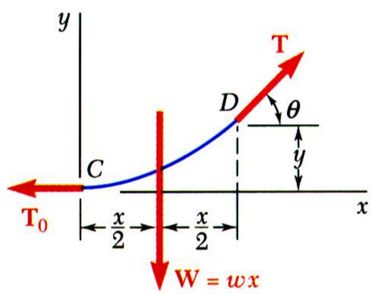
$$T\cos\theta = T_0 \qquad T\sin\theta = W$$

$$T = \sqrt{T_0^2 + W^2} \qquad \tan\theta = \frac{W}{T_0}$$

- Horizontal component of $\boldsymbol{T}$ is uniform over cable.
- Vertical component of $\boldsymbol{T}$ is equal to magnitude of *W* measured from lowest point.
- Tension is minimum at lowest point and maximum at *A* and *B*.

Parabolic Cable





- Consider a cable supporting a uniform, horizontally distributed load, e.g., support cables for a suspension bridge.

- With loading on cable from lowest point C to a point D given by $W = wx$, internal tension force magnitude and direction are

$$T = \sqrt{T_0^2 + w^2 x^2} \qquad \tan\theta = \frac{wx}{T_0}$$

- Summing moments about D,

$$\sum M_D = 0: \qquad wx\frac{x}{2} - T_0 y = 0$$

or

$$y = \frac{wx^2}{2T_0}$$

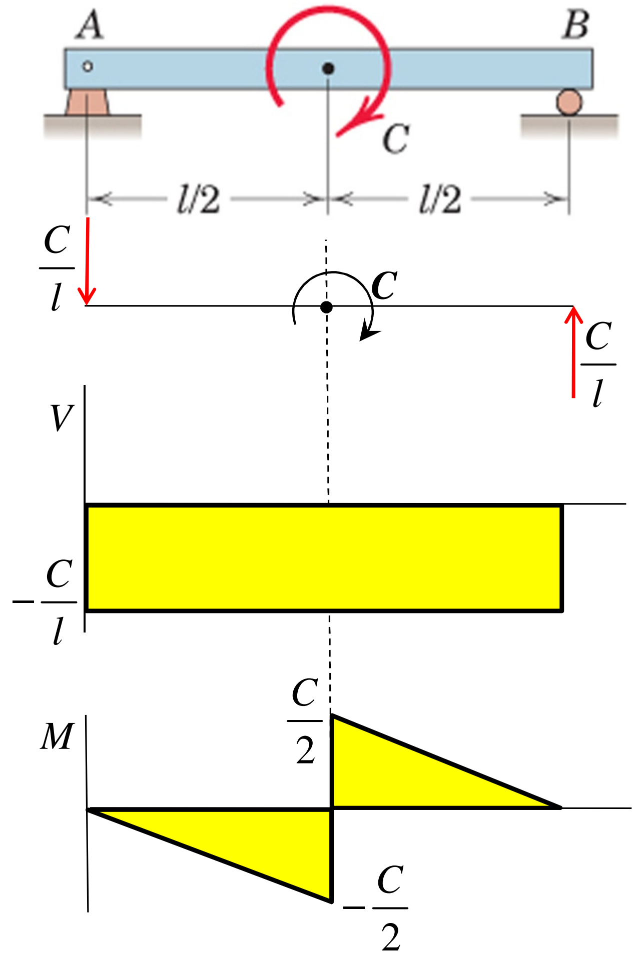
The cable forms a parabolic curve.

Beams – SFD and BMD: Example

Draw the SFD and BMD for the beam acted upon by a clockwise couple at mid point

Solution: Draw FBD of the beam and Calculate the support reactions

Draw the SFD and the BMD starting From any one end



Beams – SFD and BMD: Example

Draw the SFD and BMD for the beam

Solution: Draw FBD of the beam and

Calculate the support reactions

$\sum M_A = 0 \rightarrow R_A = 60$ N↓
$\sum M_B = 0 \rightarrow R_B = 60$ N↑

Draw the SFD and the BMD starting from any one end