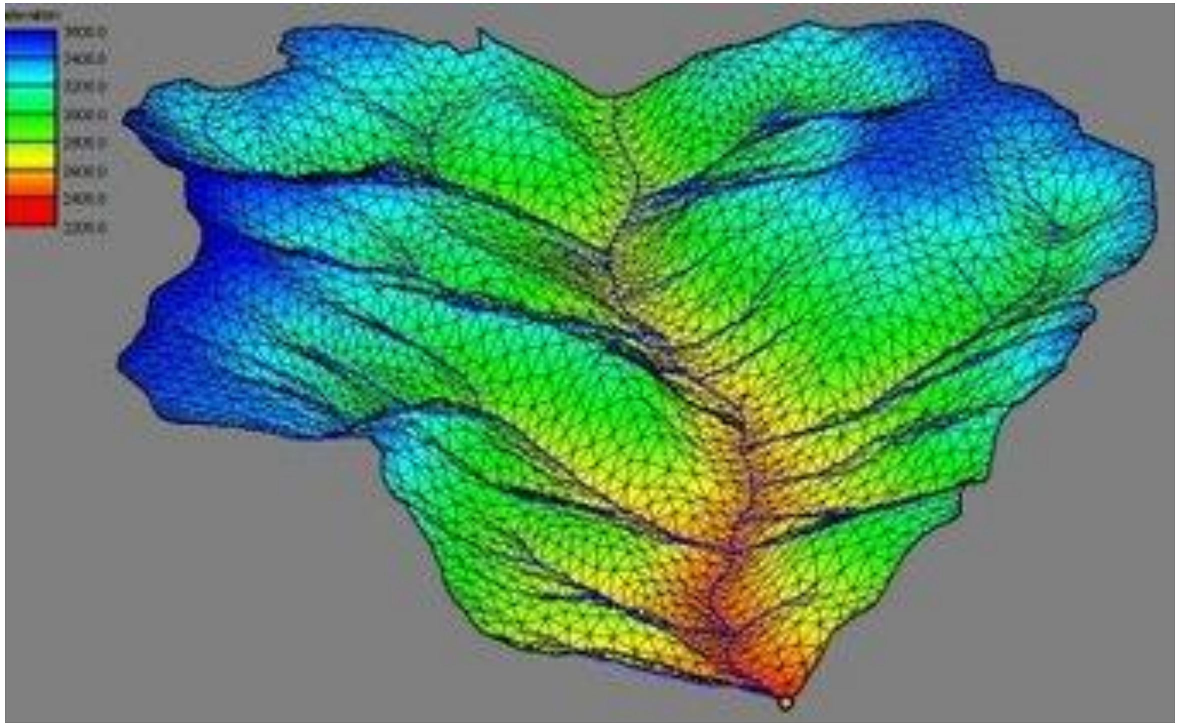
CE 501: Surface Water Hydrology

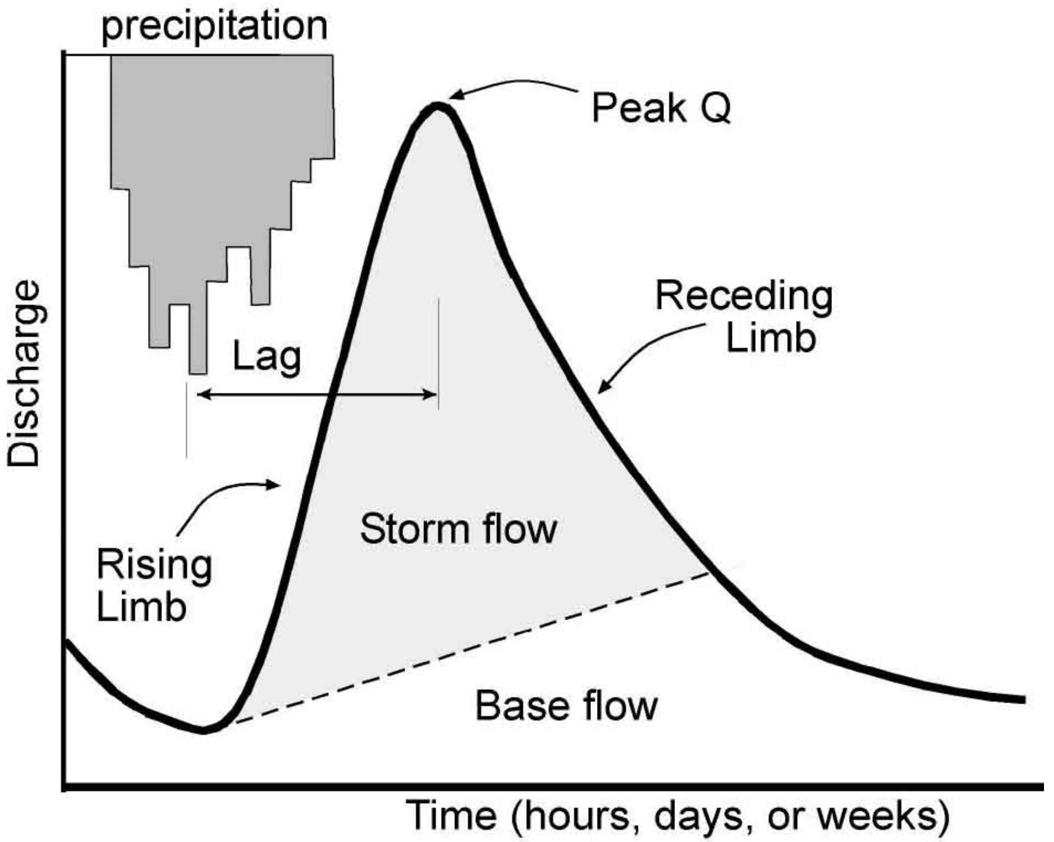
Prof. (Dr.) Rajib Kumar Bhattacharjya

Indian Institute of Technology Guwahati

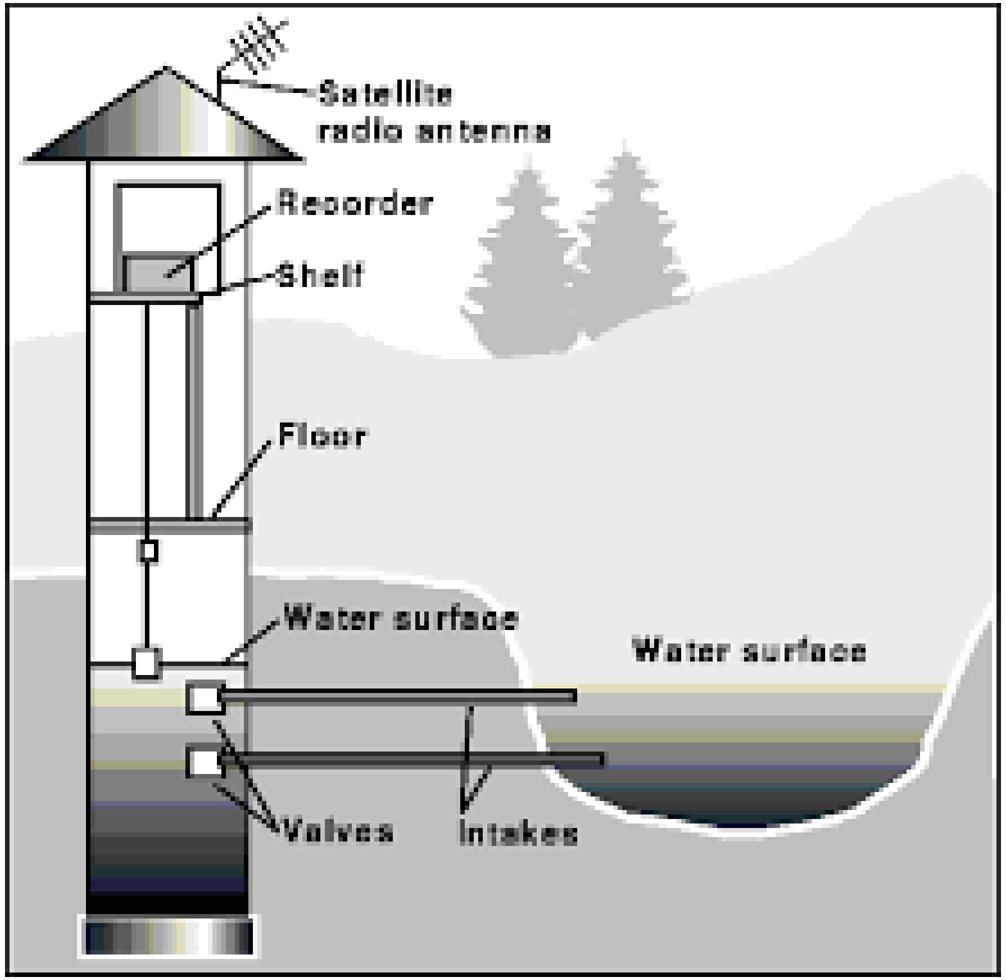
Guwahati, Assam

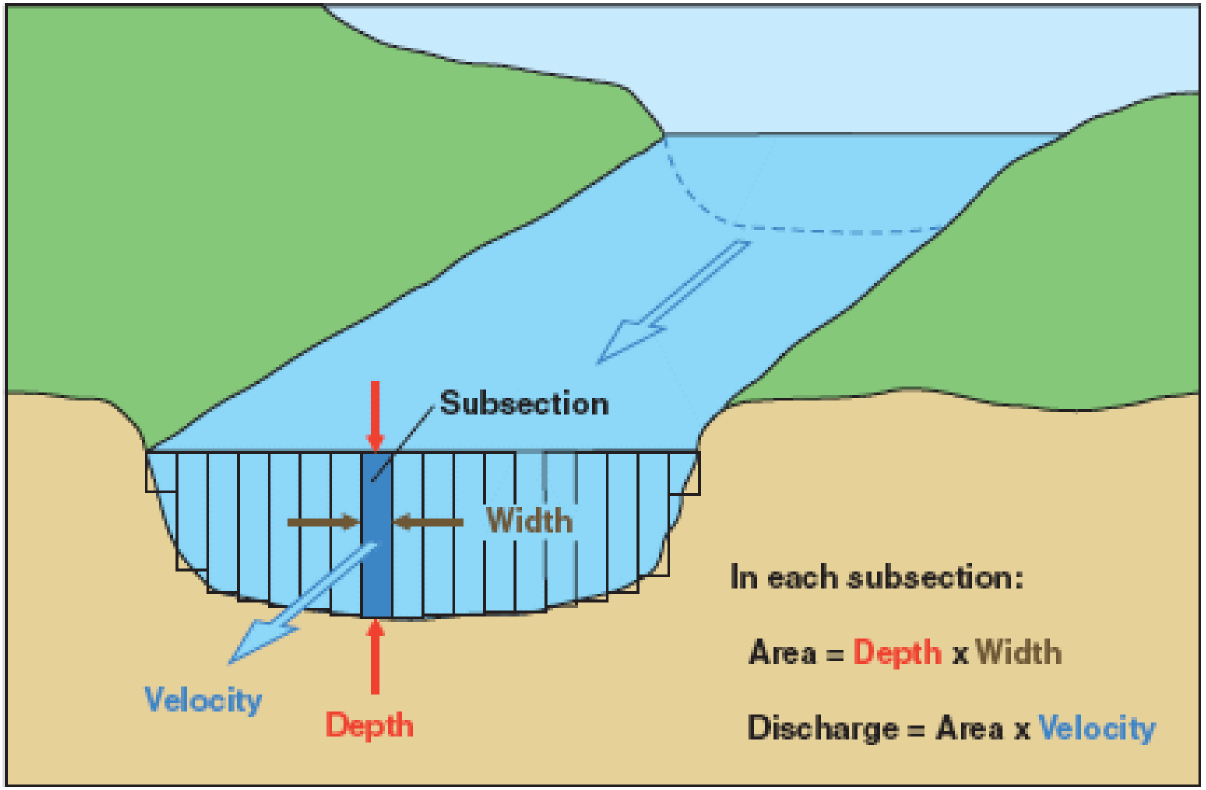
Email: rkbc@iitg.ernet.in Web: www.iitg.ernet.in/rkbc

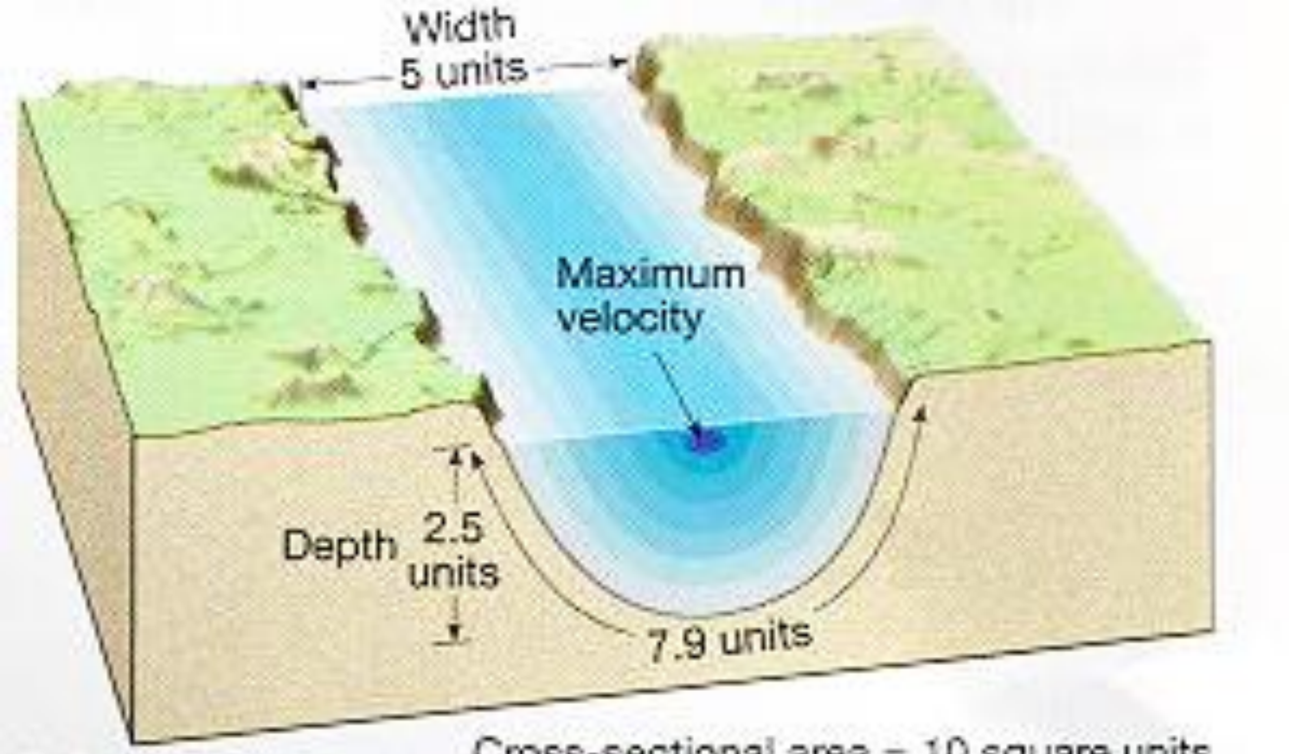



precipitation
Peak Q
Discharge
Lag
Receding
Limb
Storm flow
Rising
Limb
Base flow
Time (hours, days, or weeks)

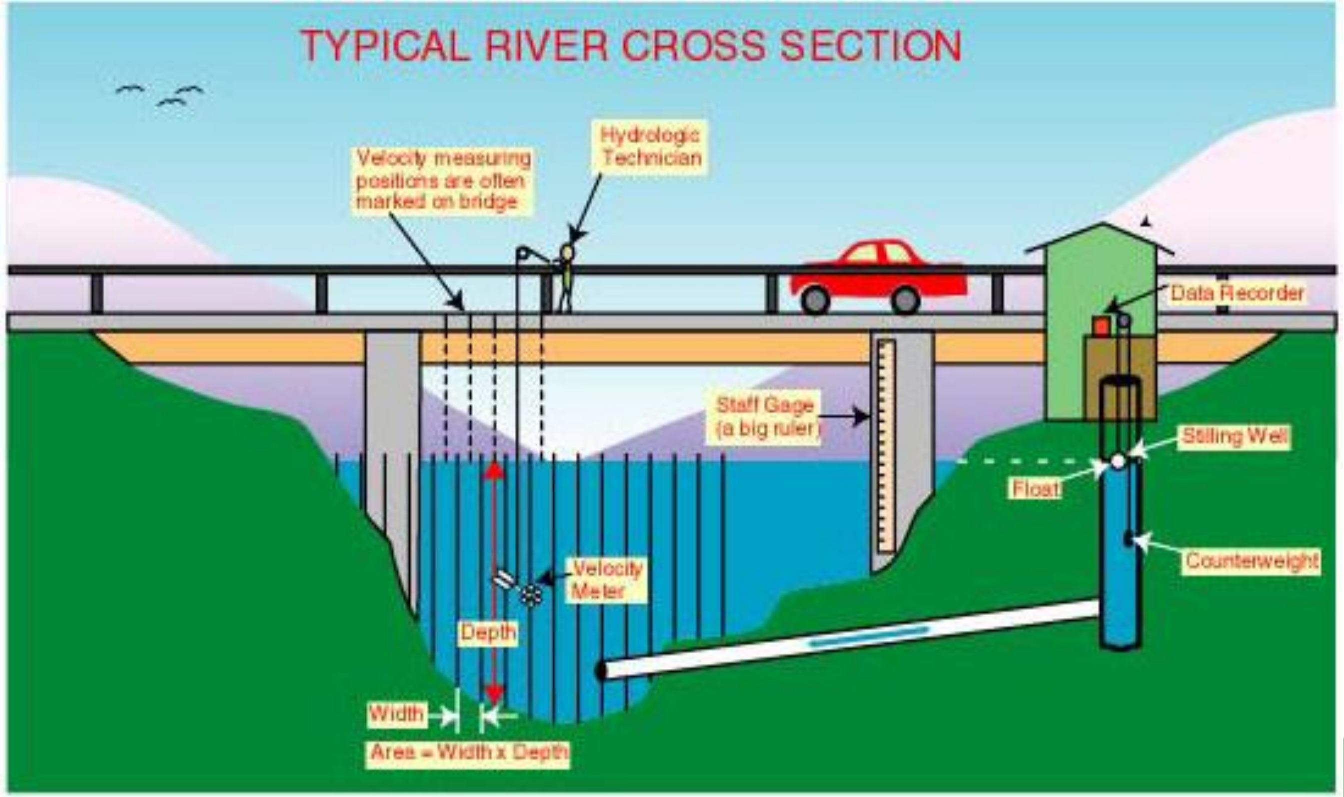
Stream gauging station

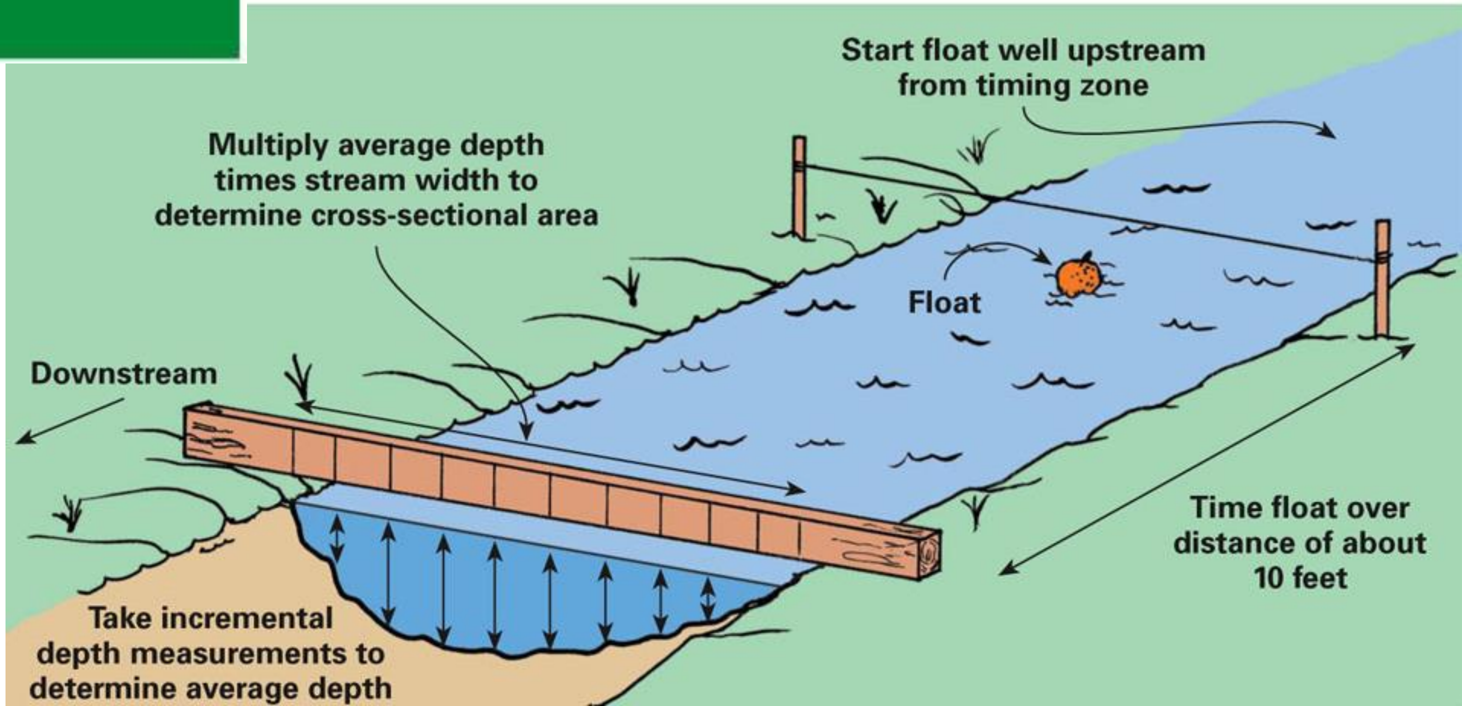


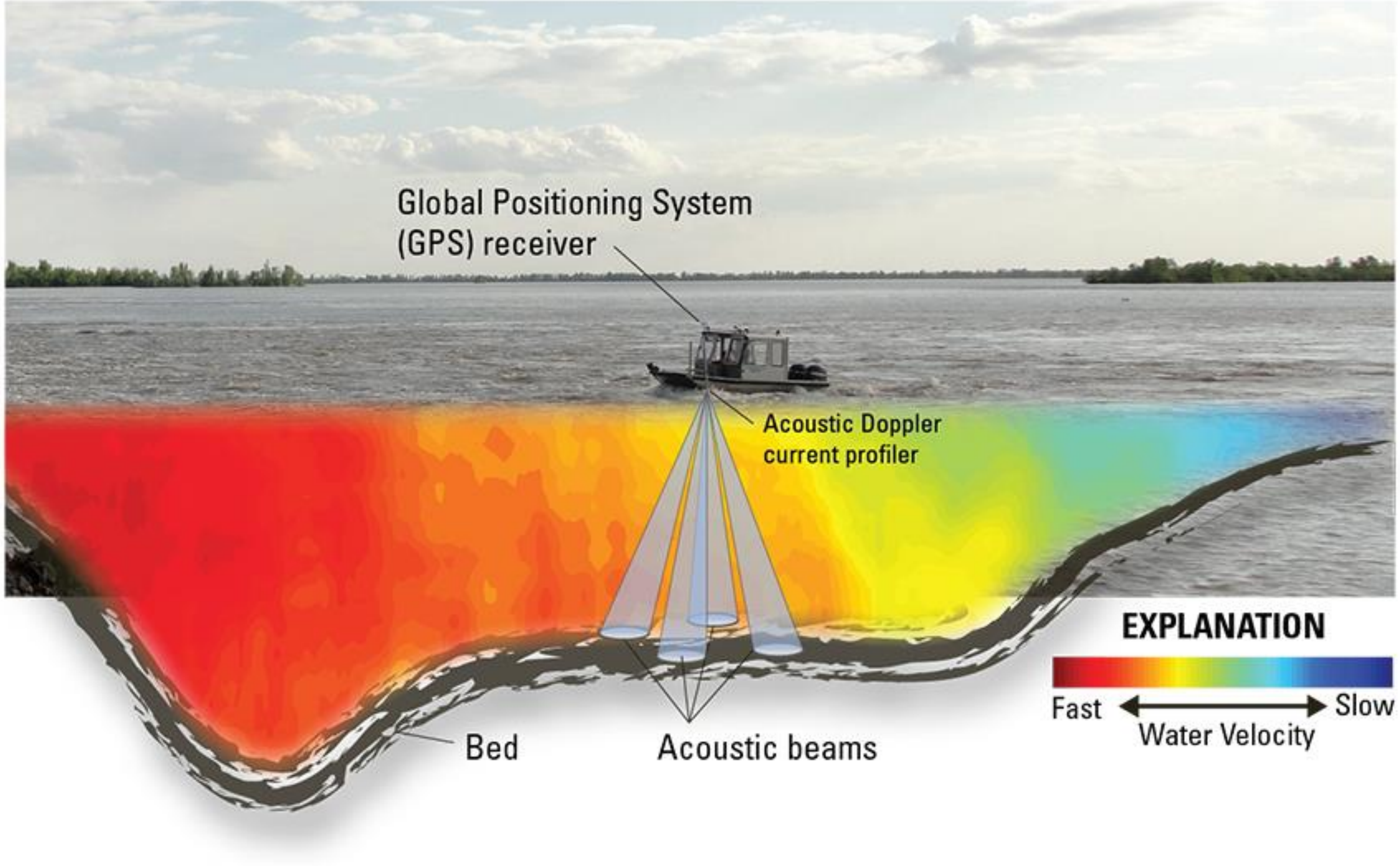


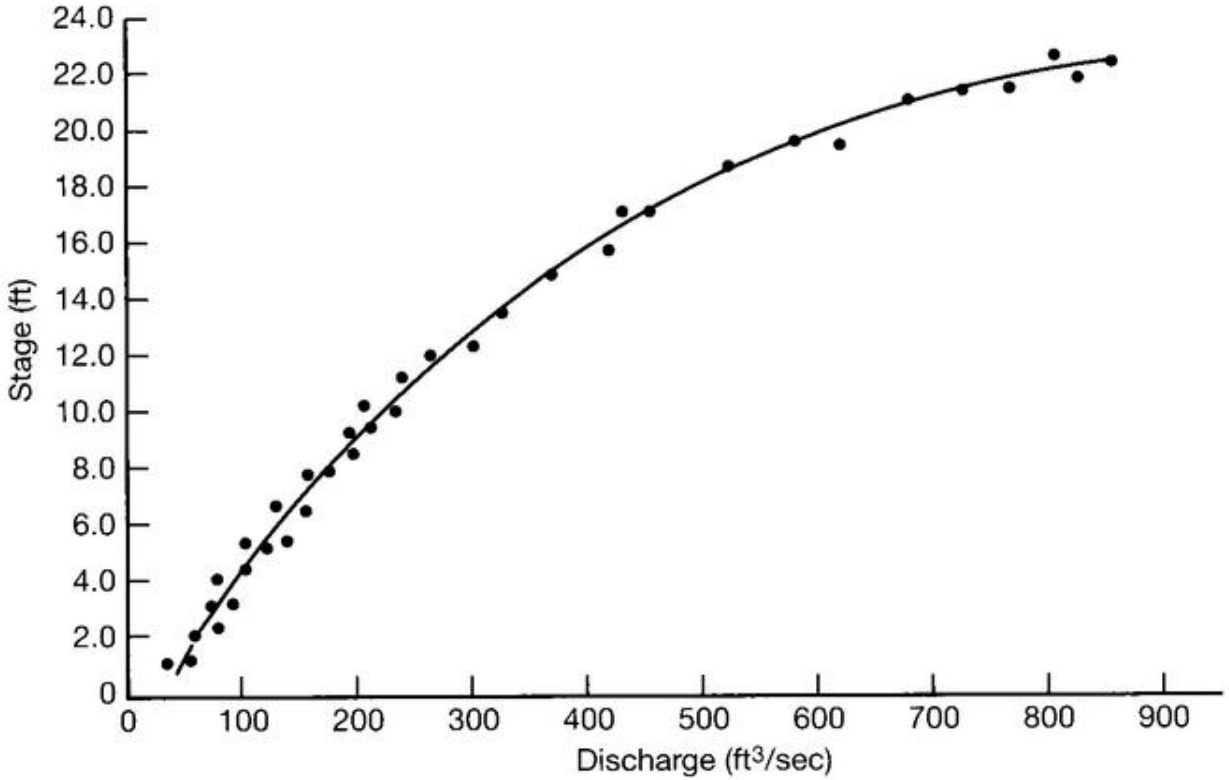


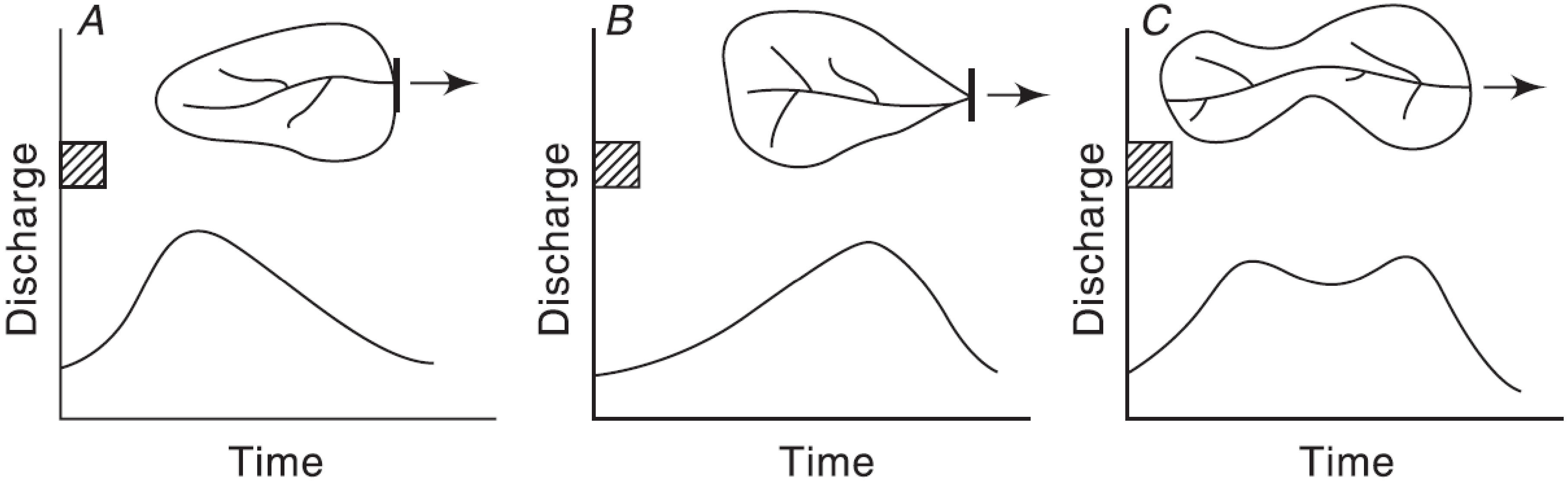
B. Semicircular channel


TYPICAL RIVER CROSS SECTION
Velocity measuring positions are often marked on bridge
Hydrologic Technician
Data Recorder
Staff Gage (a big ruler)
Stilling Well
Float
Counterweight
Velocity Meter
Depth
Width
Area = Width x Depth


Start float well upstream from timing zone
Multiply average depth times stream width to determine cross-sectional area
Float
Downstream
Time float over distance of about 10 feet
Take incremental depth measurements to determine average depth


Global Positioning System
(GPS) receiver
Acoustic Doppler
current profiler
Bed
Acoustic beams
EXPLANATION
Fast
Slow
Water Velocity


Stage (ft)
24.0
22.0
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0
0
100
200
300
400
500
600
700
800
900
Discharge (ft3/sec)


A
Discharge
Time
B
Discharge
Time
C
Discharge
Time

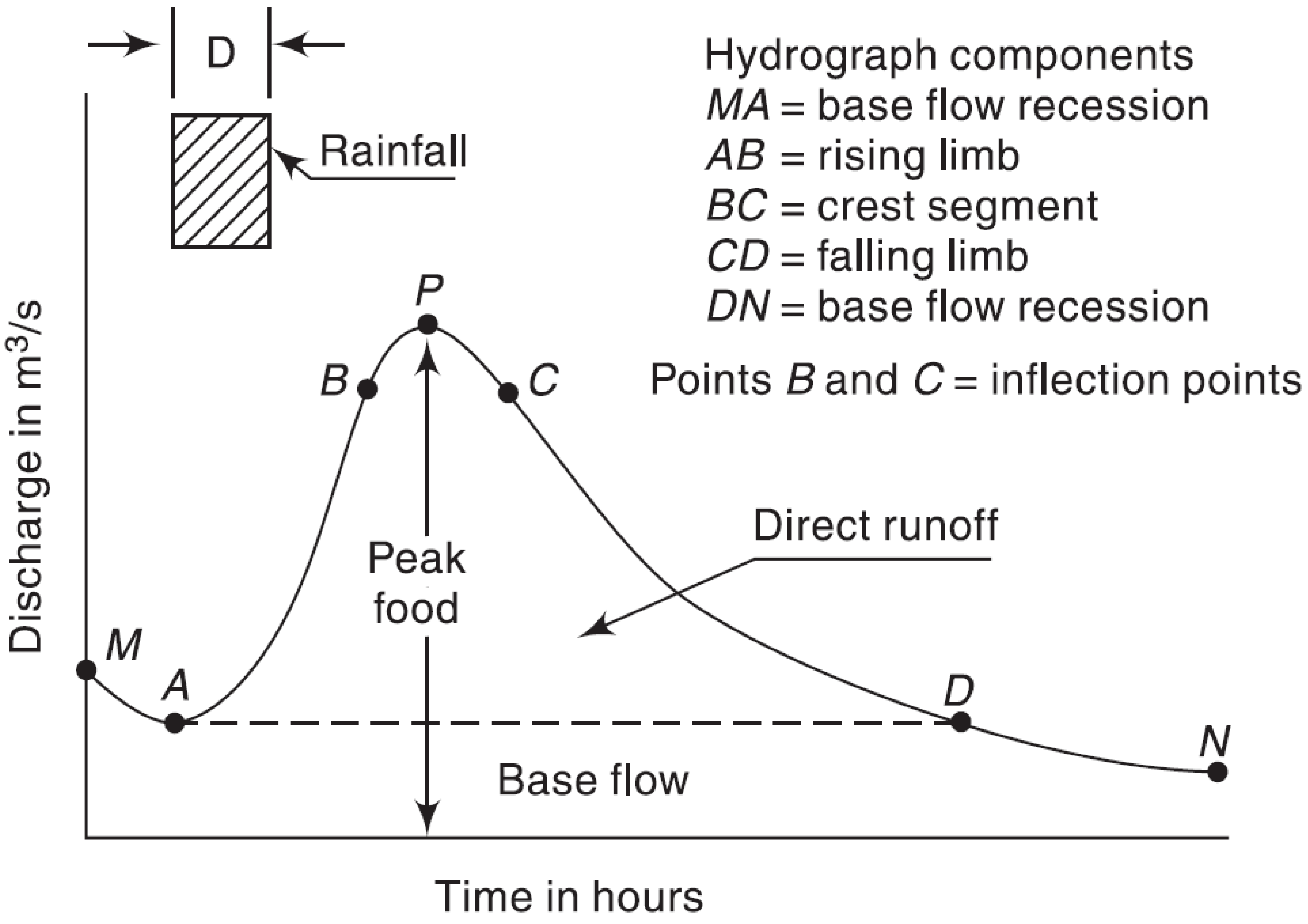


Fig. 6.1 Elements of a Flood Hydrograph

Table 6.1 Factors Affecting Flood Hydrograph

Physiographic factors	Climatic factors
1. Basin characterstics: (a) Shape (b) size (c) slope (d) nature of the valley (e) elevation (f) drainage density 2. Infiltration characteristics: (a) land use and cover (b) soil type and geological conditions (c) lakes, swamps and other storage 3. Channel characteristics: cross-section, roughness and storage capacity	1. Storm characterstics: precipitation, intensity, duration, magnitude and movement of storm. 2. Initial loss 3. Evapotranspiration

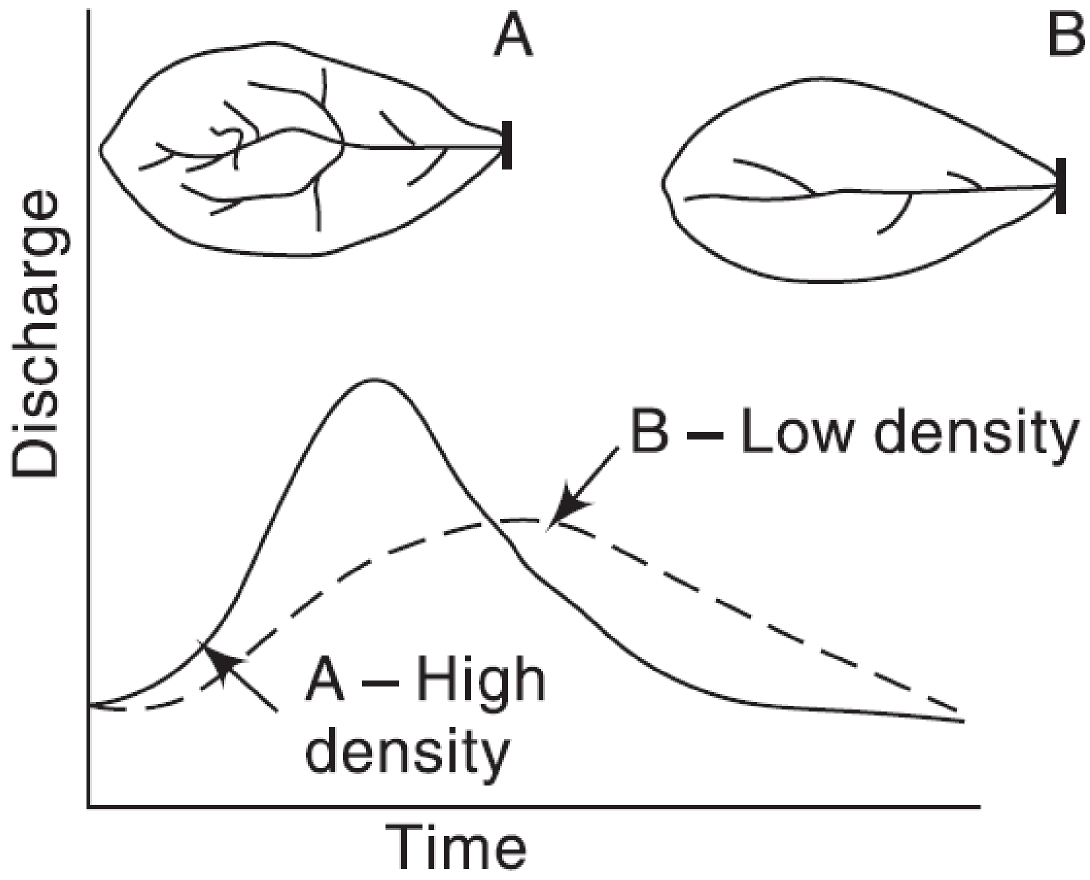


Fig. 6.3 Role of Drainage Density on the Hydrograph

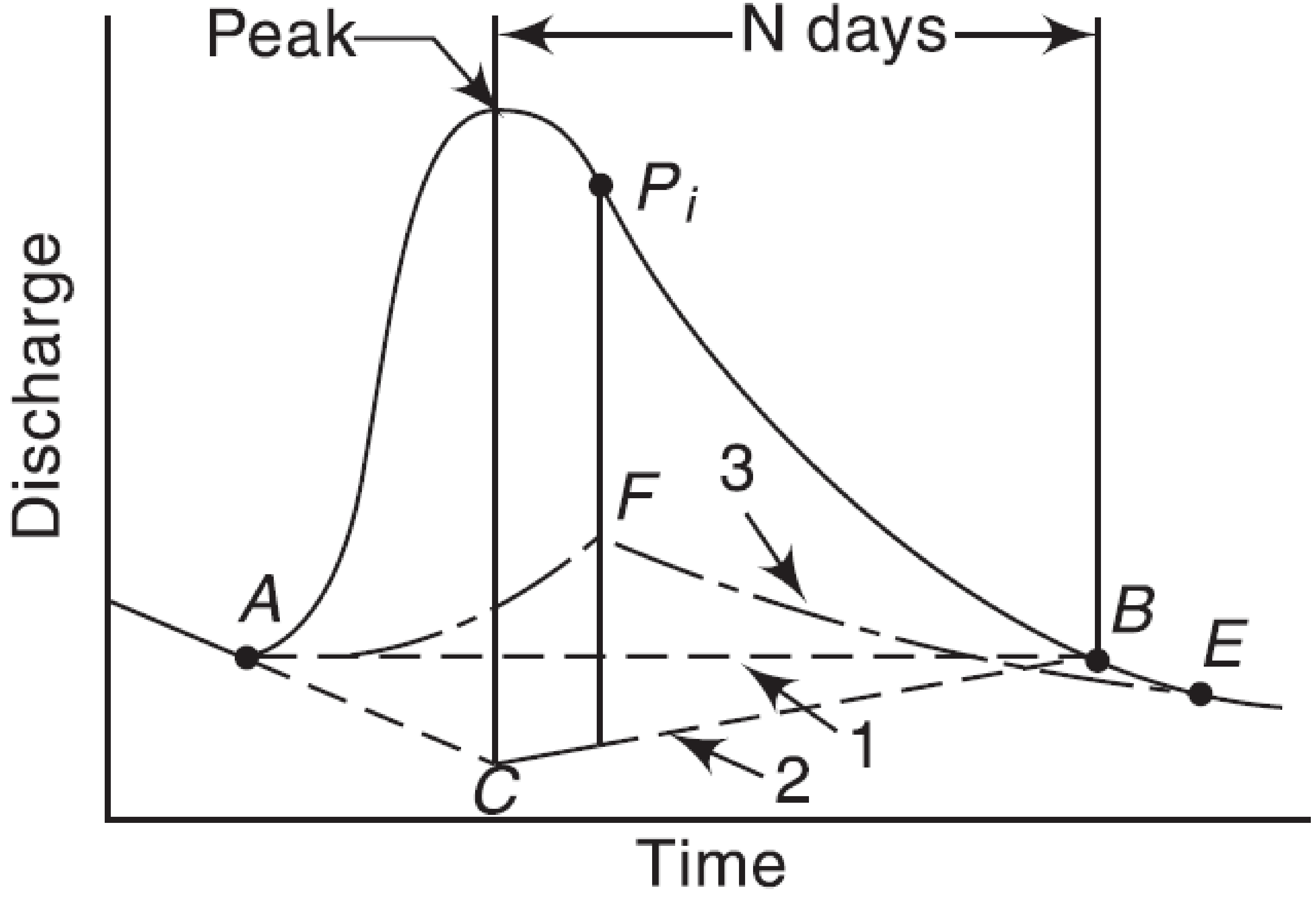


Fig. 6.5 Base Flow Seperation Methods

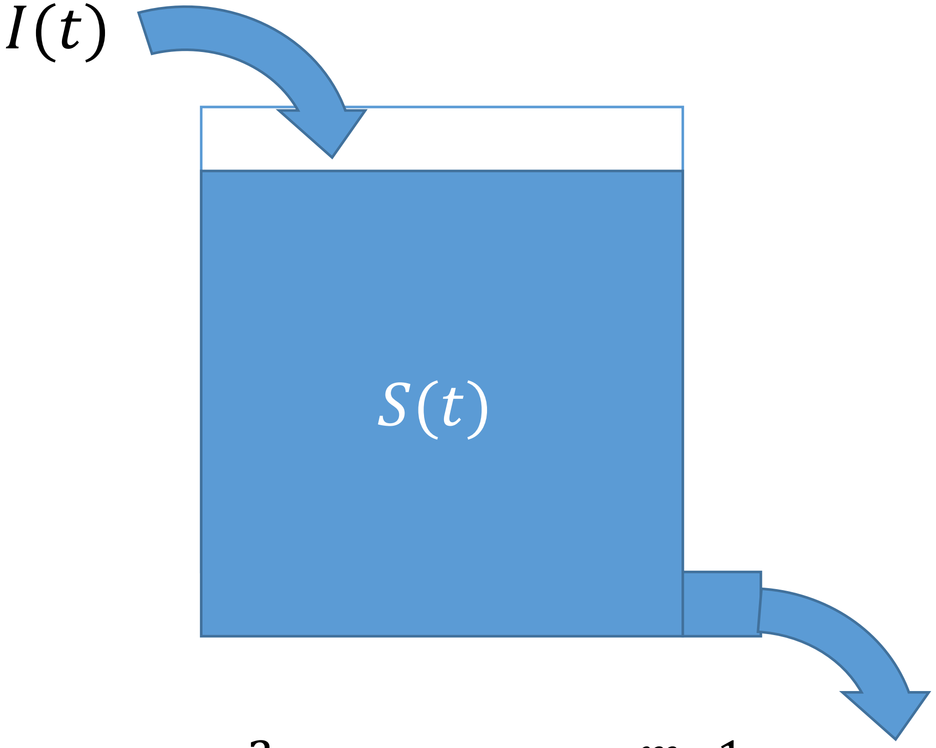
$N = aA^{0.2}$

N is time in days

A is the drainage area in km^2

$a = 0.8$

HYDROLOGICAL SYSTEM MODEL



$$\frac{dS}{dt} = I - Q$$

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, \frac{d^3I}{dt^3}, \dots, Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, \frac{d^3Q}{dt^3}, \dots, \right)$$

$$S = a_1 Q + a_2 \frac{dQ}{dt} + a_3 \frac{d^2Q}{dt^2} + \cdots + a_n \frac{d^{n-1}Q}{dt^{n-1}} + b_1 I + b_2 \frac{dI}{dt} + b_3 \frac{d^2I}{dt^2} + \cdots + b_m \frac{d^{m-1}I}{dt^{m-1}}$$

Differentiating

$$\frac{dS}{dt} = a_1 \frac{dQ}{dt} + a_2 \frac{d^2Q}{dt^2} + a_3 \frac{d^3Q}{dt^3} + \cdots + a_n \frac{d^nQ}{dt^n} + b_1 \frac{dI}{dt} + b_2 \frac{d^2I}{dt^2} + b_3 \frac{d^3I}{dt^3} + \cdots + b_m \frac{d^mI}{dt^m}$$

$$\frac{dS}{dt} = I - Q$$

$$a_1\frac{dQ}{dt} + a_2\frac{d^2Q}{dt^2} + a_3\frac{d^3Q}{dt^3} +, \dots, a_n\frac{d^nQ}{dt^n} + b_1\frac{dI}{dt} + b_2\frac{d^2I}{dt^2} + b_3\frac{d^3I}{dt^3} +, \dots, b_m\frac{d^mI}{dt^m} = I - Q$$

$$a_1\frac{dQ}{dt} + a_2\frac{d^2Q}{dt^2} + a_3\frac{d^3Q}{dt^3} +, \dots, a_n\frac{d^nQ}{dt^n} + Q = I - b_1\frac{dI}{dt} + b_2\frac{d^2I}{dt^2} + b_3\frac{d^3I}{dt^3} +, \dots, b_m\frac{d^mI}{dt^m}$$

$$N(D)Q = M(D)I$$

Where $N(D) = a_1\frac{d}{dt} + a_2\frac{d^2}{dt^2} + a_3\frac{d^3}{dt^3} +, \dots, a_n\frac{d^n}{dt^n} + 1$

$$M(D) = -b_1\frac{d}{dt} + b_2\frac{d^2}{dt^2} + b_3\frac{d^3}{dt^3} +, \dots, b_n\frac{d^n}{dt^n} + 1$$

Solving for Q $\quad Q = \frac{M(D)}{N(D)}I$

Considering $a_1 = k$ and all other coefficients zero

$$\boxed{k\frac{dQ}{dt} + Q = I}$$

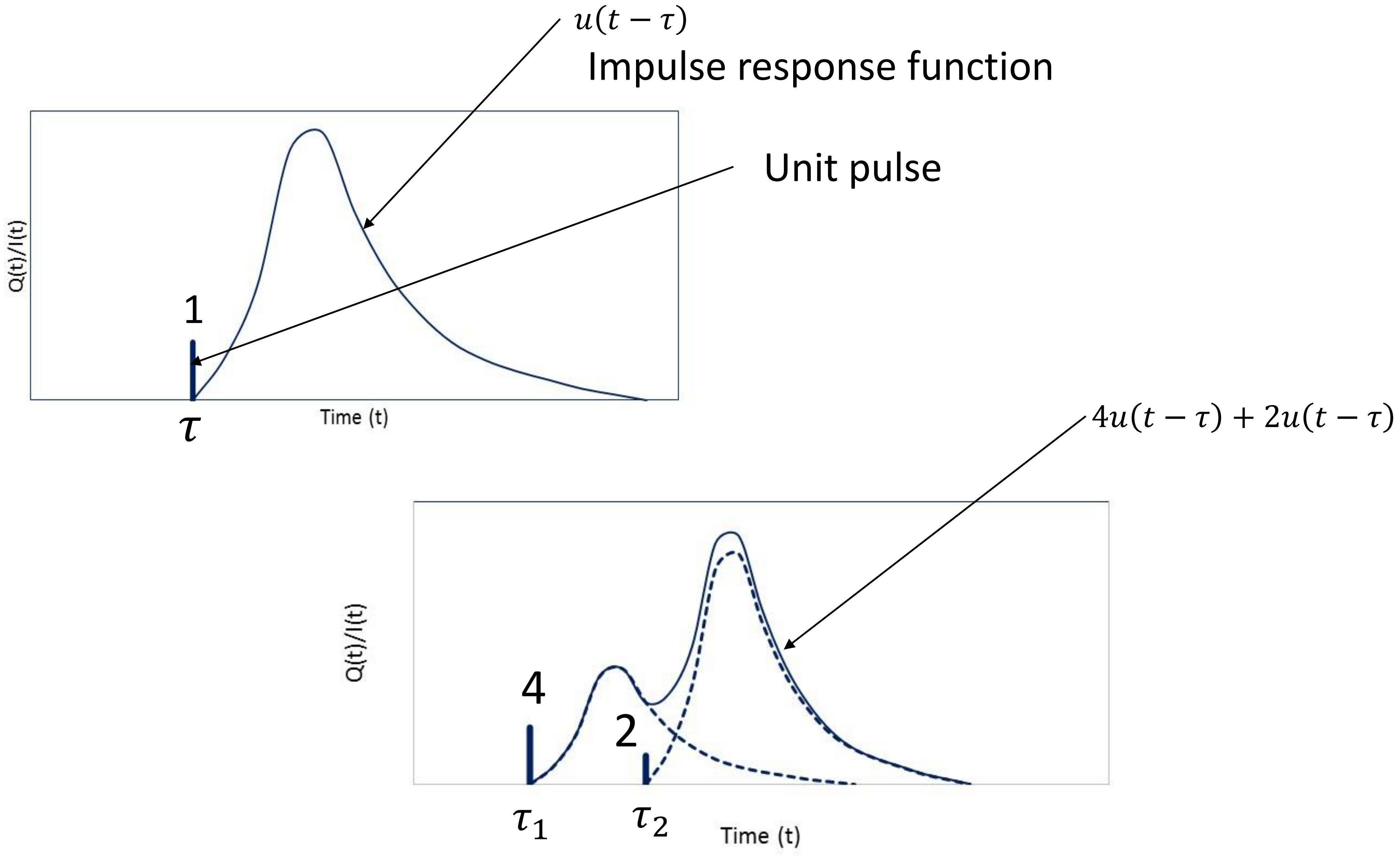
Response function of linear system

Flow the two principle of linearity: Principle of proportionality and Superposition

Principle of proportionality: If a solution of $f(x)$ is multiplied by a constant c, the resulting function $cf(x)$ is also a solution.

Superposition: If two solutions $f_1(x)$ and $f_2(x)$**,** the resulting function $f_1(x) + f_2(x)$ is also a solution

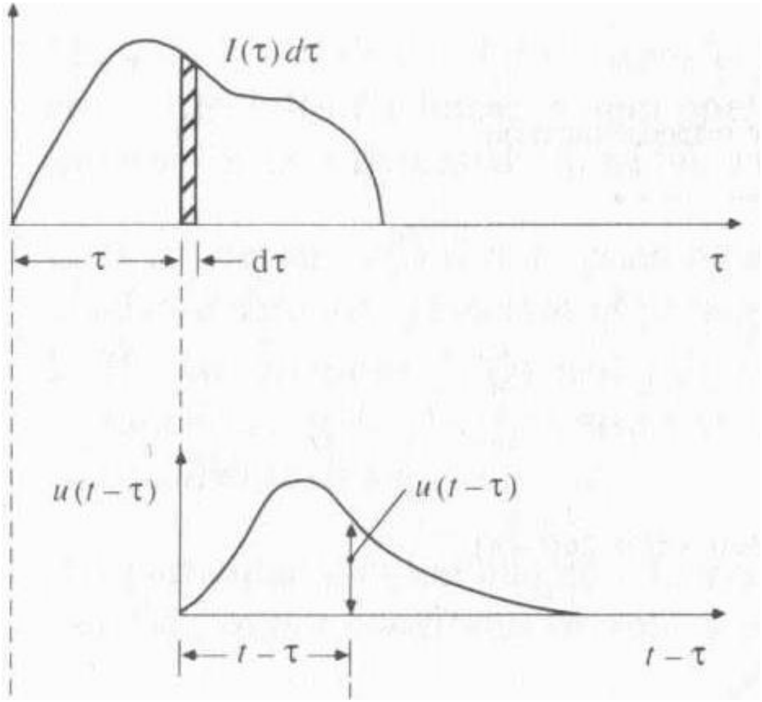
Impulse Response function

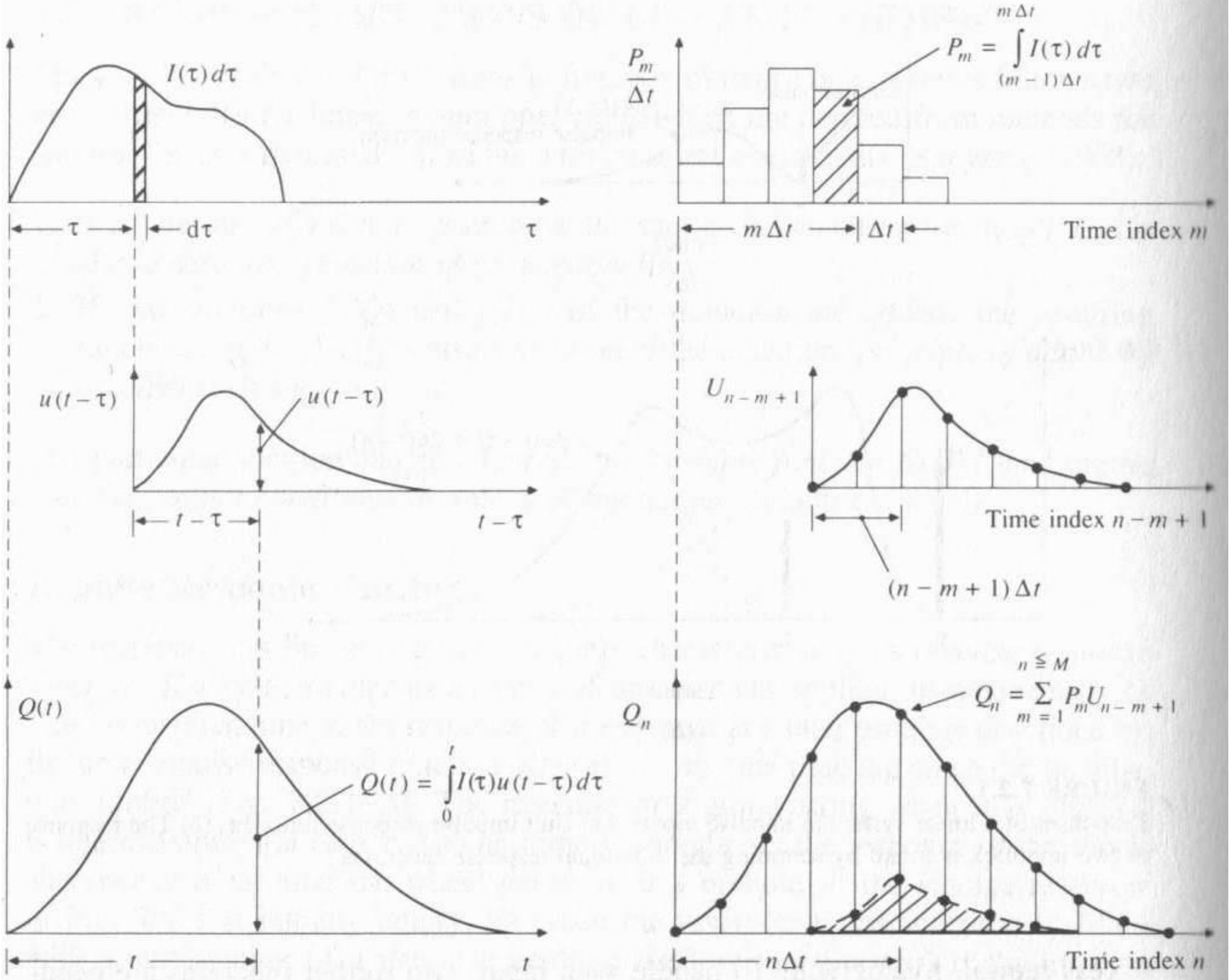


$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$

This is known as convolution integral

This is the fundamental equation for solution of linear system on a continuous time scale





(*a*) Continuous time functions (*b*) Discrete time functions

Step response function

A unit step input is an input that goes from 0 to 1 at time 0 and continuous indefinitely at that rate thereafter

$$Q(t) = g(t) = \int_0^t I(\tau)u(t-\tau)d\tau \quad I(\tau) = 1$$

$$Q(t) = g(t) = \int_0^t u(t-\tau)d\tau$$

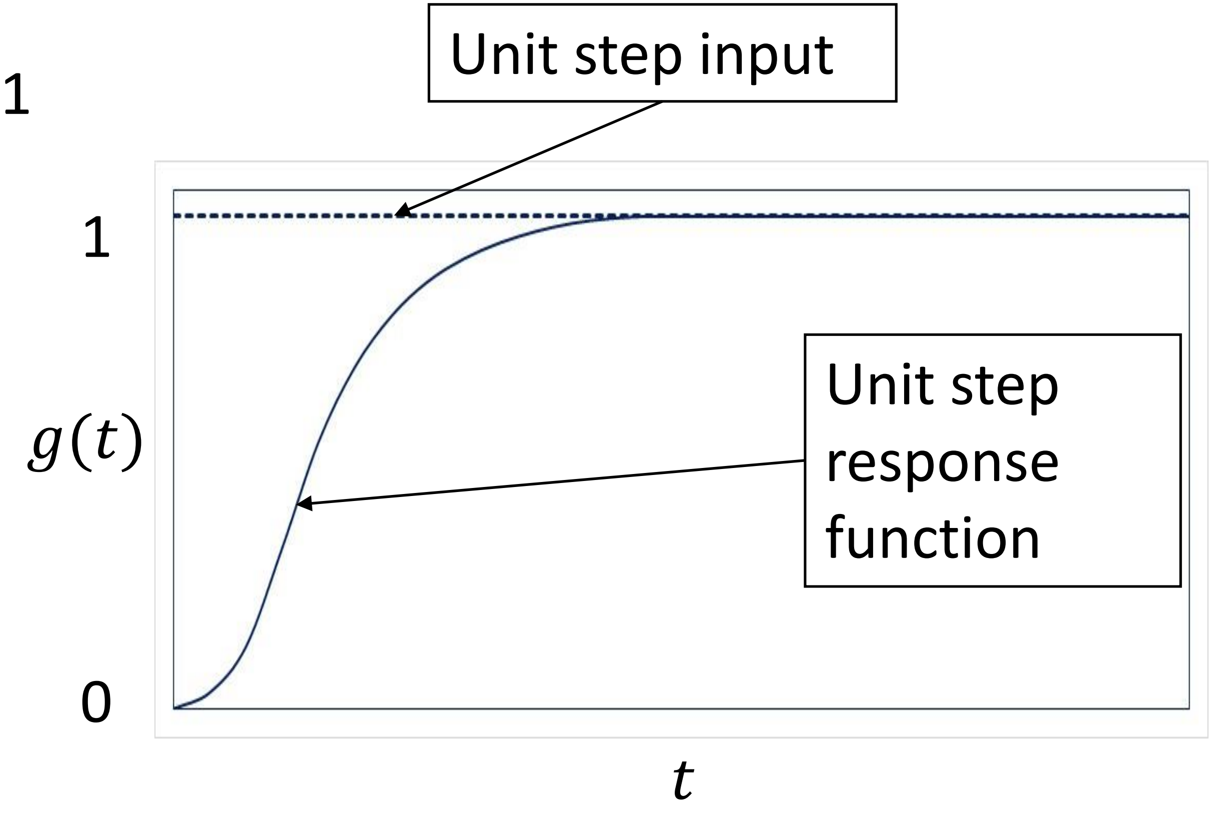
Substituting $l = t - \tau$ $\quad d\tau = -dl$

The limit $\tau = t$, becomes $l = t - t = 0$

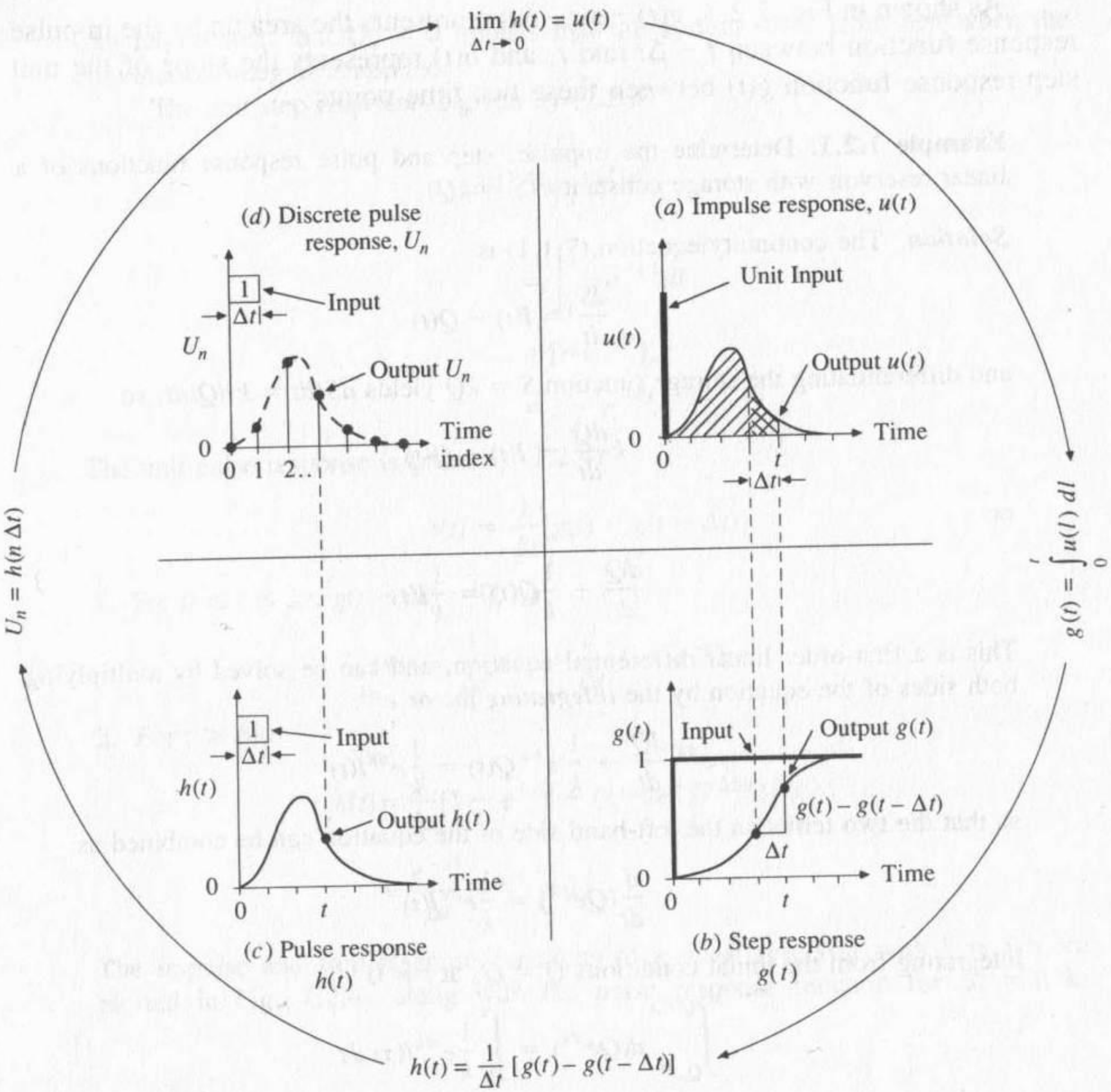
The limit $\tau = 0$, becomes $l = t - 0 = t$

$$g(t) = -\int_t^0 u(l)dl$$

$$g(t) = \int_0^t u(l)dl$$



Thus the unit step response function at time t is the integral of the impulse response function up to the time


$\lim_{\Delta t \to 0} h(t) = u(t)$
(a) Impulse response, $u(t)$
Unit Input
$u(t)$
Output $u(t)$
Time
0
t
Δt
$g(t) = \int_0^t u(l)\, dl$
$g(t)$
Input
Output $g(t)$
1
$g(t) - g(t - \Delta t)$
Δt
Time
(b) Step response $g(t)$
$h(t) = \frac{1}{\Delta t}[g(t) - g(t - \Delta t)]$
1
Input
Δt
$h(t)$
Output $h(t)$
Time
(c) Pulse response $h(t)$
$U_n = h(n \Delta t)$
(d) Discrete pulse response, U_n
1
Input
Δt
U_n
Output U_n
Time index
0 1 2..

Pulse response function

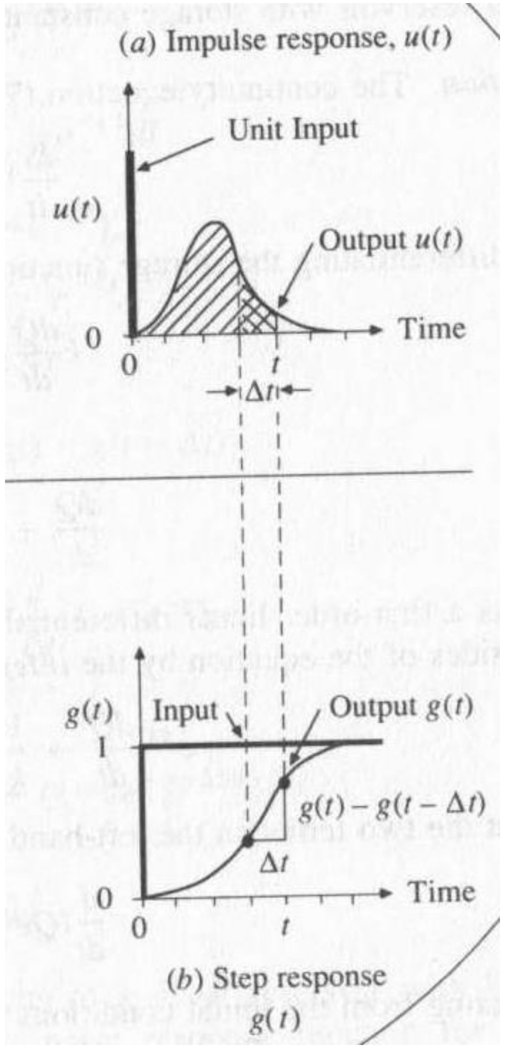
A unit pulse input is an input of unit amount occurring in duration Δt. The rate is
$I(\tau) = 1/\Delta t$, between $0 \leq \tau \leq \Delta t$ and 0 elsewhere

Response to a unit step input of rate $1/\Delta t$ beginning at time 0 is $(1/\Delta t)$g(t)

Response to a unit step input of rate $1/\Delta t$ beginning at time Δt is $(1/\Delta t)g(t-\Delta t)$

Unit pulse response function

$$h(t) = \frac{1}{\Delta t}\left(g(t) - g(t-\Delta t)\right)$$



Pulse response function

$$h(t) = \frac{1}{\Delta t}(g(t) - g(t - \Delta t))$$

$$= \frac{1}{\Delta t}\left(\int_0^t u(l)dl - \int_0^{t-\Delta t} u(l)dl\right)$$

$$= \frac{1}{\Delta t}\int_{t-\Delta t}^t u(l)dl$$

Step response function $g(t) = \int_0^t u(l)dl$

Pulse response function $h(t) = \frac{1}{\Delta t}\int_{t-\Delta t}^t u(l)dl$

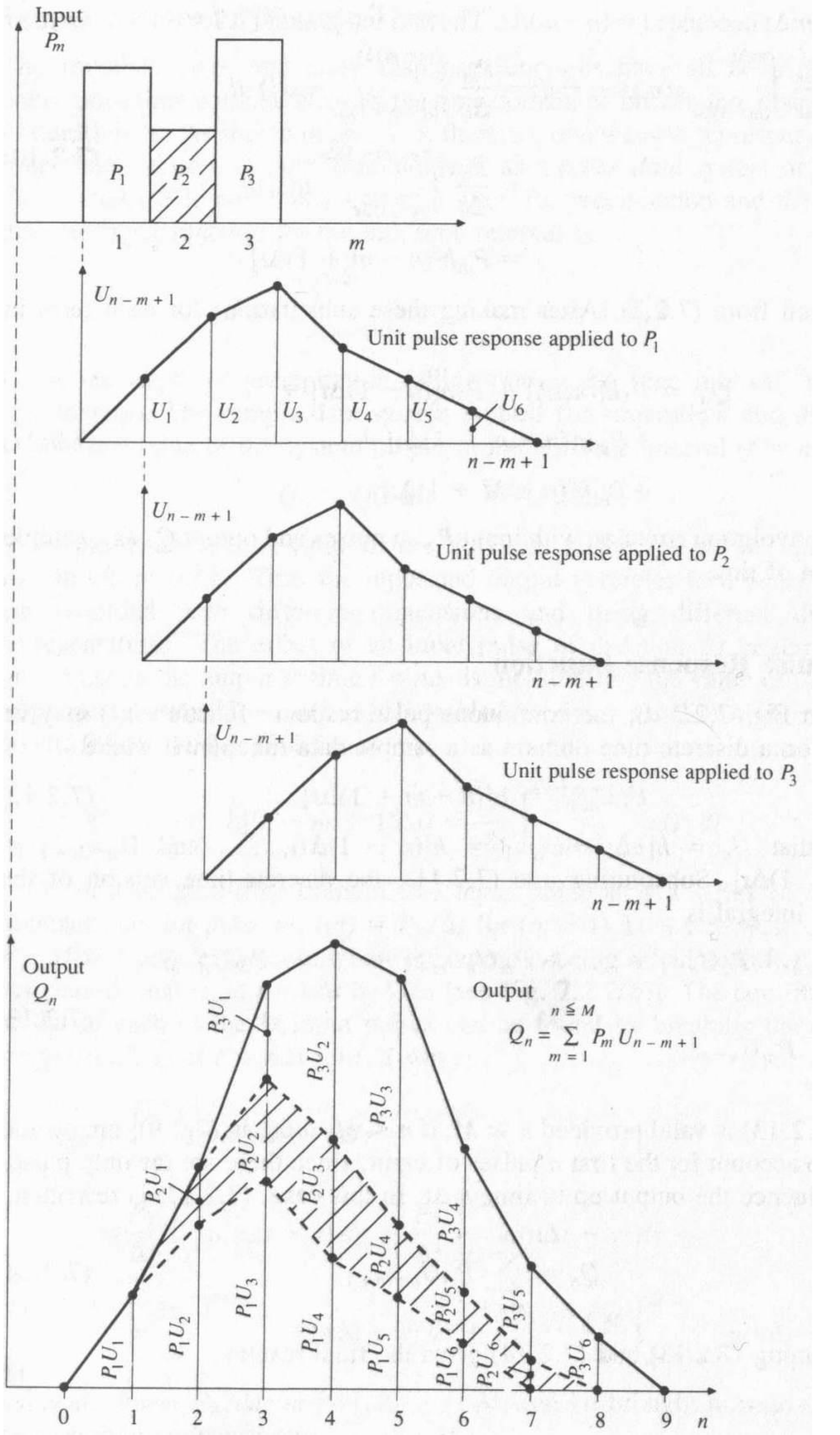
Unit Hydrograph

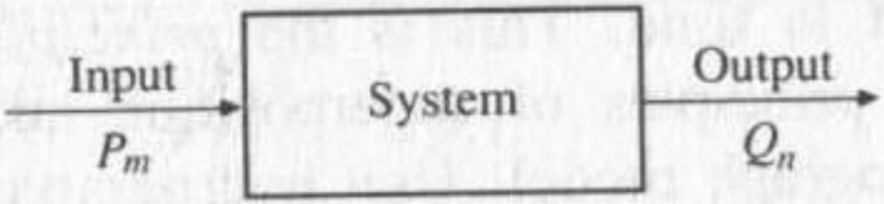
The unit hydrograph is the unit pulse response function of a linear hydrologic system

It was first proposed by Sherman (1932), the unit hydrograph of a watershed is defined as a direct runoff hydrograph resulting from 1 cm of excess rainfall generated uniformly over the drainage area.

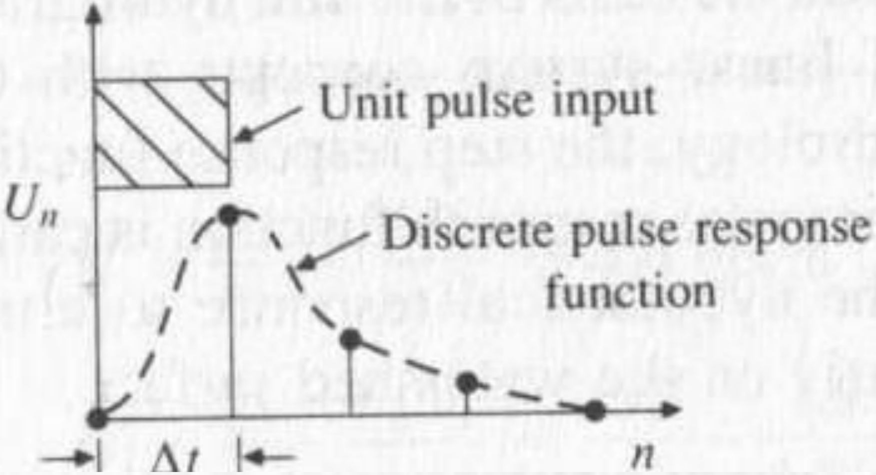
- The excess rainfall has a constant intensity within the effective duration
- The excess rainfall is uniformly distributed throughout the whole drainage area
- The base time of the DRH resulting from an excess rainfall of given duration is constant
- The hydrograph resulting from a given excess rainfall reflects the unchanging characteristics of the watershed
- The ordinate of all DRH's of a common base time are directly proportional to the total amount of direct runoff represented by each hydrograph

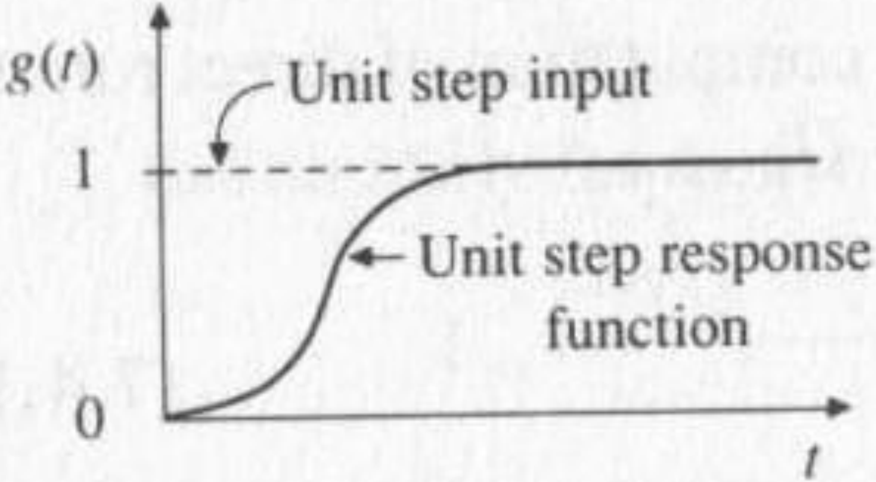
$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

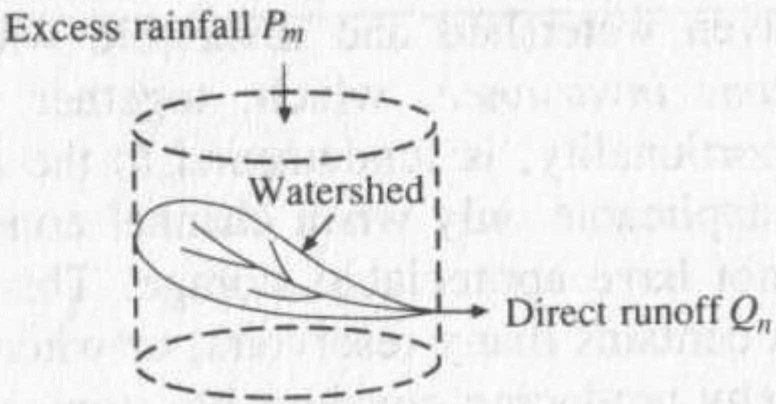


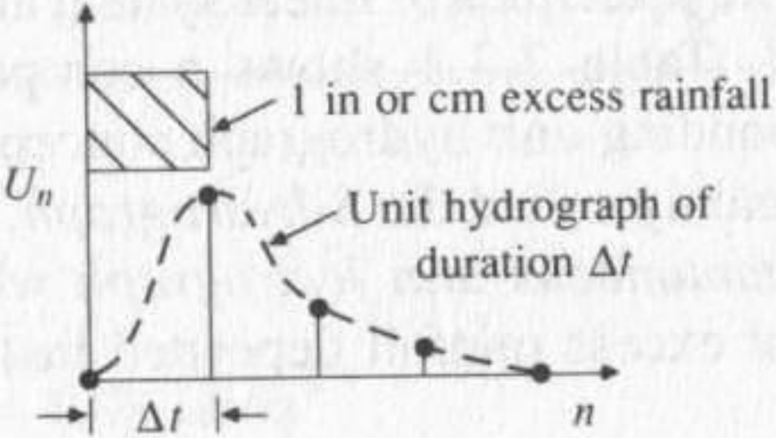


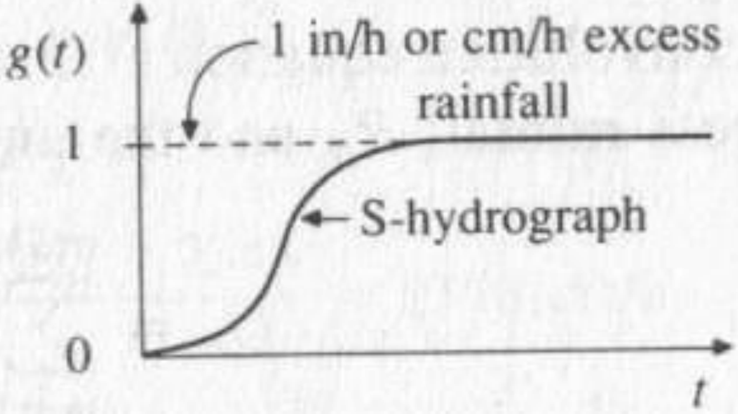
$$Q_n = \sum_{m=1}^{n \leqq M} P_m U_{n-m+1}$$

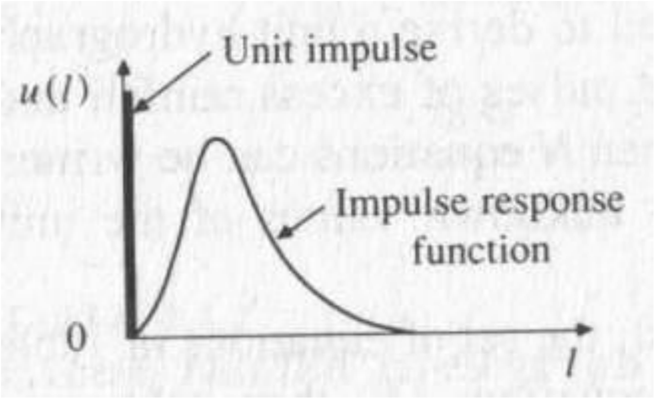




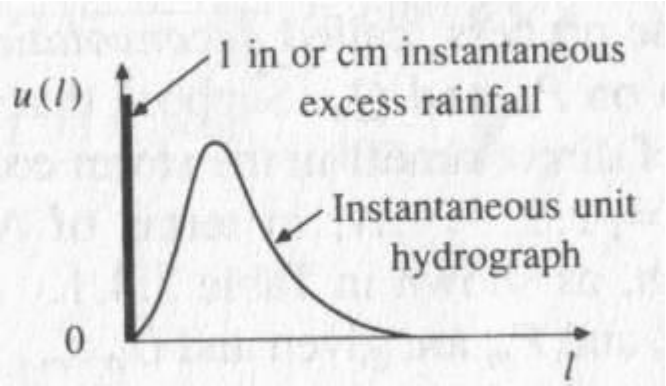








Unit impulse
u(l)
Impulse response function
0
l

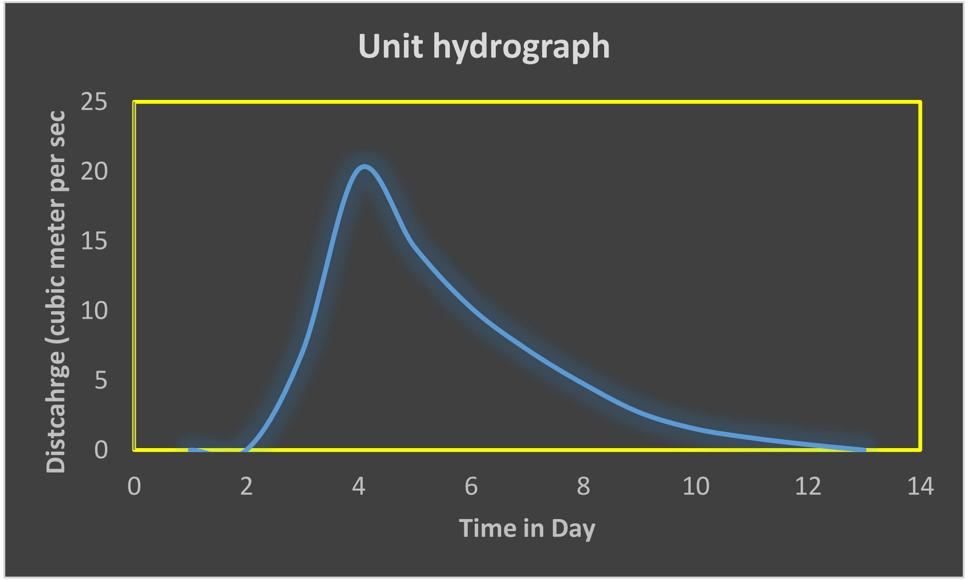


1 in or cm instantaneous excess rainfall
u(l)
Instantaneous unit hydrograph
0
l

Estimate Unit hydrograph of the catchment

Time (Days)	Total Runoff (m3/s)	Base Flow (m3/s)	Direct runoff (m3/s)	Unit Hydrograph (m3/s) per mm
1	168	168	0	0
2	160	160	0	0
3	500	160	340	7
4	1130	160	970	20
5	860	160	700	15
6	650	158	492	10
7	500	153	347	7
8	380	151	229	5
9	280	150	130	3
10	220	147	73	2
11	185	143	42	1
12	160	141	19	0
13	140	140	0	0

Catchment area is 6500 sq. km



UNIT HYDROGRAPH OF DIFFERENT DURATION

$$h(t) = \frac{1}{\Delta t}[g(t) - g(t - \Delta t)]$$

$$h(t - \Delta t) = \frac{1}{\Delta t}[g(t - \Delta t) - g(t - 2\Delta t)]$$

$$h(t - 2\Delta t) = \frac{1}{\Delta t}[g(t - 2\Delta t) - g(t - 3\Delta t)]$$

$$h(t - 3\Delta t) = \frac{1}{\Delta t}[g(t - 3\Delta t) - g(t - 4\Delta t)]$$

$$\vdots$$

$$g(t) = \Delta t[h(t) + h(t - \Delta t) + h(t - 2\Delta t) + \cdots]$$

$$g(t) = g(t - \Delta t')$$

$$\boxed{h'(t) = \frac{1}{\Delta t'}[g(t) - g(t - \Delta t')]}$$