CE 311: Hydrology & Water Resources Engineering

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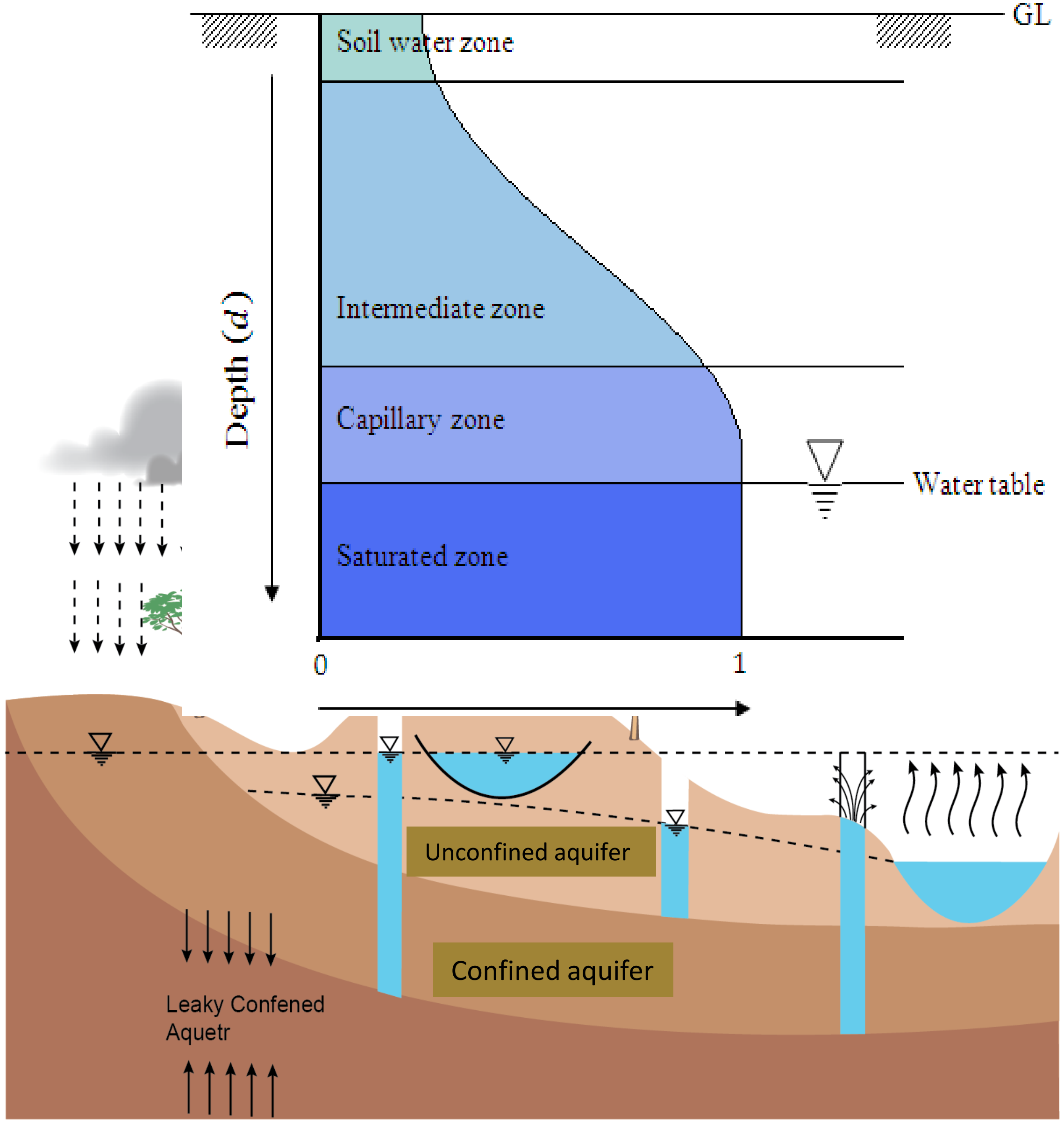
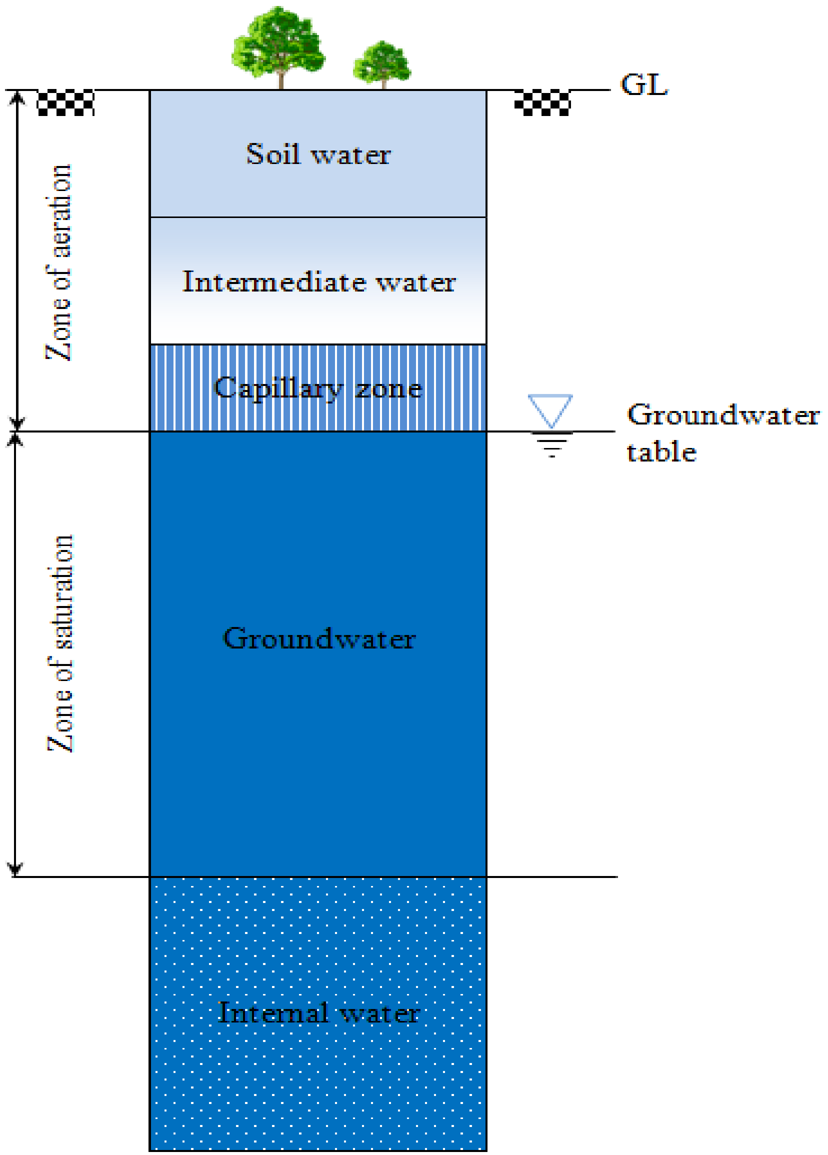
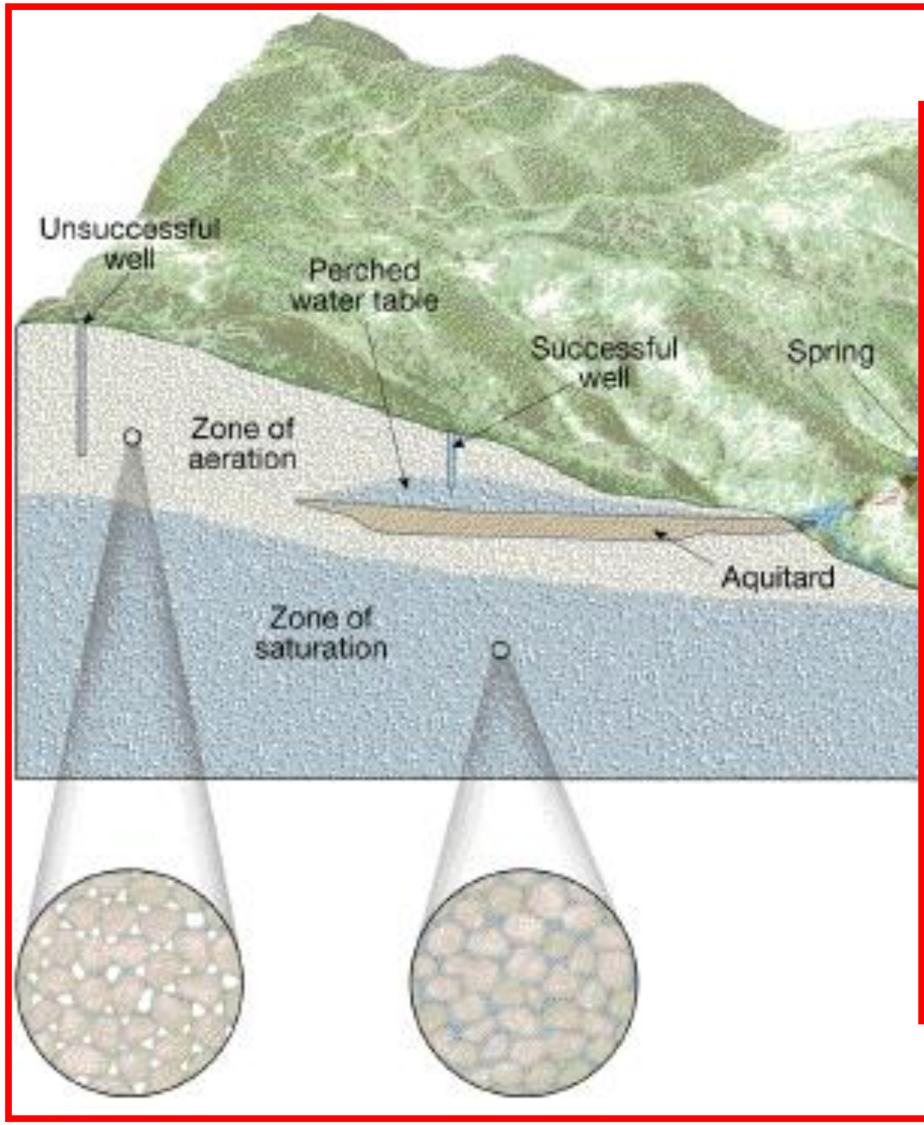
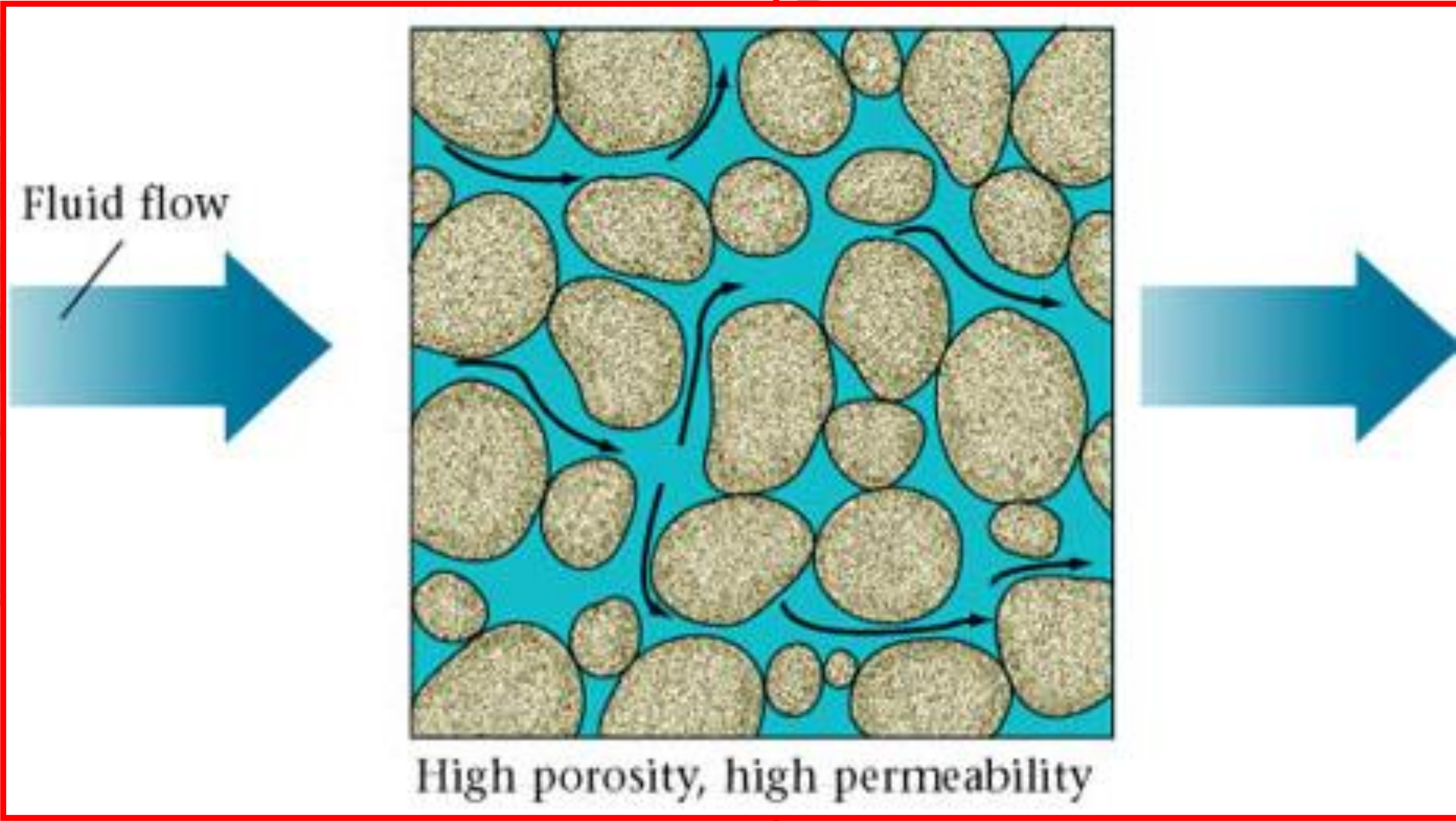
Groundwater : Introduction





Groundwater





QUIZ

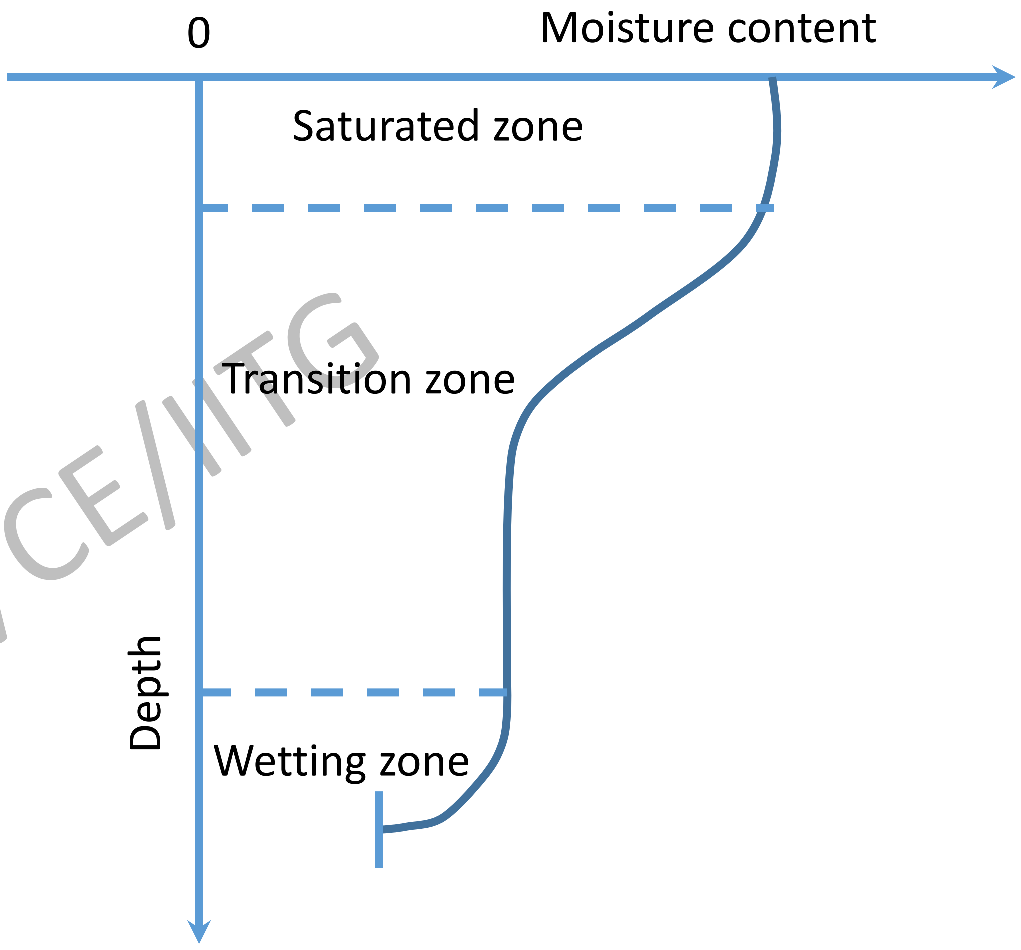
Infiltration

If water is ponded on the surface, the infiltration occurs at the potential infiltration rate

If f is the infiltration rate, the cumulative infiltration F

$$F(t) = \int_0^t f(\tau)d\tau$$

$$f(t) = \frac{dF(t)}{dt}$$



Horton's equation

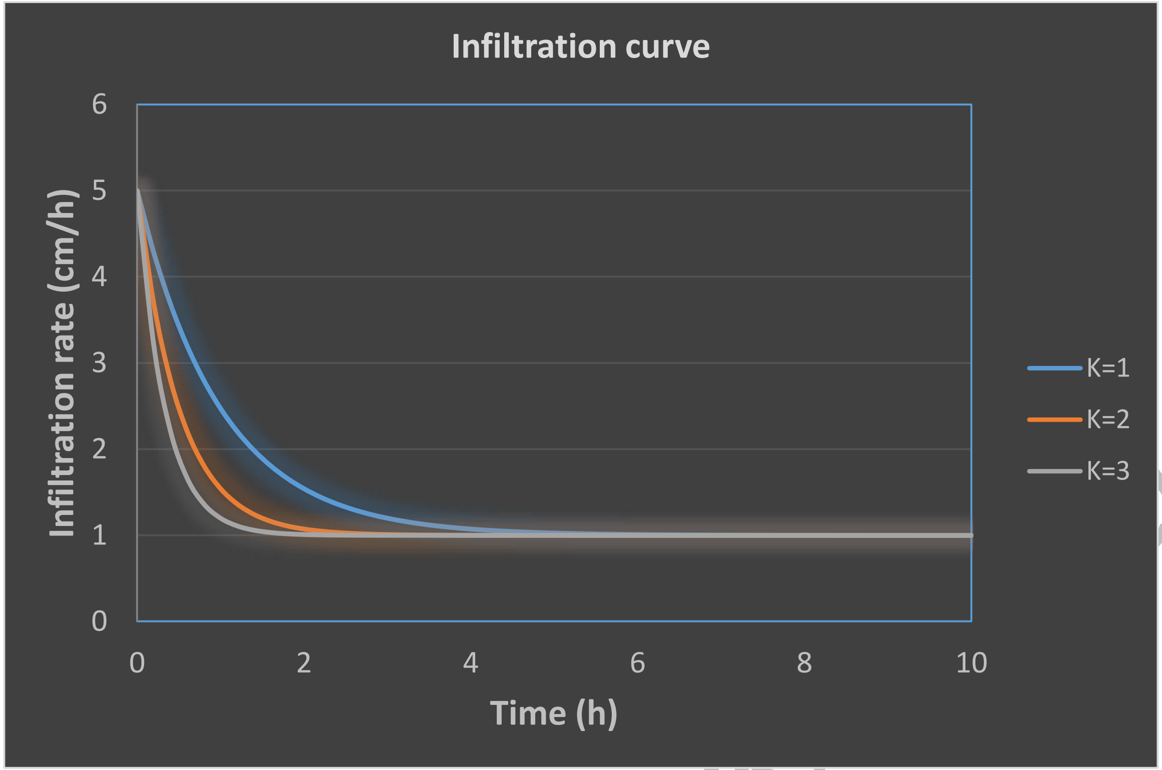
$$f(t) = f_c + (f_o - f_c)e^{-kt}$$

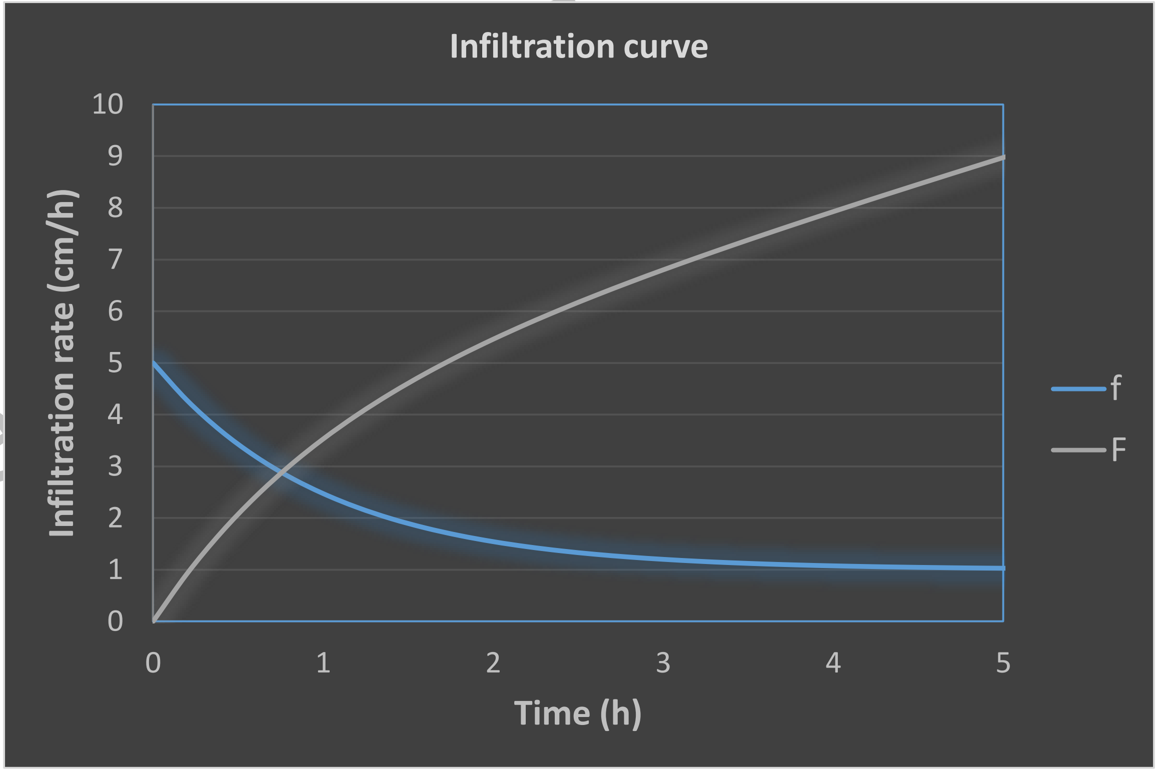
f_c constant infiltration rate
f_o infiltration at the beginning
k is the decay constant

Horton's equation

$$f(t) = f_c + (f_o - f_c)e^{-kt}$$

$$F = f_c t + \frac{(f_o - f_c)}{k}\left(1 - e^{-kt}\right)$$





Phillip’s Equation

$$F(t) = St^{1/2} + Kt$$

Where S is a parameter called sorptivity which is a function of soil suction potential and K is the hydraulic conductivity

$$f(t) = \frac{1}{2}St^{-1/2} + K$$

S is the suction term

K is the gravity term

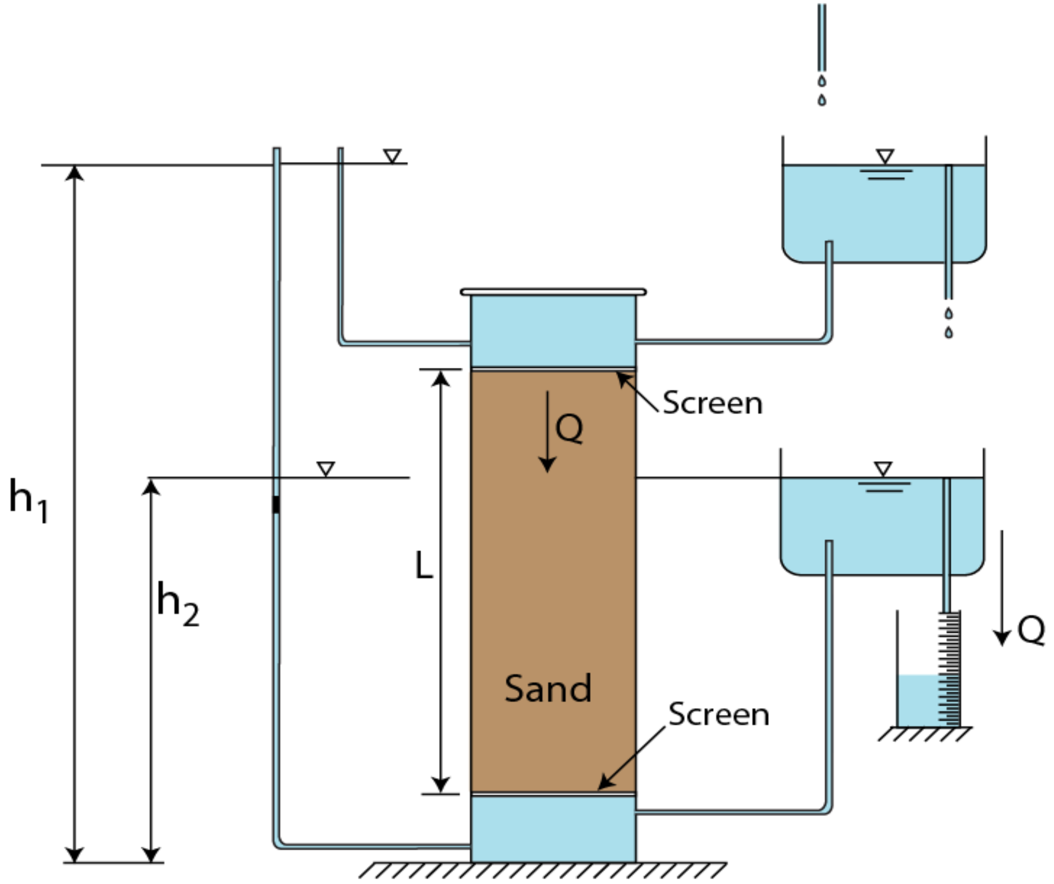
At $t \rightarrow \infty$ $\qquad f(t) = K$

For horizontal column

$$f(t) = \frac{1}{2}St^{-1/2}$$

MOVEMENT OF GROUNDWATER

- proportional to the cross sectional area (A) of the filter
- proportional to the difference in piezometric heads
- inversely proportional to the length (L) of the filter



After combining these conclusions, we have

$$Q = KA\left(\frac{h_1 - h_2}{L}\right)$$

Where,

Q is the flow rate, *i.e.* the volume of water that flows through the sand filter per unit time.

K is the coefficient of proportionality and is termed as hydraulic conductivity of the medium. It is a measure of the permeability of the porous medium. It is also known as coefficient of permeability.

h_1 and h_2 are the piezometric heads.

Now, defining $J = \frac{h_1 - h_2}{L}$ and $q = \frac{Q}{A}$

Where J is the hydraulic gradient and q is the specific discharge, *i.e.* the discharge per unit area.

The equation 3.1 can also be written as,

$q = KJ$

$$q = -K \frac{\partial h}{\partial x}$$ Darcy's law

$Q = KA(\frac{\varphi_1 - \varphi_2}{L})$

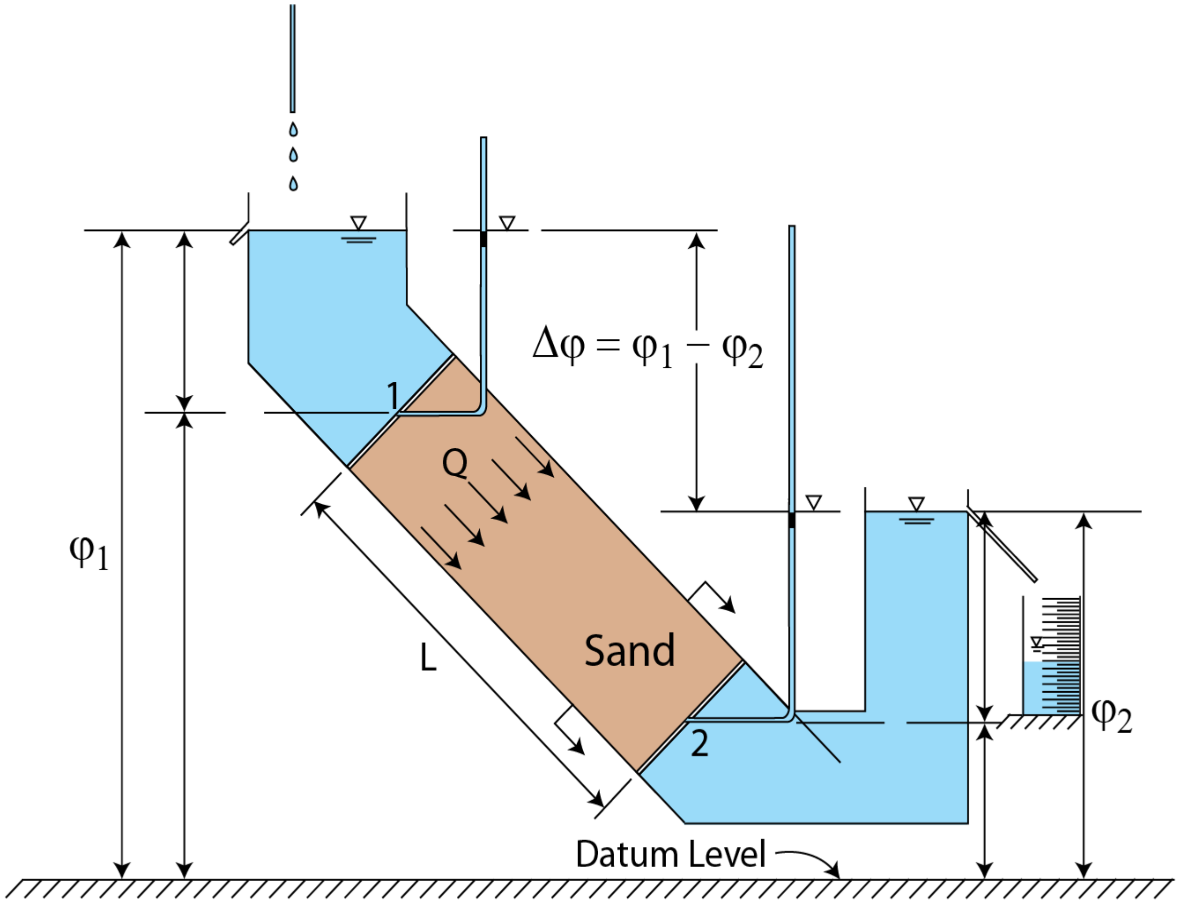
Or, $q = K(\frac{\varphi_1 - \varphi_2}{L})$

Or, $q = KJ$

Where, $J = \frac{\varphi_1 - \varphi_2}{L}$ and $\varphi_1 = z_1 + \frac{p_1}{\gamma}$ and $\varphi_2 = z_2 + \frac{p_2}{\gamma}$

z_1 and z_2 are the datum head or elevation head

p_1/γ and p_2/γ are the pressure head



It should be noted here that q and K have the same dimension with the velocity. The value of q will be equal to K for unit hydraulic gradient. As such for the case of isotropic medium, the hydraulic conductivity (K) may be defined as the specific discharge (q) occurs under unit hydraulic gradient $(J = 1)$.

The hydraulic conductivity is dependent on both porous matrix properties and fluid properties and can be expressed as

$$K = \frac{k\rho g}{\mu} = \frac{kg}{\upsilon}$$

Where, ρ is the density of the fluid,

The Darcy's Law can be written as

$$q = \left(\frac{k\rho g}{\mu}\right) \mathrm{J}$$

In Darcy's law, we have neglected the kinetic energy of water. The velocity of water in case of porous medium is very low and along the flow path, the change in piezometric head is much smaller than the change in kinetic energy. Hence, kinetic energy can be neglected.

Further, it may be noted that the flow takes place from higher piezometric head to lower piezometric head and not from higher pressure to lower pressure. Only in case of horizontal flow $(z_1 = z_2)$, the flow takes place from higher pressure to lower pressure. Thus incase of horizontal flow, the Darcy's formula can be written as,

$$q = K\left(\frac{P_1 - P_2}{\gamma L}\right)$$

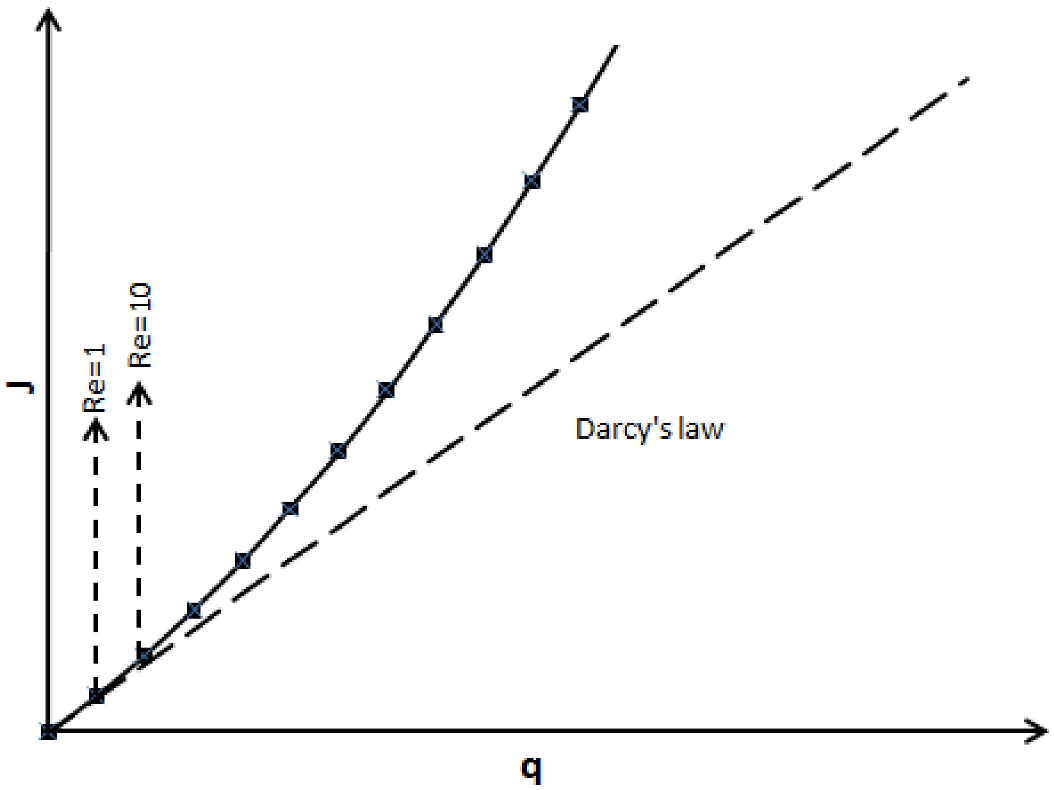
In case of flow through porous medium, the flow takes place only through the pores of the medium. Therefore, the cross sectional area through which the flow actually takes place is ηA.Where η is the porosity of the porous medium. As such, the average velocity of the flow can be expressed as

$$V = \frac{Q}{\eta A} = \frac{q}{\eta} = \frac{KJ}{\eta}$$

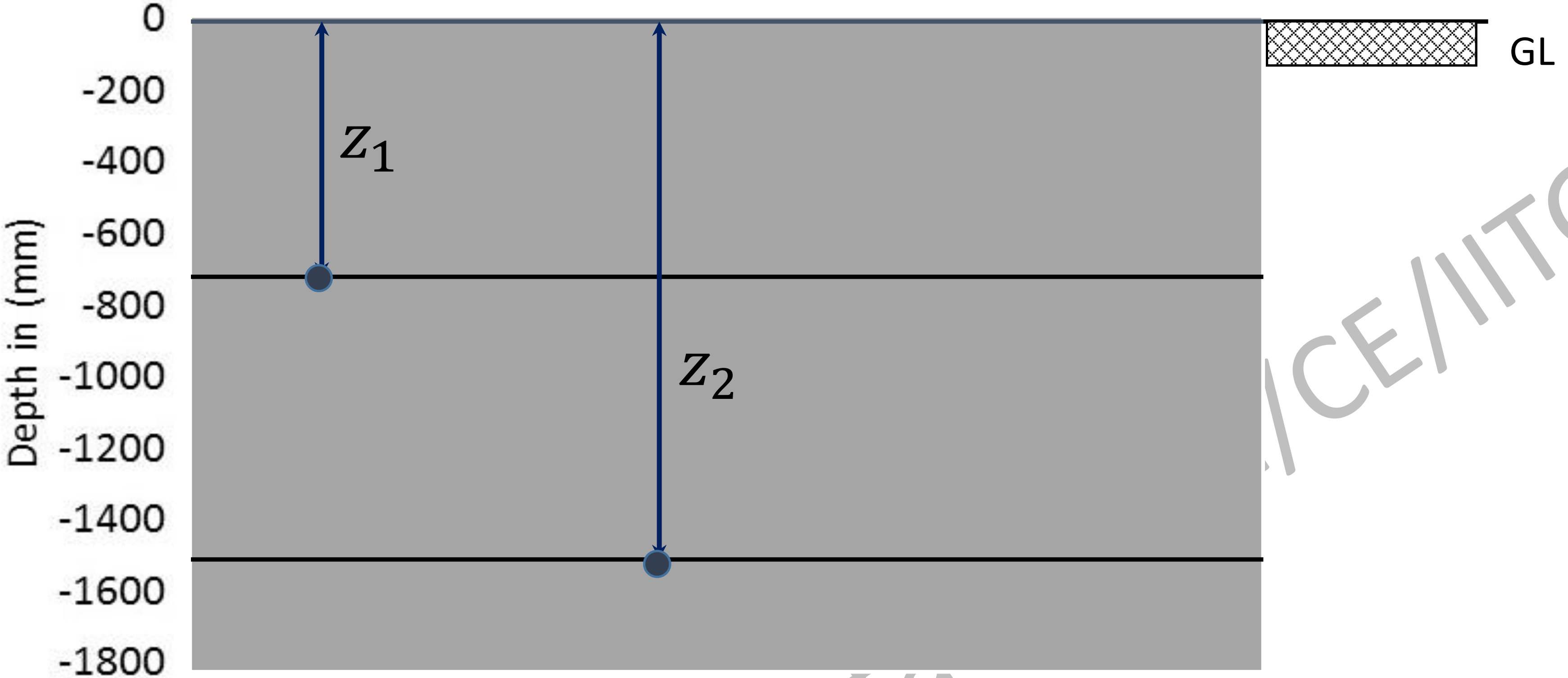
Validity of Darcy's Law

The Reynolds number, which is expressed as the ratio between inertial force and viscous force acting on the fluid, can be written as,

$$Re = \frac{qd}{\vartheta}$$

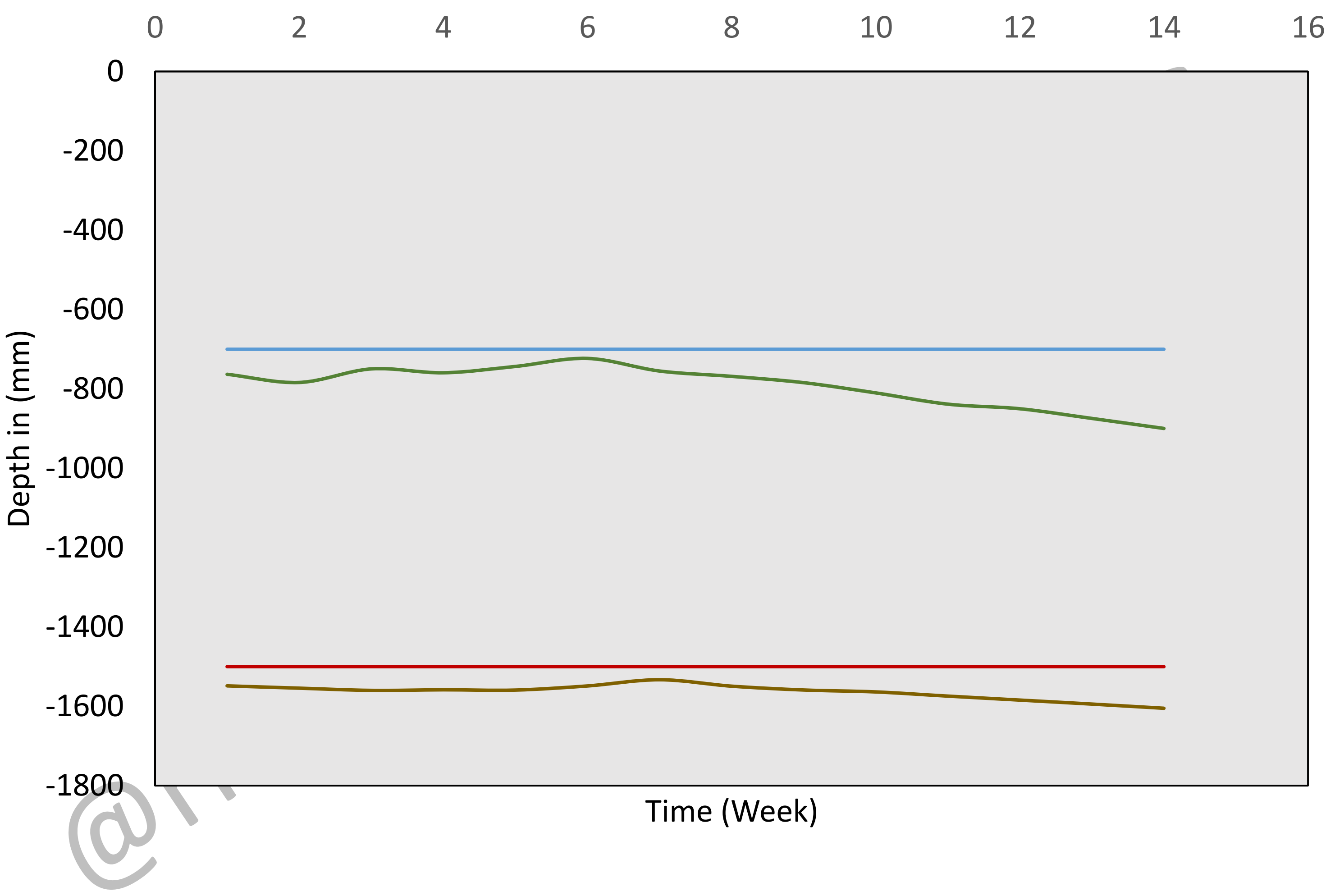


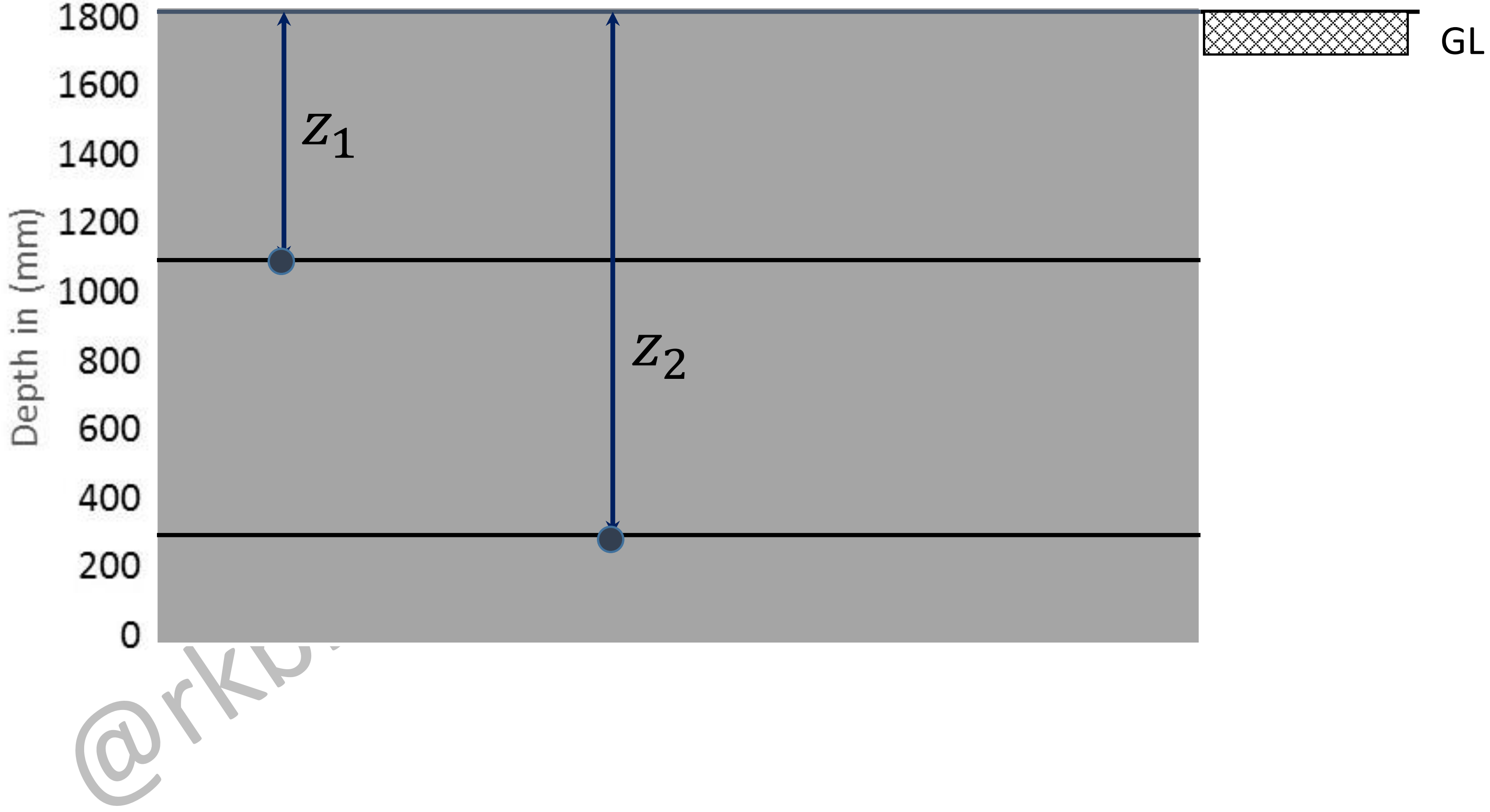
Darcy's law also stated that the flow is proportional to the hydraulic gradient. As per the law, if there is a small hydraulic gradient, there should have a very small specific discharge. However, in real situation there is a threshold value of hydraulic gradient. The flow will take place only when the hydraulic gradient is more than a threshold gradient.



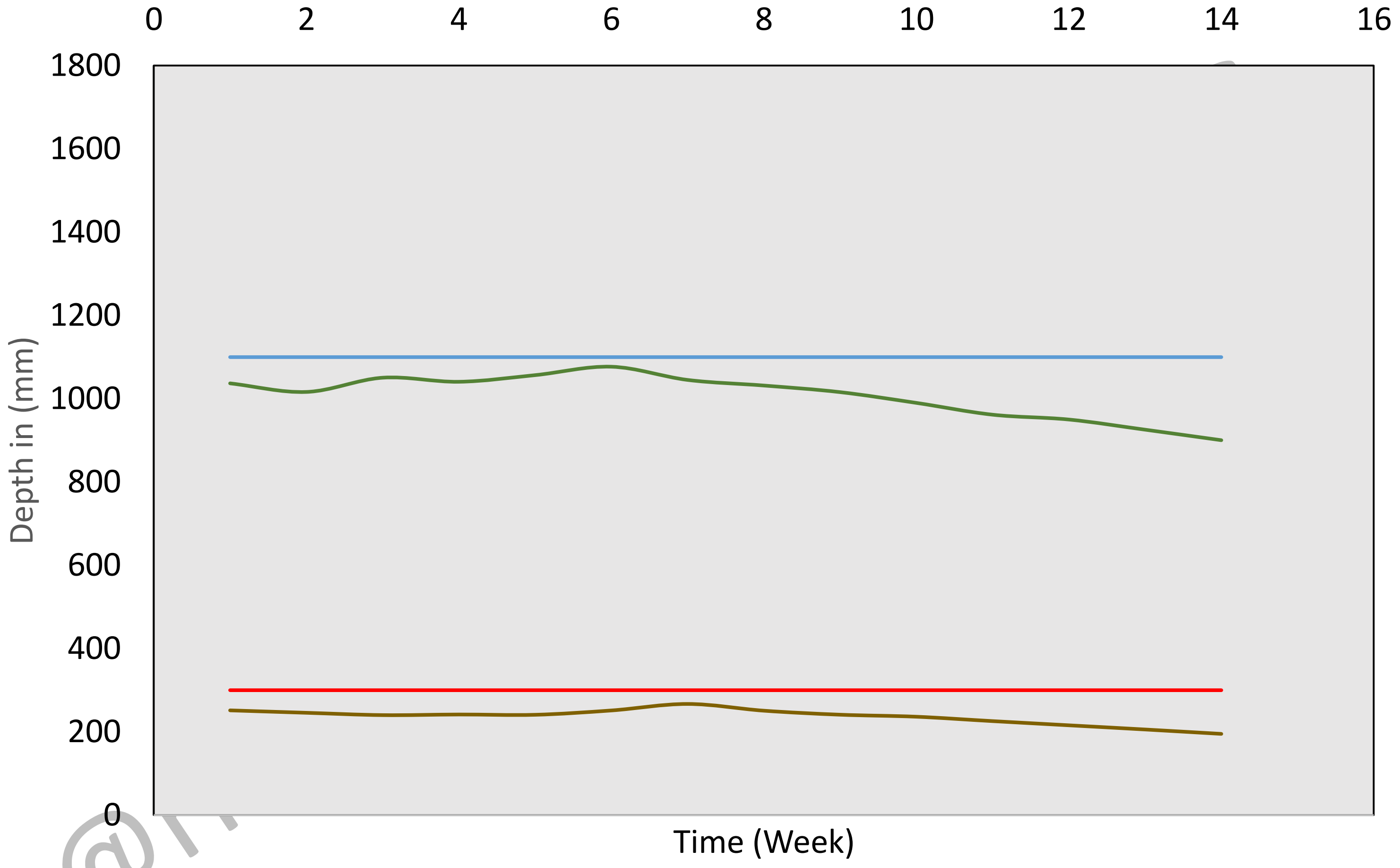
Suction head (ψ): The part of total energy possessed by the fluid due to the soil suction force is referred to as the suction head.

Time (Week)	z_1(mm)	z_2(mm)	ψ_1 (mm)	ψ_2 (mm)	$h_1 = z_1 + \psi_1$ (mm)	$h_2 = z_2 + \psi_2$ (mm)	$h_1 - h_2$ (mm)
1	-700	-1500	-63	-48	-763	-1548	785
2	-700	-1500	-84	-54	-784	-1554	771
3	-700	-1500	-49	-60	-749	-1560	810
4	-700	-1500	-59	-58	-759	-1558	799
5	-700	-1500	-43	-59	-743	-1559	816
6	-700	-1500	-23	-49	-723	-1549	826
7	-700	-1500	-55	-33	-755	-1533	778
8	-700	-1500	-68	-49	-768	-1549	781
9	-700	-1500	-84	-59	-784	-1559	775
10	-700	-1500	-110	-64	-810	-1564	754
11	-700	-1500	-138	-74	-838	-1574	736
12	-700	-1500	-150	-84	-850	-1584	734
13	-700	-1500	-174	-94	-874	-1594	720
14	-700	-1500	-200	-105	-900	-1605	705


0
2
4
6
8
10
12
14
16
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
Depth in (mm)
Time (Week)


1800
1600
1400
1200
1000
800
600
400
200
0
Depth in (mm)
z_1
z_2
GL

Time	z_1(mm)	z_2(mm)	ψ_1 (mm)	ψ_2 (mm)	$h_1 = z_1 + \psi_1$ (mm)	$h_2 = z_2 + \psi_2$ (mm)	$h_1 - h_2$ (mm)
1	1100	300	-63	-48	1037	252	785
2	1100	300	-84	-54	1016	246	771
3	1100	300	-49	-60	1051	240	810
4	1100	300	-59	-58	1041	242	799
5	1100	300	-43	-59	1057	241	816
6	1100	300	-23	-49	1077	251	826
7	1100	300	-55	-33	1045	267	778
8	1100	300	-68	-49	1032	251	781
9	1100	300	-84	-59	1016	241	775
10	1100	300	-110	-64	990	236	754
11	1100	300	-138	-74	962	226	736
12	1100	300	-150	-84	950	216	734
13	1100	300	-174	-94	926	206	720
14	1100	300	-200	-105	900	195	705



0
2
4
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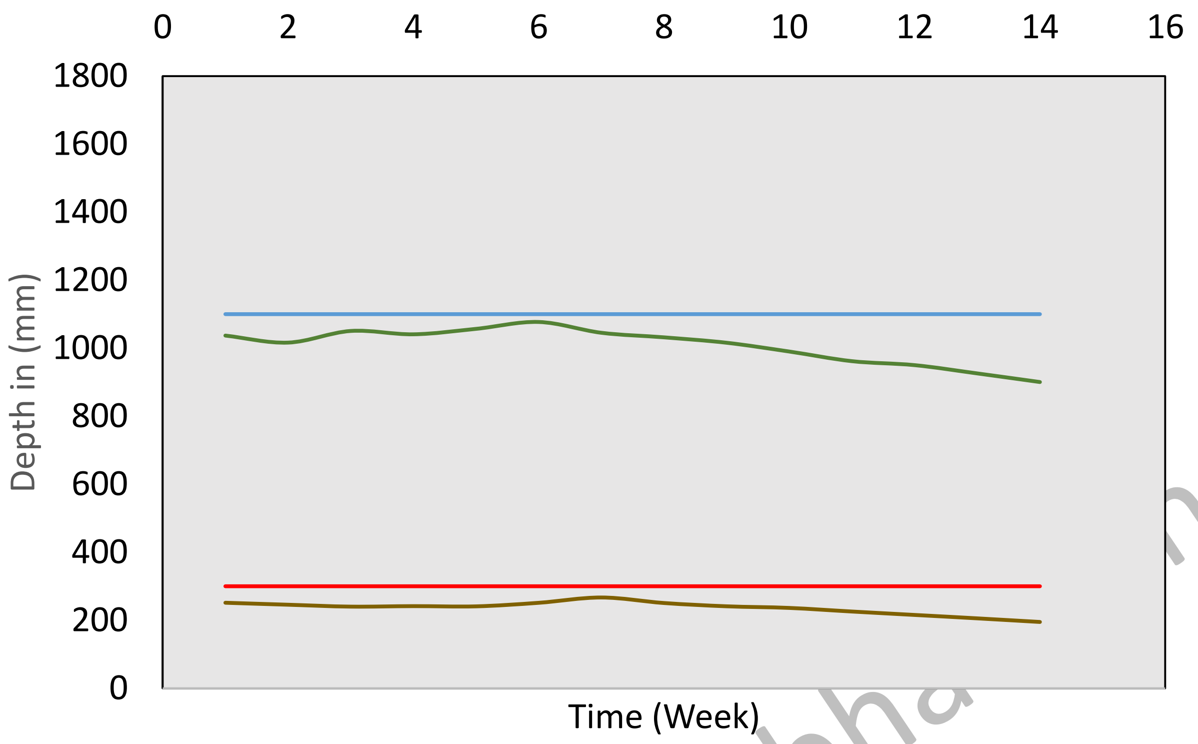
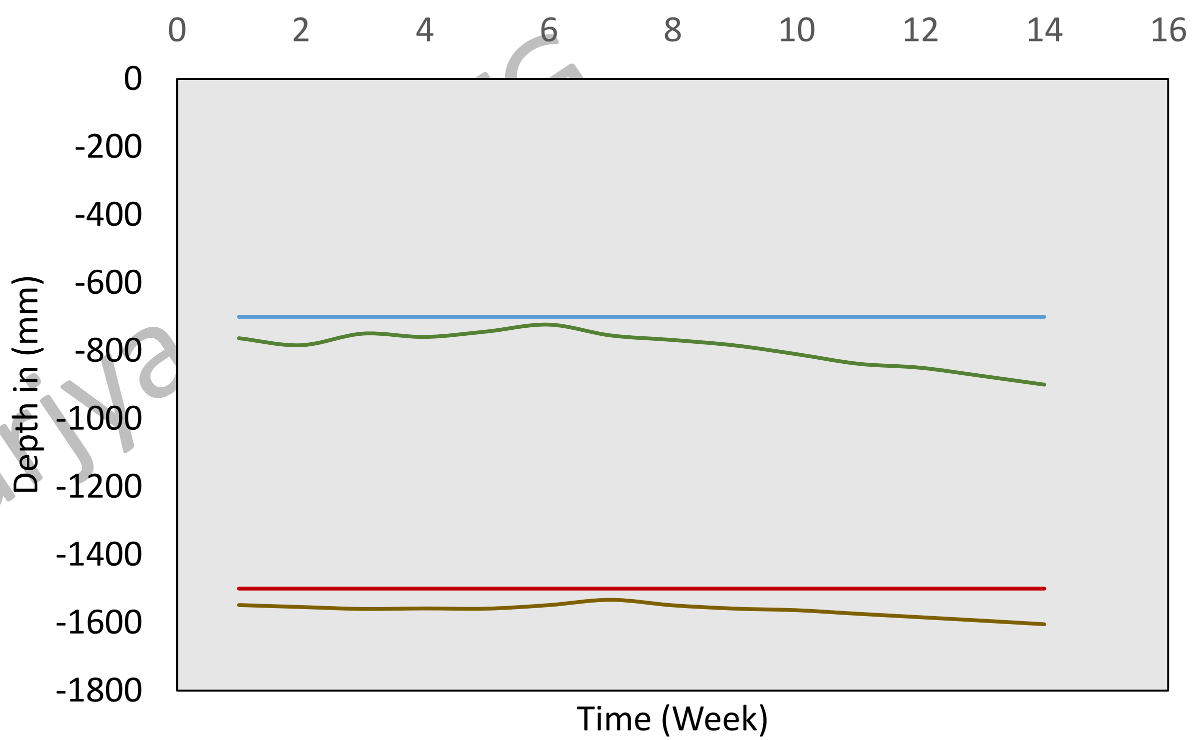
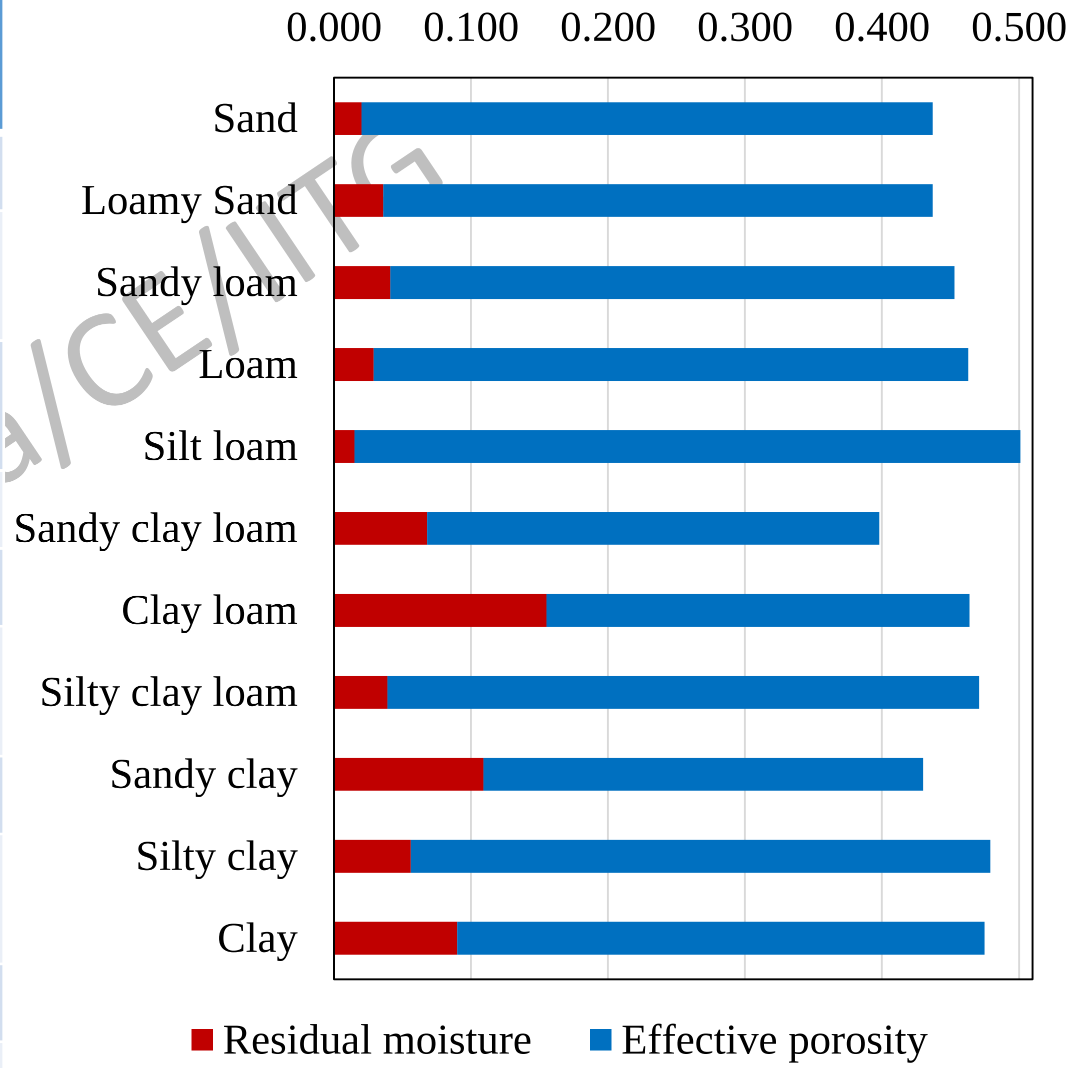
1800
1600
1400
1200
1000
800
600
400
200
0
Depth in (mm)
Time (Week)


0 2 4 6 8 10 12 14 16
1800 1600 1400 1200 1000 800 600 400 200 0
Depth in (mm)
Time (Week)

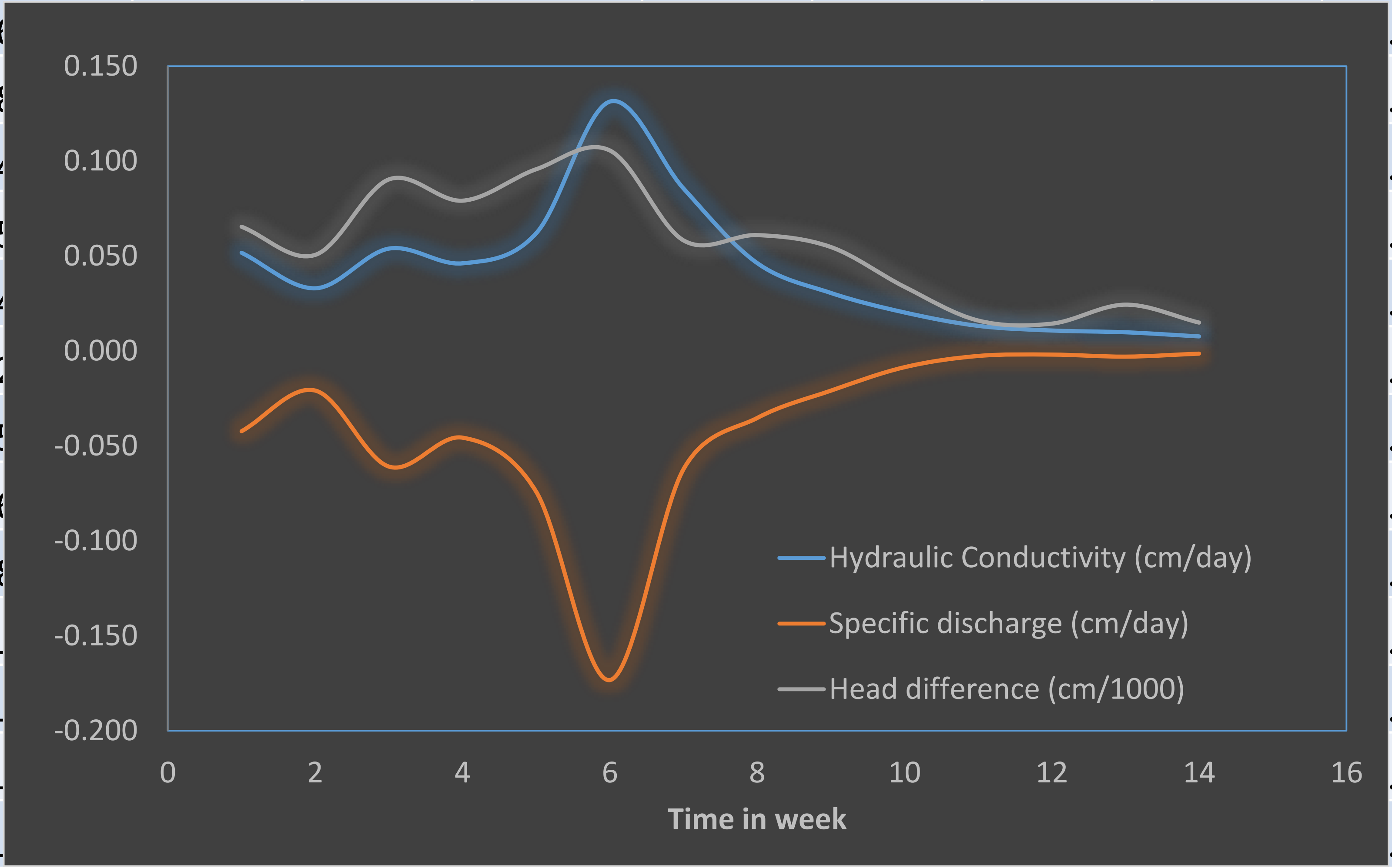

0 2 4 6 8 10 12 14 16
0 -200 -400 -600 -800 -1000 -1200 -1400 -1600 -1800
Depth in (mm)
Time (Week)

Soil Type	Porosity (η)	Residual moisture (θ_r)	Effective porosity (θ_e)	Wetting front soil suction head (ψ in cm)	Hydraulic conductivity (K in cm/h)
Sand	0.437	0.020	0.417	4.950	11.780
Loamy Sand	0.437	0.036	0.401	6.130	2.990
Sandy loam	0.453	0.041	0.412	11.010	1.090
Loam	0.463	0.029	0.434	8.890	0.340
Silt loam	0.501	0.015	0.486	16.680	0.650
Sandy clay loam	0.398	0.068	0.330	21.850	0.150
Clay loam	0.464	0.155	0.309	20.880	0.100
Silty clay loam	0.471	0.039	0.432	27.300	0.100
Sandy clay	0.430	0.109	0.321	23.900	0.060
Silty clay	0.479	0.056	0.423	29.220	0.050
Clay	0.475	0.090	0.385	31.630	0.030



Calculation of soil moisture flux

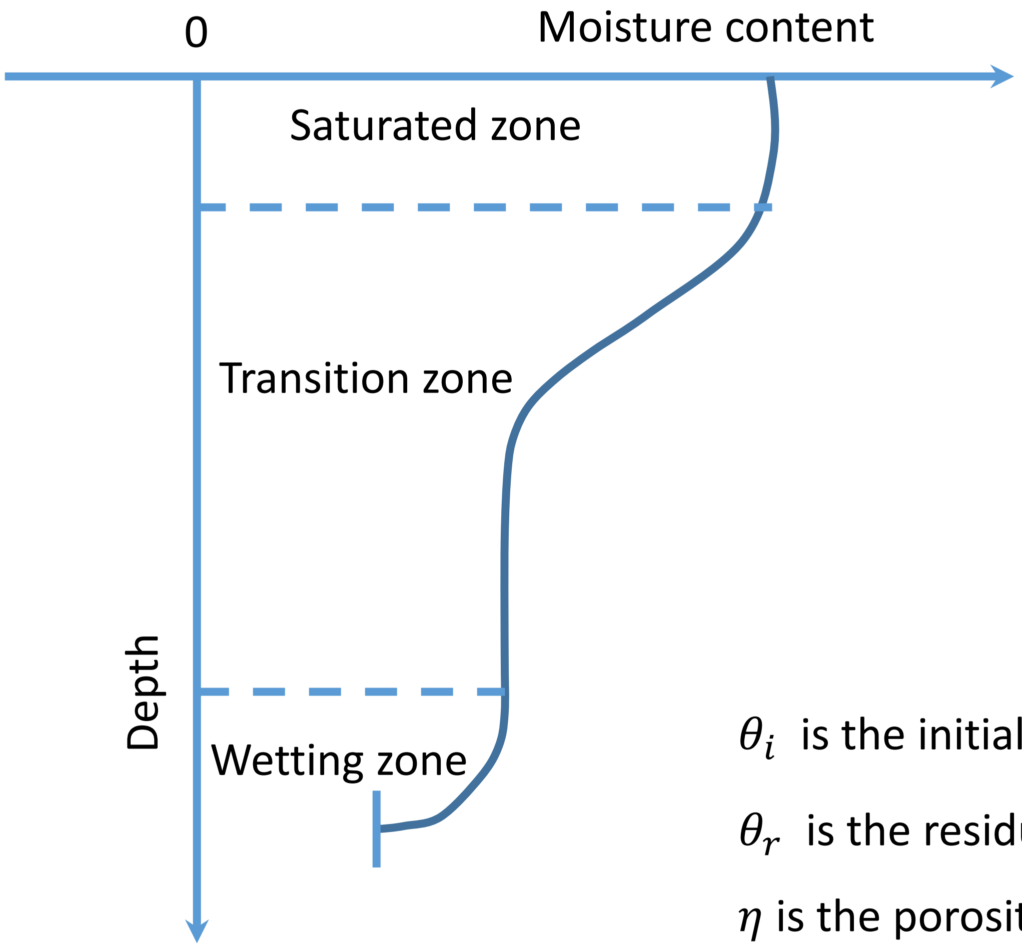
Time (week)	ψ_1 (cm)	ψ_2 (cm)	z_1 (cm)	z_2 (cm)	h_1 (cm)	h_2 (cm)	$h_1 - h_2$ (cm)	K (cm/day)	q (cm/day)
1	-6								.042
2	-8								.021
3	-4								.061
4	-5								.046
5	-4								.074
6	-2								.173
7	-5								.062
8	-6								.035
9	-8								.021
10	-1								.009
11	-1								.003
12	-1								.002
13	-1								.003
14	-170.000	-104.934	-70	-150	-240.00	-254.93	14.93	0.008	-0.001



$$q = -K \frac{h_1 - h_2}{z_1 - z_2}$$

$$K = 250(-\psi)^{-2.11}$$

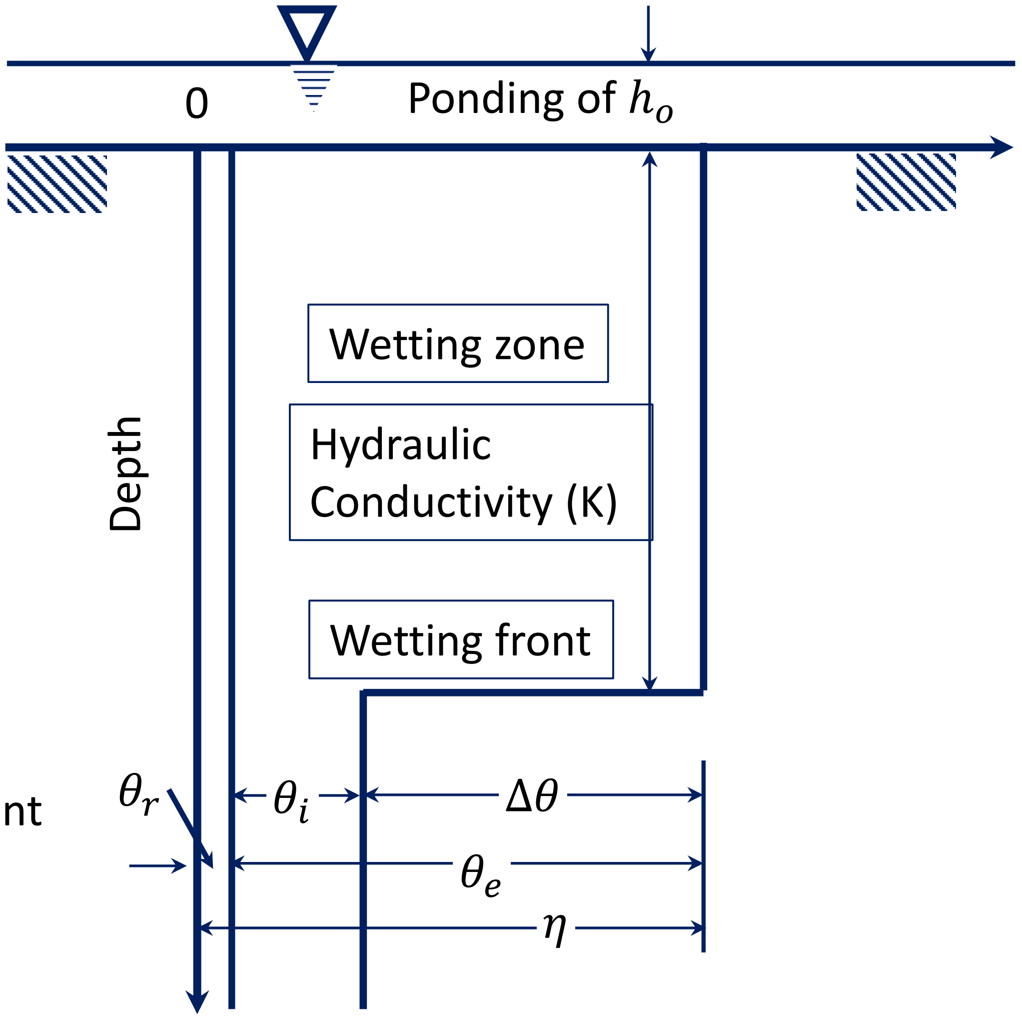
Green Ampt Model



θ_i is the initial moisture content

θ_r is the residual moisture content

η is the porosity of the soil



Green Ampt Model

Increase in water in the control volume is $L(\eta - \theta_i)$

The increase in water in the control volume is equal to the total infiltration

$$F(t) = L(\eta - \theta_i) = L\Delta\theta$$

$$L = F(t)/\Delta\theta$$

Darcy's law

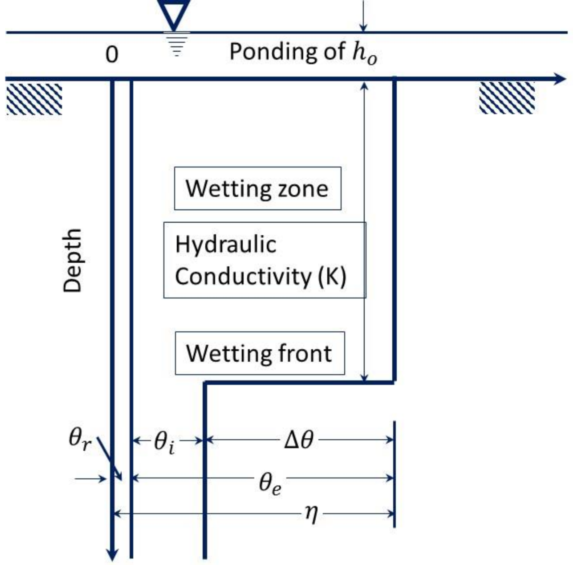
$$q = -K\frac{\partial h}{\partial z}$$

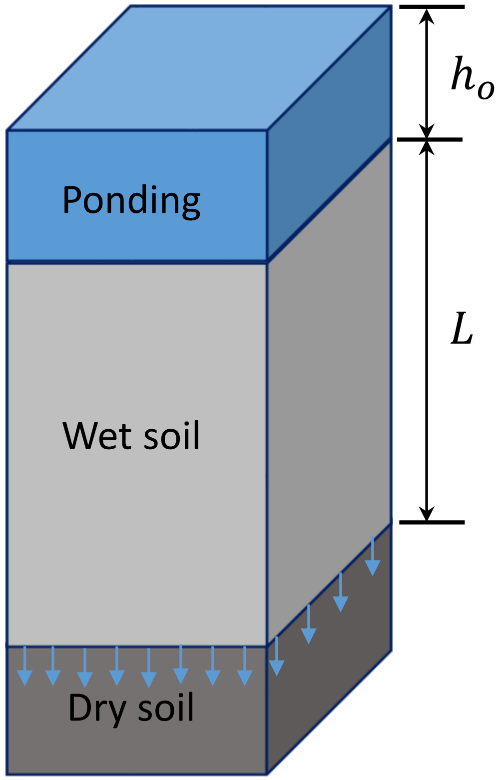
In this case, the Darcy's flux q is constant throughout the depth and is equal to $-f$

$$f = K\frac{\partial h}{\partial z} = K\frac{h_1 - h_2}{z_1 - z_2} = K\frac{h_o - (-\psi - L)}{L}$$

For small value of h_o, h_o can be neglected

$$f = K\frac{\psi + L}{L} = K\frac{\psi + F(t)/\Delta\theta}{F(t)/\Delta\theta} = K\frac{\psi\Delta\theta + F(t)}{F(t)}$$





$$f = K\left[\frac{\psi\Delta\theta + F(t)}{F(t)}\right]$$

$$\frac{dF(t)}{dt} = K\left[\frac{\psi\Delta\theta + F(t)}{F(t)}\right]$$

$$\left[\frac{F(t)}{\psi\Delta\theta + F(t)}\right] dF = kdt$$

$$\left[\frac{F + \psi\Delta\theta}{\psi\Delta\theta + F} - \frac{\psi\Delta\theta}{\psi\Delta\theta + F}\right] dF = kdt$$

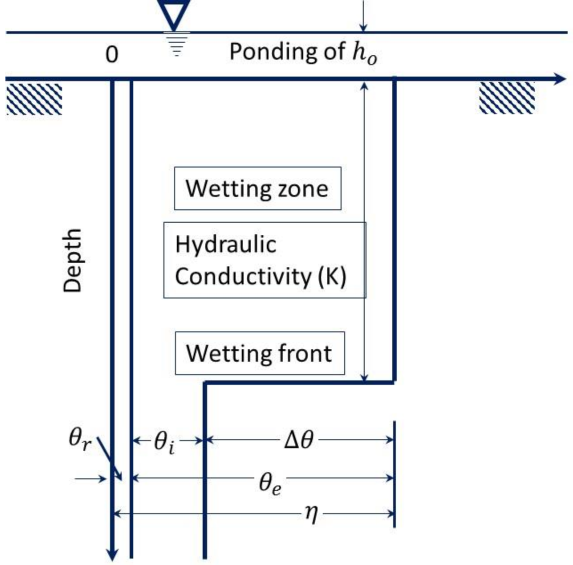
$$\int_0^{F(t)} \left[1 - \frac{\psi\Delta\theta}{\psi\Delta\theta + F}\right] dF = \int_0^t kdt$$

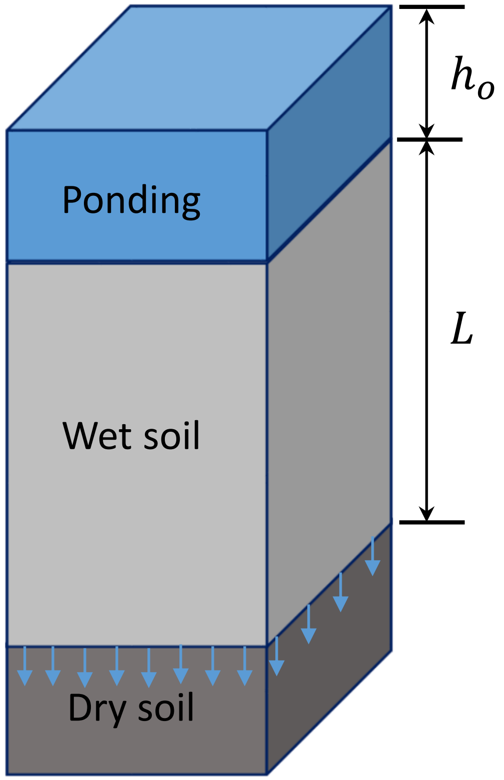
$$F - \psi\Delta\theta ln(F + \psi\Delta\theta)\Big]_0^{F(t)} = kdt\Big]_0^t$$

$$F - \psi\Delta\theta[ln(F + \psi\Delta\theta) - ln(\psi\Delta\theta)] = kt$$

$$F - \psi\Delta\theta\left[ln\left(1 + \frac{F}{\psi\Delta\theta}\right)\right] = kt$$

$$\int \frac{1}{ax + b} dx = \frac{1}{a} ln(ax + b)$$





$$F - \psi\Delta\theta\left[ln\left(1 + \frac{F}{\psi\Delta\theta}\right)\right] = Kt$$

This is the Green Ampt equation for the cumulative infiltration. Once F is obtained, the infiltration can be estimated

$$f = K\left[\frac{\psi\Delta\theta + F(t)}{F(t)}\right] = K\left[1 + \frac{\psi\Delta\theta}{F(t)}\right]$$

The equation for cumulative infiltration can be obtained using successive substitution method

$$F = Kt + \psi\Delta\theta\left[ln\left(1 + \frac{F}{\psi\Delta\theta}\right)\right]$$

- Prior to time of ponding, all rainfall is infiltrated
- Potential infiltration is a function of cumulative infiltration
- Ponding occurs when potential infiltration rate is less than intensity

Ponding time

$$f = K\left[1 + \frac{\psi\Delta\theta}{F(t)}\right]$$

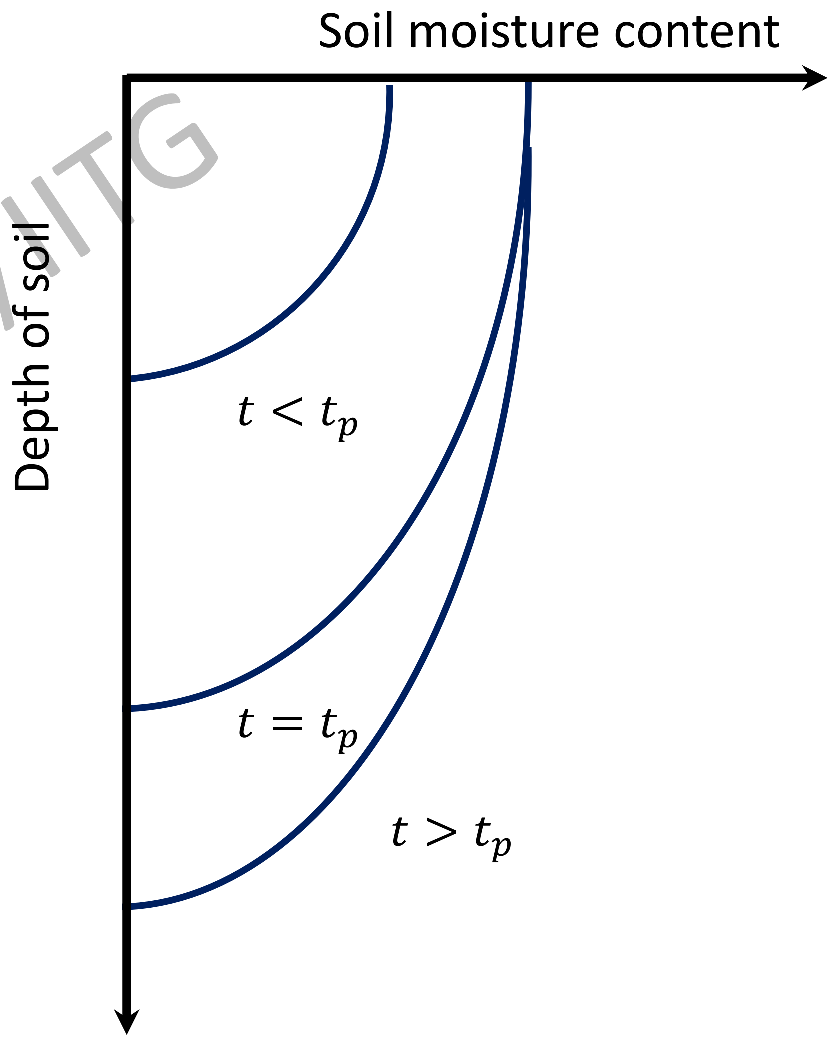
Cumulative infiltration at the time of ponding is

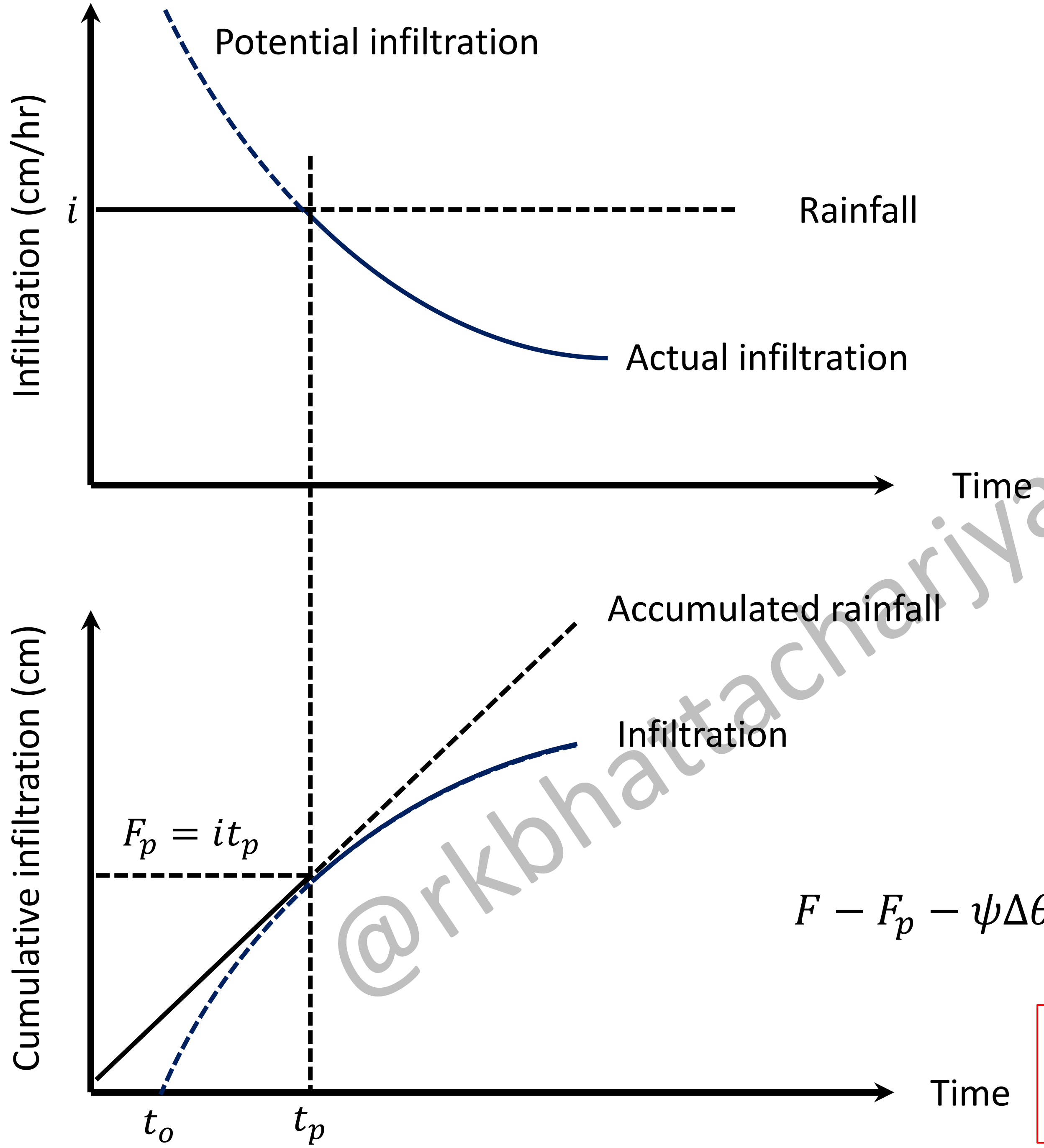
$F_p = it_p$ Where F_p is the cumulative infiltration at the time of ponding, t_p is the ponding time and i is the rainfall intensity

$$f = i$$

$$i = K\left[1 + \frac{\psi\Delta\theta}{it_p}\right]$$

Solving $$\boxed{t_p = \frac{K\psi\Delta\theta}{i(i-K)}}$$





$$F - \psi\Delta\theta\left[ln\left(1 + \frac{F}{\psi\Delta\theta}\right)\right] = Kt$$

Substituting $t = t_p - t_o$ and $F = F_p$

$$F_p - \psi\Delta\theta\left[ln\left(1 + \frac{F_p}{\psi\Delta\theta}\right)\right] = K(t_p - t_o) \qquad (1)$$

For $t > t_p$

$$F - \psi\Delta\theta\left[ln\left(1 + \frac{F}{\psi\Delta\theta}\right)\right] = K(t - t_o) \qquad (2)$$

Subtracting (1) from (2), we have

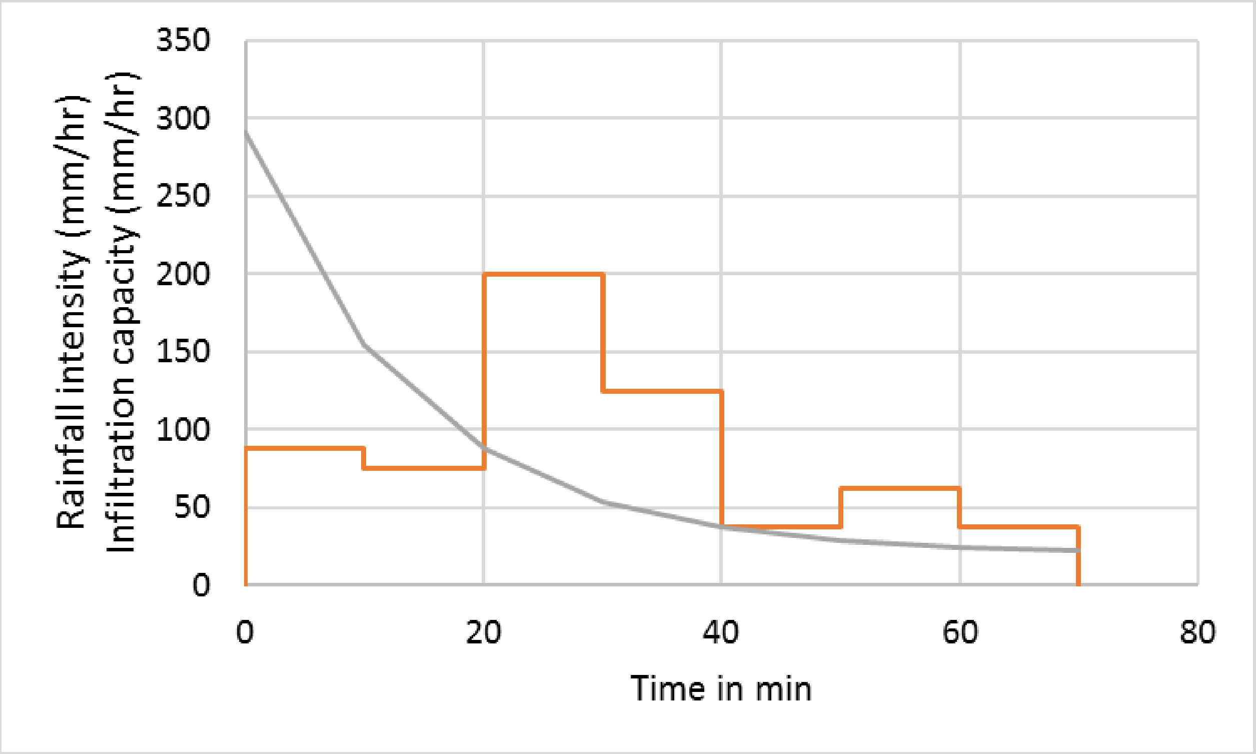
$$F - F_p - \psi\Delta\theta\left[ln\left(1 + \frac{F}{\psi\Delta\theta}\right) - ln\left(1 + \frac{F_p}{\psi\Delta\theta}\right)\right] = K(t - t_p)$$

$$\boxed{F - F_p - \psi\Delta\theta\left[ln\left(\frac{\psi\Delta\theta + F}{\psi\Delta\theta + F_p}\right)\right] = K(t - t_p)}$$

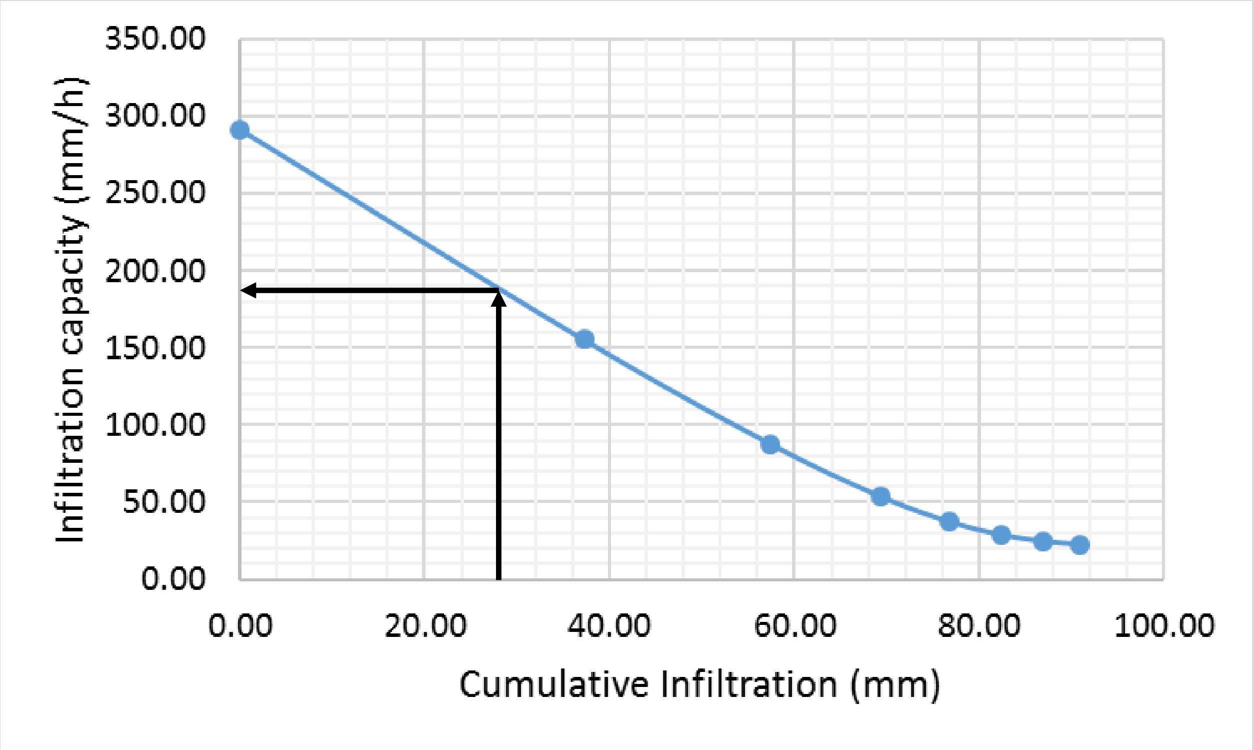
Example: The infiltration capacity curve for a watershed is given by $f_p = (291.5 - 20.75)e^{-0.07t} + 20.75$ where t is in min and f_p is in mm/hr. The storm pattern is as follows.

Time	Intensity (mm)
0-10	87.5
10-20	75
20-30	200
30-40	125
40-50	37.5
50-60	62.5
60-70	37.5

Determine the rainfall excess for the successive 10-min period



Time	Potential Infiltration Capacity (f_p) (mm/hr)	Δt	Average f_p	ΔF (mm)	Cumulative F (mm)
0	291.50				0.00
10	155.20	10	223.35	37.23	37.23
20	87.52	10	121.36	20.23	57.45
30	53.91	10	70.71	11.79	69.24
40	37.21	10	45.56	7.59	76.83
50	28.93	10	33.07	5.51	82.34
60	24.81	10	26.87	4.48	86.82
70	22.77	10	23.79	3.96	90.78



Total rainfall during first two interval 27.08 mm
Cumulative infiltration after 20 min 27.08 mm

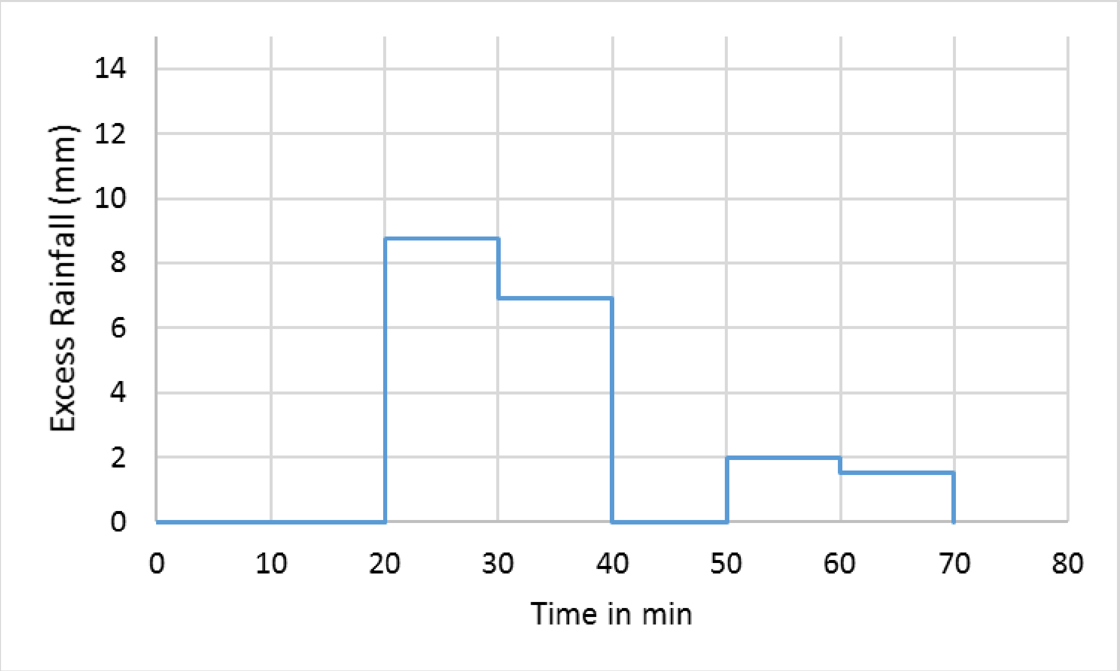
Potential infiltration for F = 27.08 is 190 mm

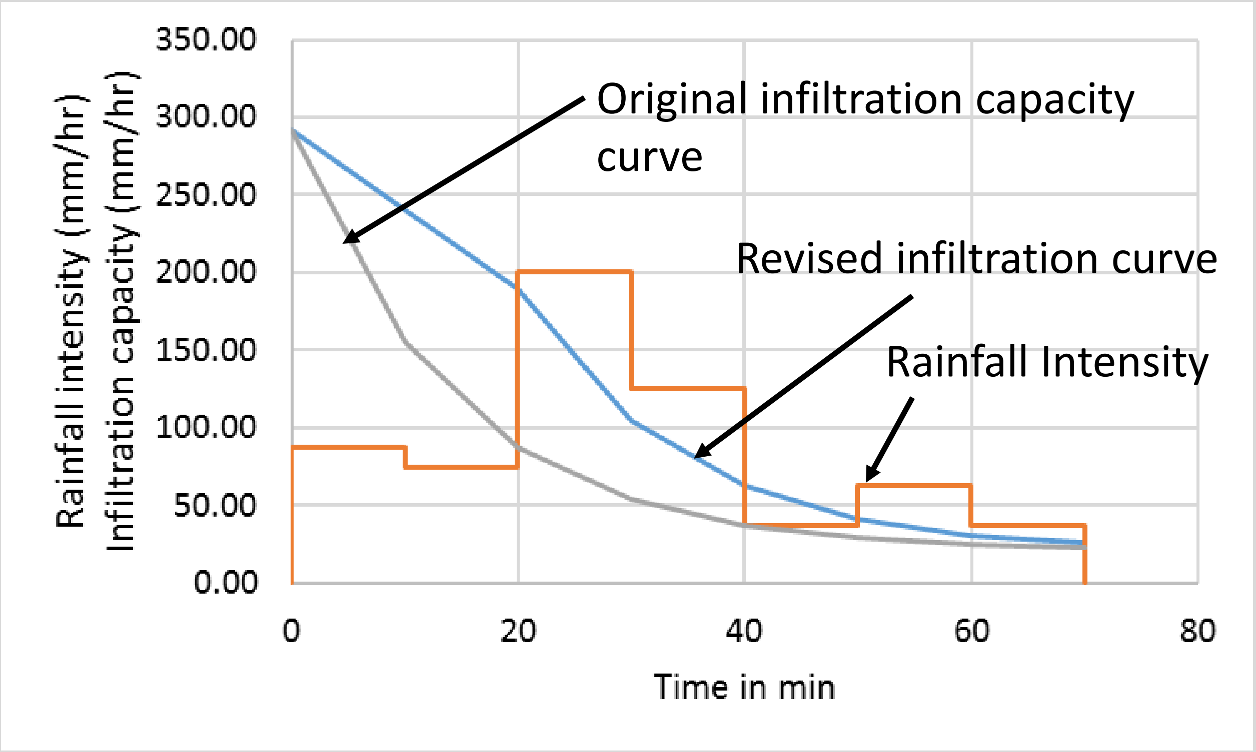
Considering $f_o = 190$ mm

$$f_p = (190.00 - 20.75)e^{-0.07t} + 20.75$$

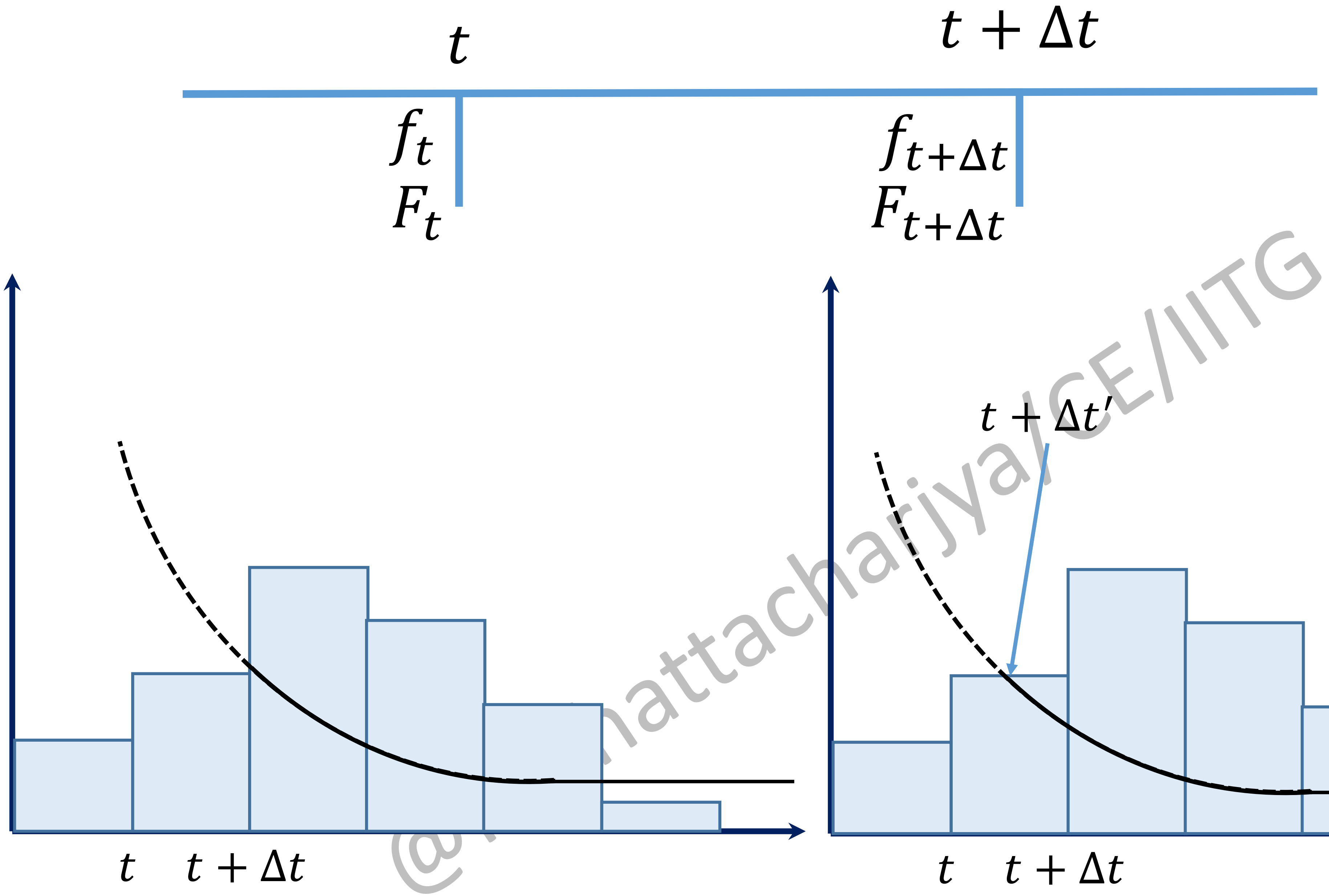
Time	Intensity of Rainfall (i) (mm)	Potential Infiltration Capacity (f_p) (mm/hr)	Δt	Average f_p	ΔF (mm)	$\Delta P = i\Delta t$ (mm)	$ER = \Delta P - \Delta F$	Revised ER
0								
10	87.50		10			14.58		
20	75.00	190.00	10			12.50		
30	200.00	104.80	10	147.40	24.57	33.33	8.77	8.77
40	125.00	62.49	10	83.64	13.94	20.83	6.89	6.89
50	37.50	41.48	10	51.98	8.66	6.25	-2.41	0.00
60	62.50	31.04	10	36.26	6.04	10.42	4.37	4.37+(-2.41)=1.96
70	37.50	25.86	10	28.45	4.74	6.25	1.51	1.51

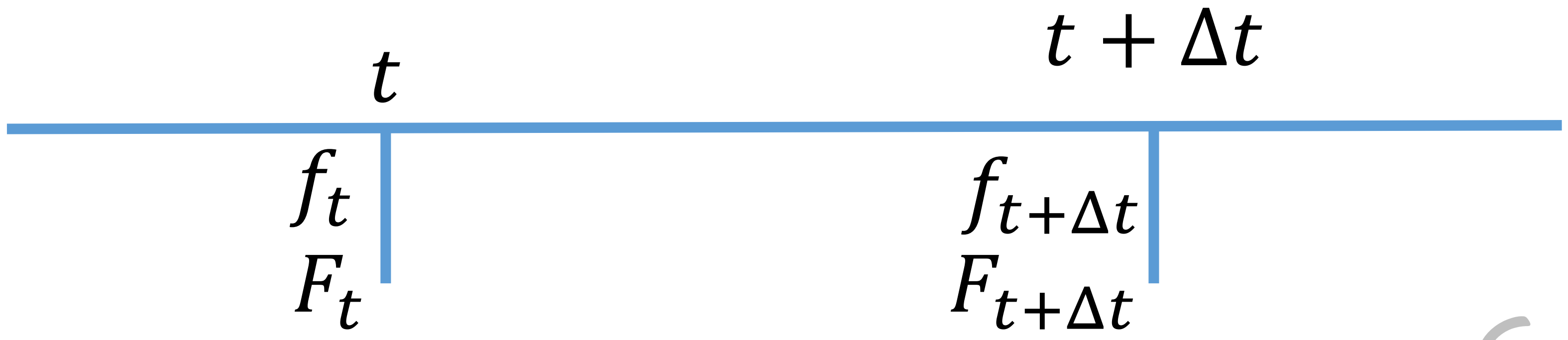
$$f_p = (190.00 - 20.75)e^{-0.07t} + 20.75$$





Excess rainfall using Green-Ampt Method



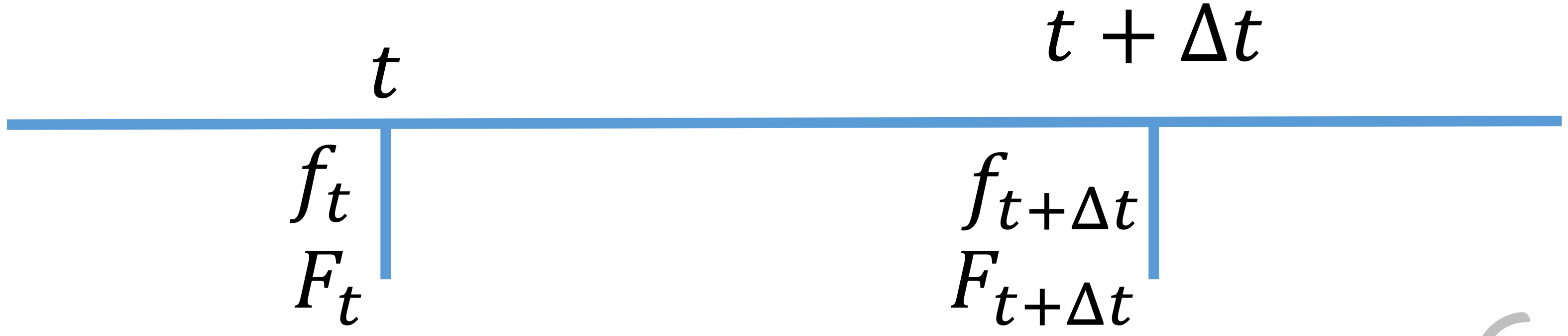


F_t is known, so f_t can be calculated $\quad f_t = K\left[1 + \dfrac{\psi\Delta\theta}{F(t)}\right]$

If f_t is less than i_t, then there is a ponding at the beginning of the time period and $F_{t+\Delta t}$ can be calculated using the following equation

$$\boxed{F_{t+\Delta t} - F_t - \psi\Delta\theta\left[ln\left(\frac{\psi\Delta\theta + F_{t+\Delta t}}{\psi\Delta\theta + F_t}\right)\right] = K\Delta t}$$

If f_t is greater than i_t, then there is no ponding at the beginning of the time period



If f_t is greater than i_t, then there is no ponding at the beginning of the time period

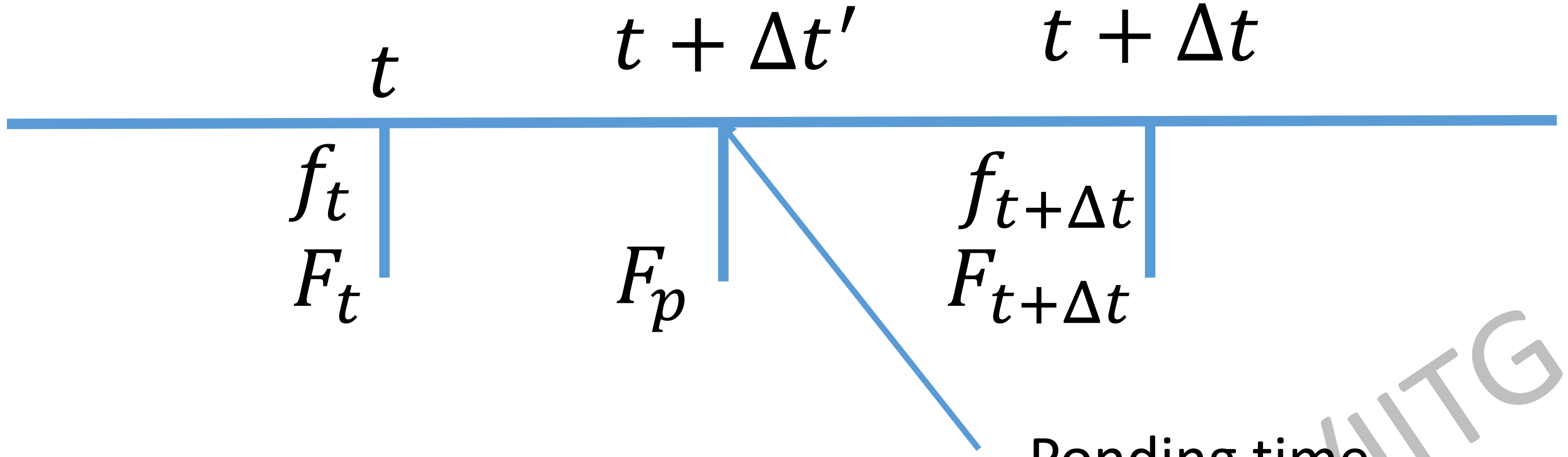
Assume that there is no ponding throughout the interval, the tentative value of cumulative infiltration at $t + \Delta t$ is

$$F'_{t+\Delta t} = F_t + i_t \Delta t$$

Now the corresponding infiltration at $t + \Delta t$ is $$f'_{t+\Delta t} = K\left[1 + \frac{\psi \Delta \theta}{F'_{t+\Delta t}}\right]$$

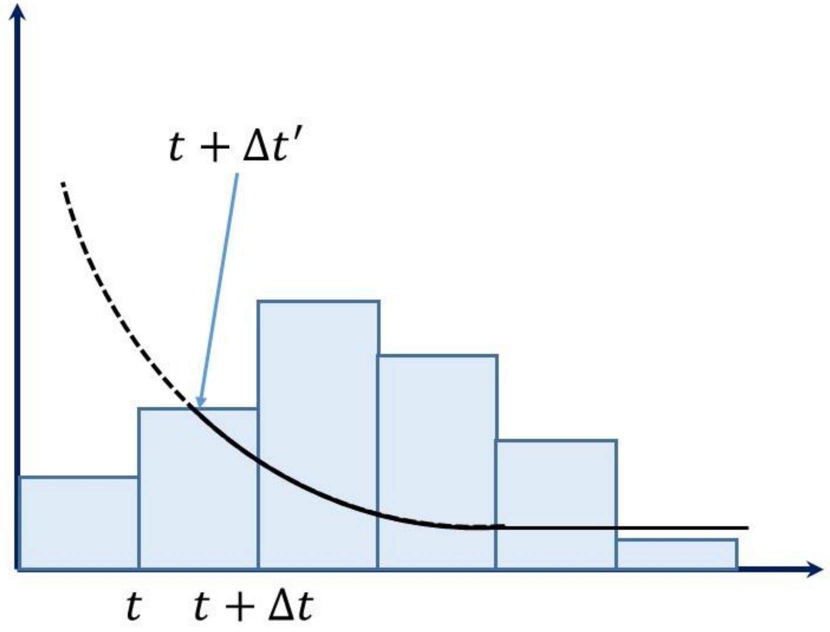
If $f'_{t+\Delta t}$ is greater than i_t, then no ponding occurs during the interval, hence $F_{t+\Delta t} = F'_{t+\Delta t}$

If $f'_{t+\Delta t}$ is less than i_t, then ponding occurs during the interval



If $f'_{t+\Delta t}$ is less than i_t, then ponding occurs during the interval. The cumulative infiltration F_p at ponding time is found by setting $f_t = i_t$ and $F_t = F_p$

$$F_p = \frac{K\psi\Delta\theta}{(i_t - K)}$$



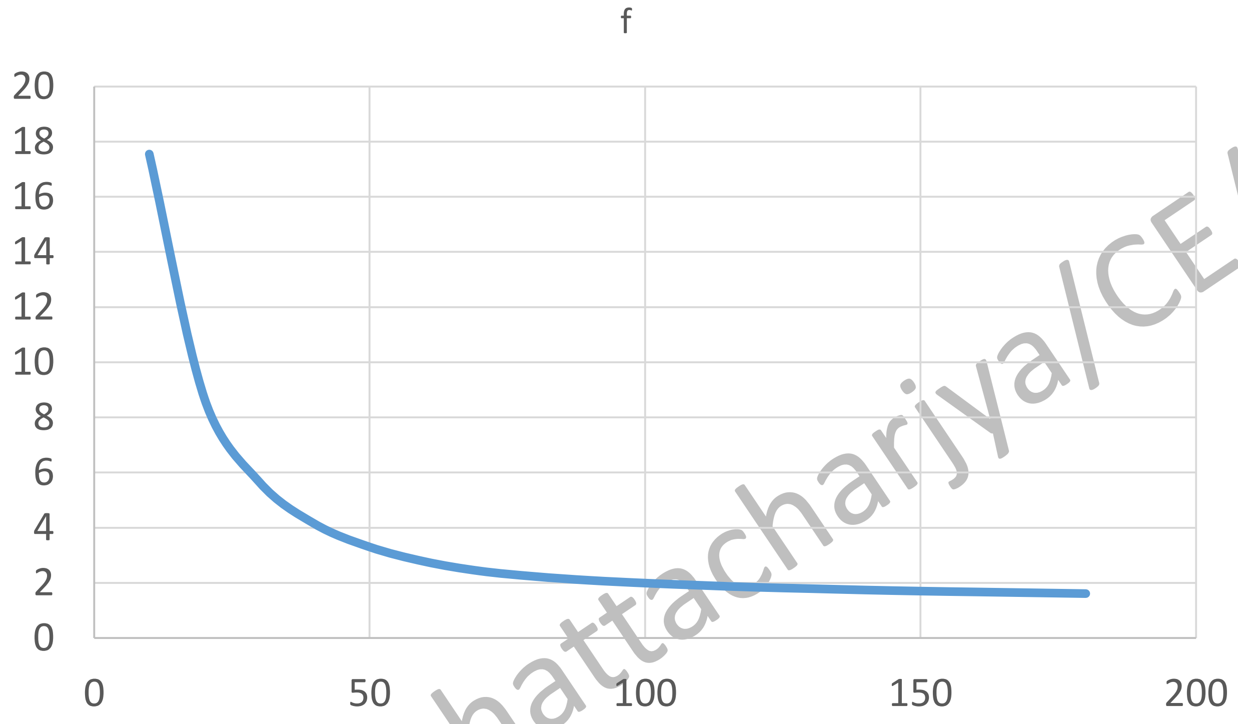
The ponding time is now $t + \Delta t'$

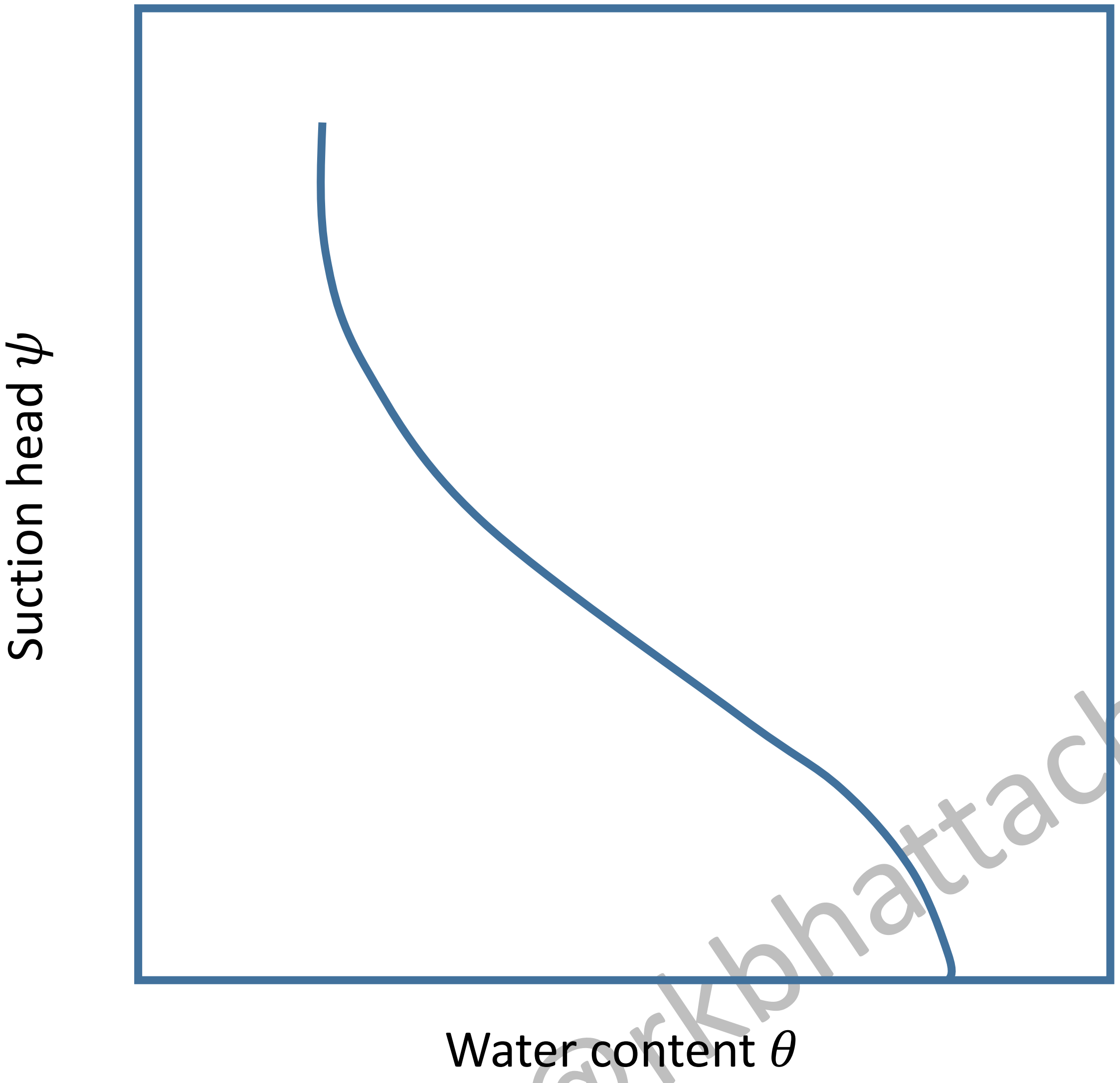
Where $\Delta t' = \frac{F_p - F_t}{i_t}$

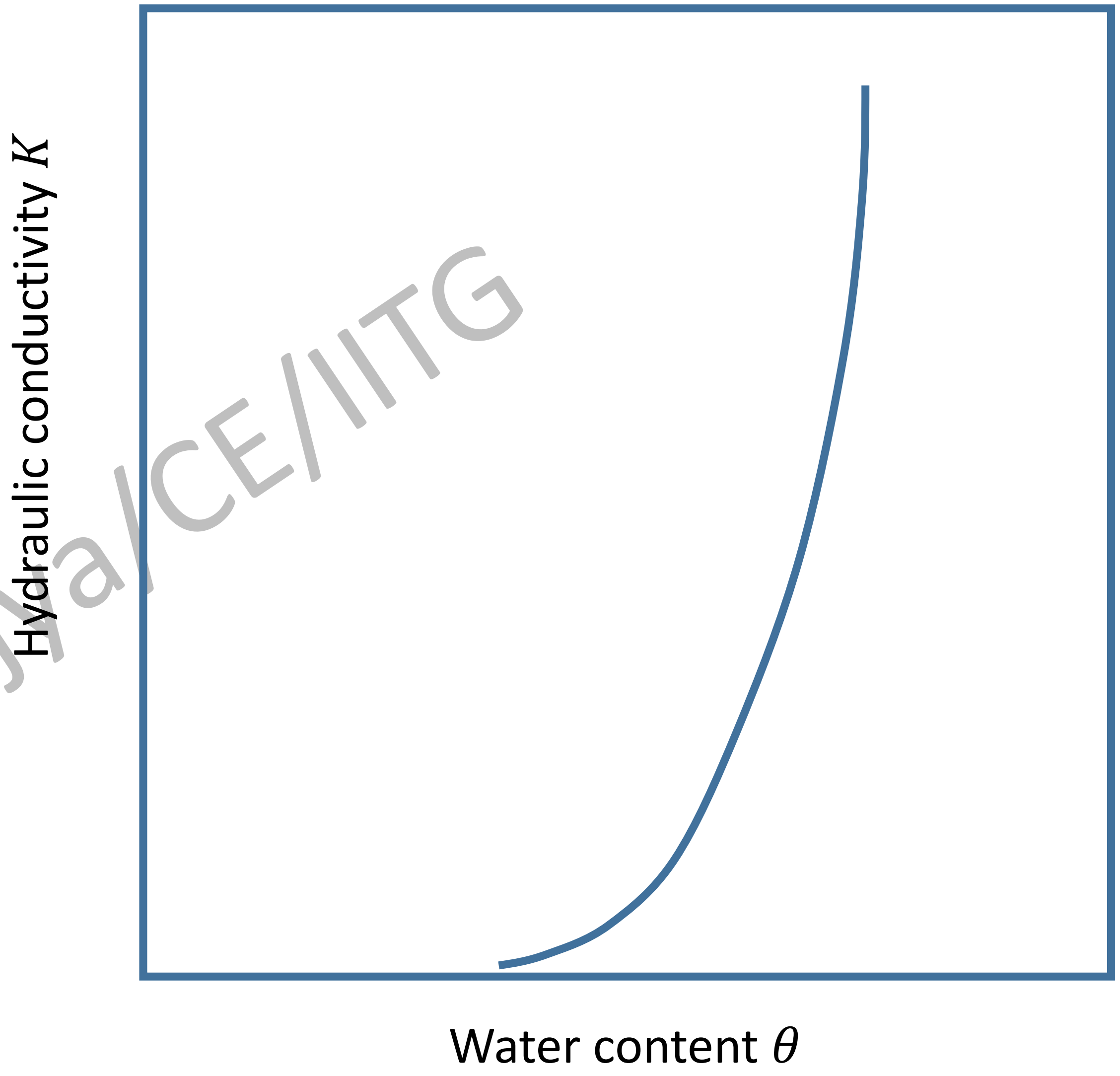
Q. Rainfall hyetograph is given in Table. Estimate infiltration and excess rainfall using Green-Ampt Equation. Take $K = 1.09$ cm/h, $\psi = 11.01$ cm, $\Delta\theta = 0.247$.

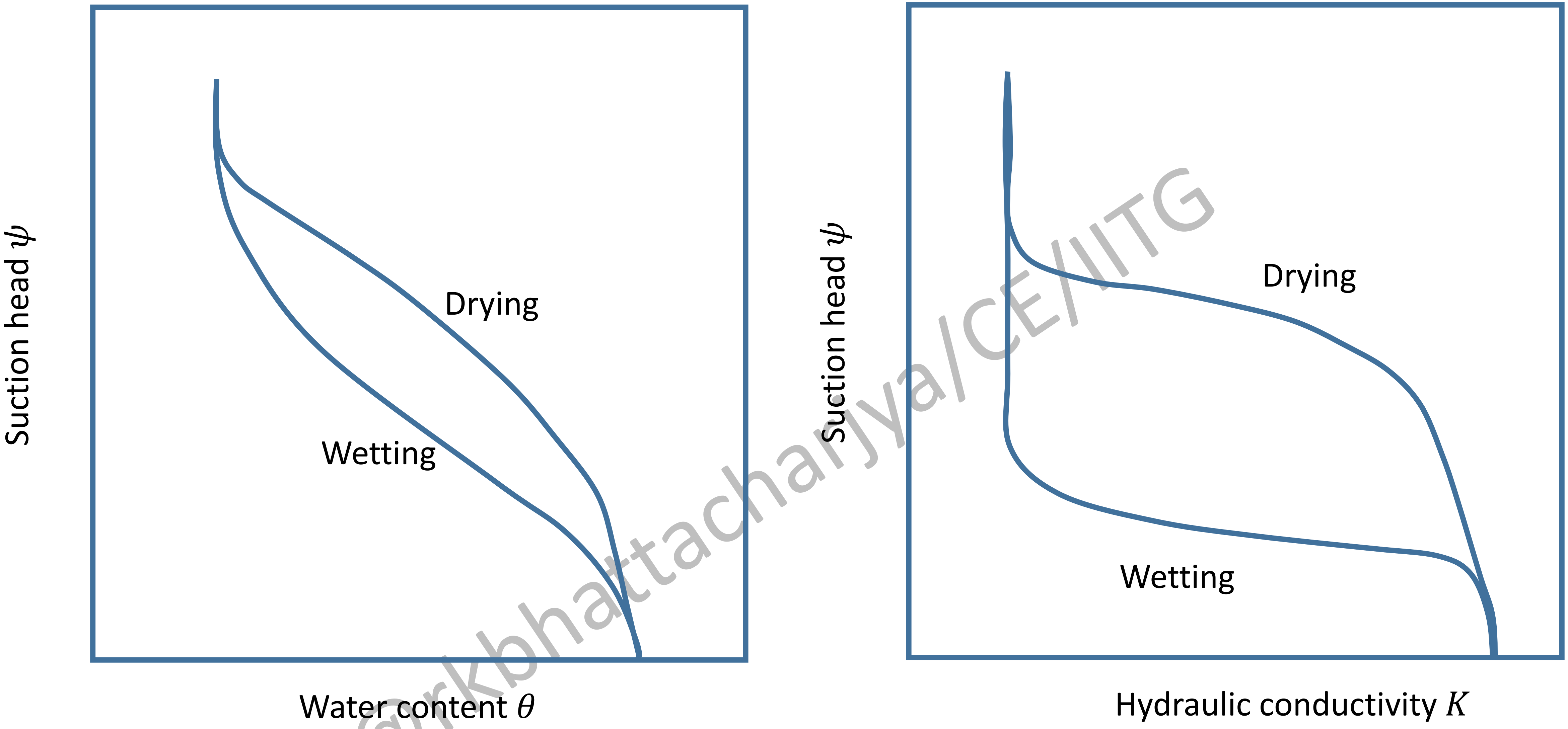
Time (min)	Incremental rainfall (cm)
0	0
10	0.18
20	0.21
30	0.26
40	0.32
50	0.37
60	0.43
70	0.64
80	1.14
90	3.18
100	1.65
110	0.81
120	0.52
130	0.42
140	0.36
150	0.28
160	0.24
170	0.19
180	0.17

Time	Rainfall			Infiltration		Excess Rainfall		
	Incremental rainfall (cm)	Cumulative Rainfall (cm)	Intensity (cm/h)	f (cm/h)	F (cm)	Cumulative (cm)	Incremental (cm)	
0	0	0	1.08		0	0	0	
10	0.18	0.18	1.26	17.55	0.18	0	0	
20	0.21	0.39	1.56	8.69	0.39	0	0	
30	0.26	0.65	1.92	5.65	0.65	0	0	
40	0.32	0.97	2.22	4.14	0.97	0	0	
50	0.37	1.34	2.58	3.3	1.34	0	0	
60	0.43	1.77	3.84	2.77	1.77	0	0	
70	0.64	2.41	6.84	2.43	2.21	0.2	0.2	Ponding
80	1.14	3.55	19.08	2.23	2.59	0.96	0.76	
90	3.18	6.73	9.9	2.09	2.95	3.78	2.82	
100	1.65	8.38	4.86	1.99	3.29	5.09	1.31	
110	0.81	9.19	3.12	1.91	3.61	5.58	0.49	
120	0.52	9.71	2.52	1.84	3.92	5.79	0.21	
130	0.42	10.13	2.16	1.79	4.22	5.91	0.12	
140	0.36	10.49	1.68	1.74	4.51	5.98	0.07	
150	0.28	10.77	1.44	1.7	4.79	5.98	0	
160	0.24	11.01	1.14	1.67	5.07	5.94	0	
170	0.19	11.2	1.02	1.64	5.35	5.85	0	
180	0.17	11.37	0	1.61	5.62	5.75	0	


f
20
18
16
14
12
10
8
6
4
2
0
0
50
100
150
200

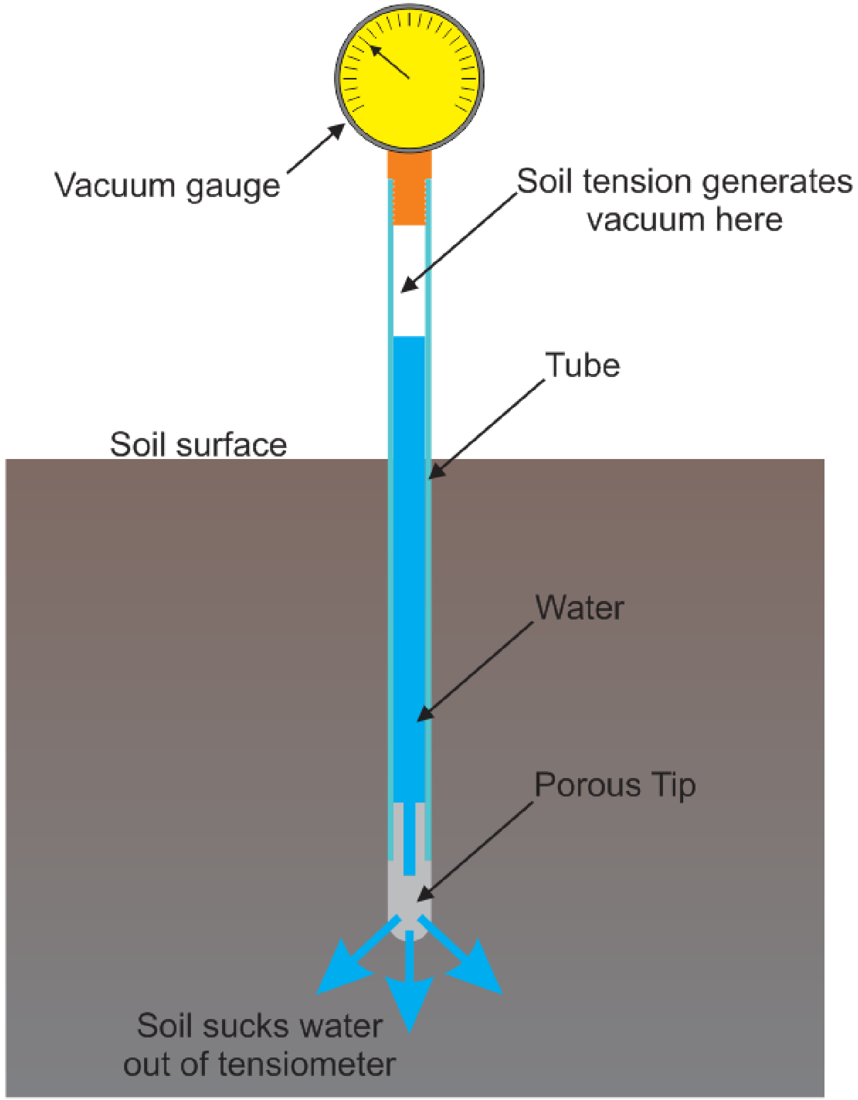

Suction head ψ
Water content θ


Hydraulic conductivity K
Water content θ


Suction head ψ
Drying
Wetting
Water content θ
Suction head ψ
Drying
Wetting
Hydraulic conductivity K

Hysteresis

Tensiometer



Net flow = Rate of change in storage

$$q\,dxdy - \left(q + \frac{\partial q}{\partial z}dz\right)dxdy = \frac{\partial \theta}{\partial t}dxdydz$$

$$\frac{\partial q}{\partial z} + \frac{\partial \theta}{\partial t} = 0$$

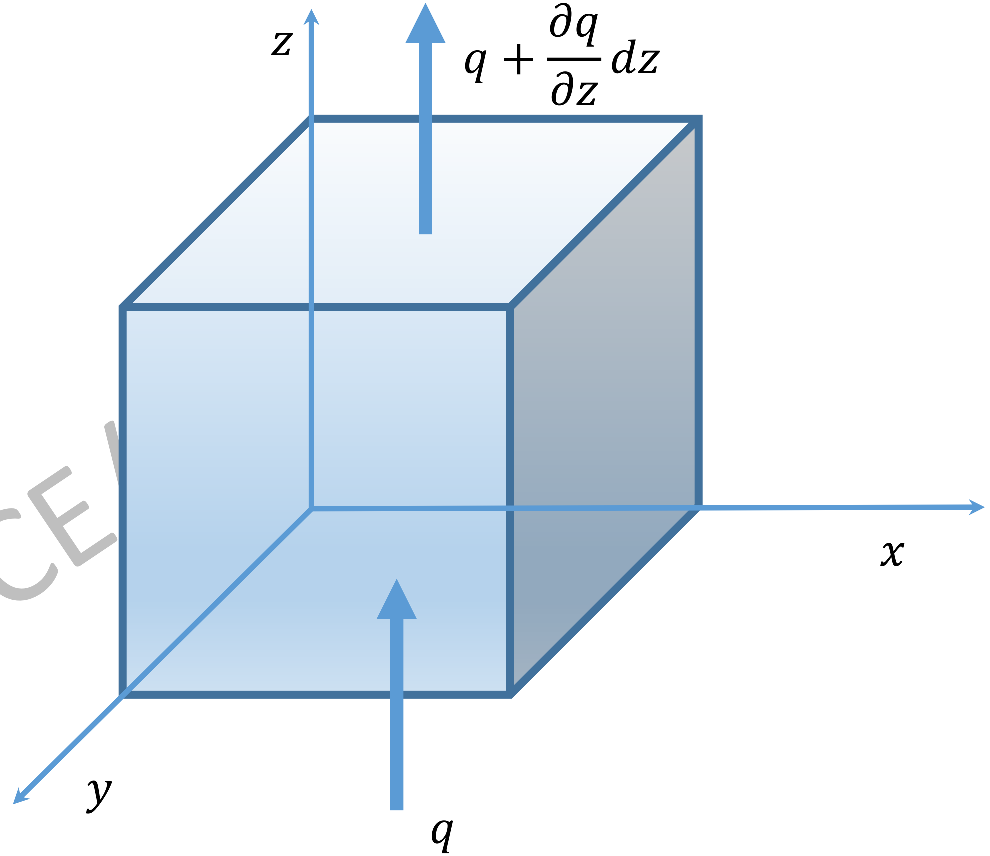
This is the continuity equation for one dimensional unsteady flow in unsaturated flow in porous medium.

Darcy's law

$$q = -K\frac{\partial h}{\partial z}$$

$$h = z + \psi$$

$$q = -K\frac{\partial (z+\psi)}{\partial z}$$



$$q = -K\frac{\partial \psi}{\partial z} - K$$

$$q = -K\left[\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z} + 1\right]$$

$$q = -\left[K\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z} + K\right] = -\left[D\frac{\partial \theta}{\partial z} + K\right]$$

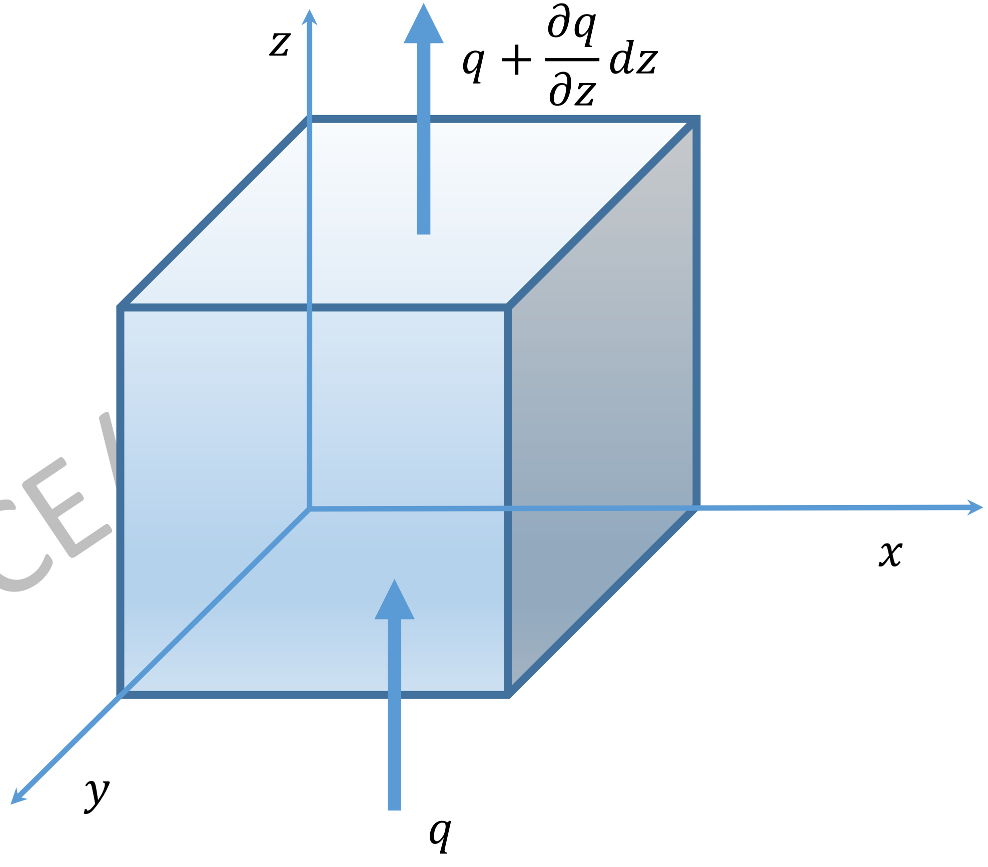
D is the soil water diffusivity $K\frac{\partial\psi}{\partial\theta}$ which has dimension of $[L^2/T]$

The continuity equation

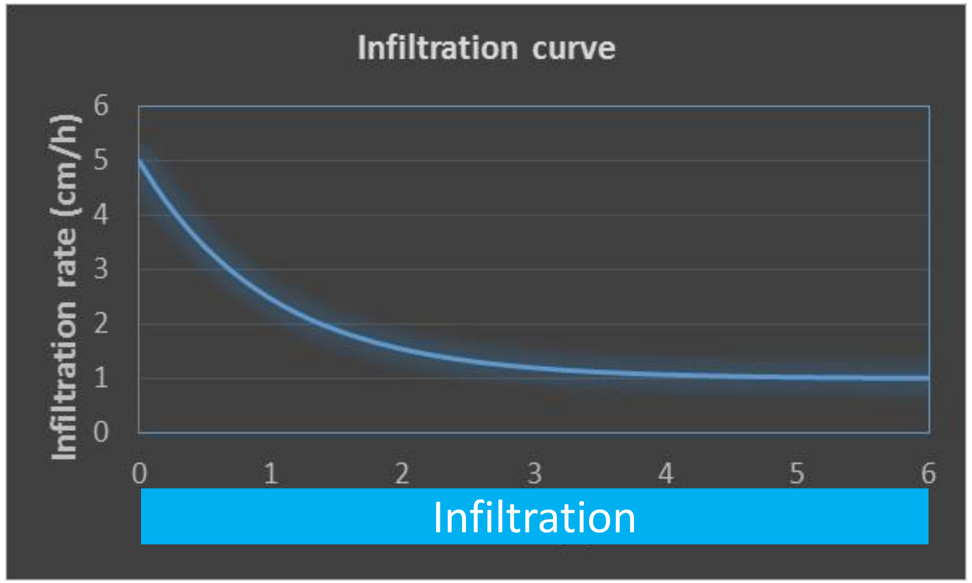
$$\frac{\partial q}{\partial z}+\frac{\partial\theta}{\partial t}=0$$

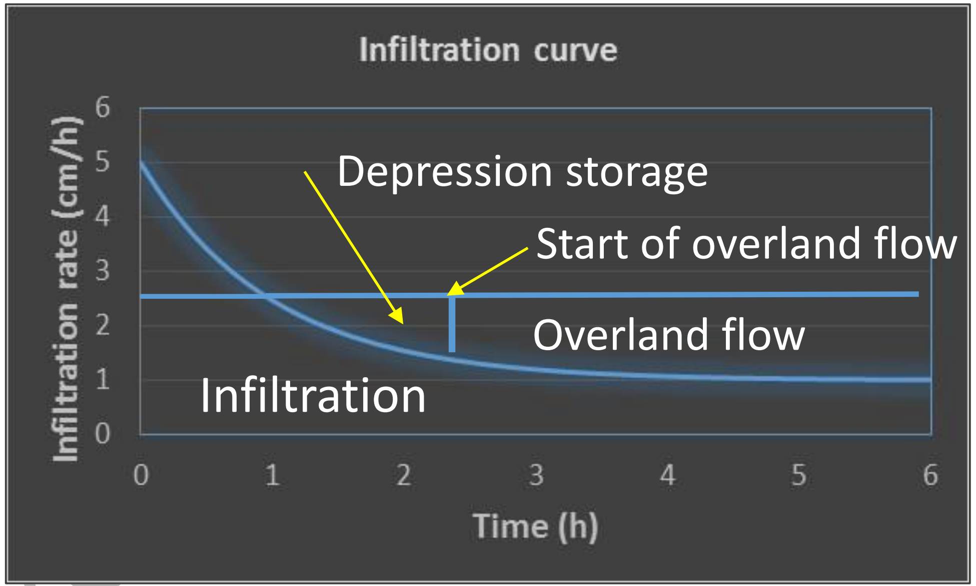
$$\frac{\partial\theta}{\partial t}=-\frac{\partial}{\partial z}\left[-\left(D\frac{\partial\theta}{\partial z}+K\right)\right]$$

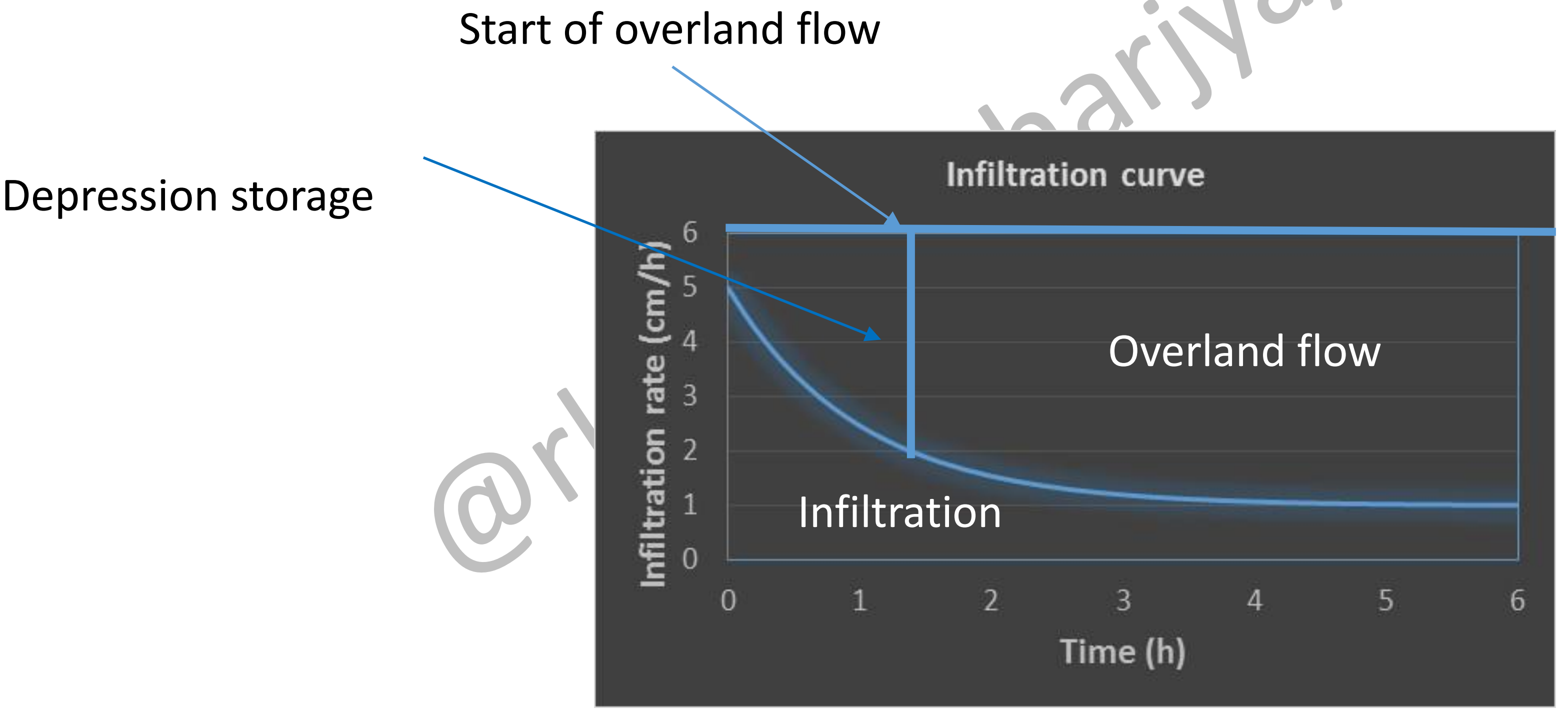
$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left[\left(D\frac{\partial\theta}{\partial z}+K\right)\right]$$

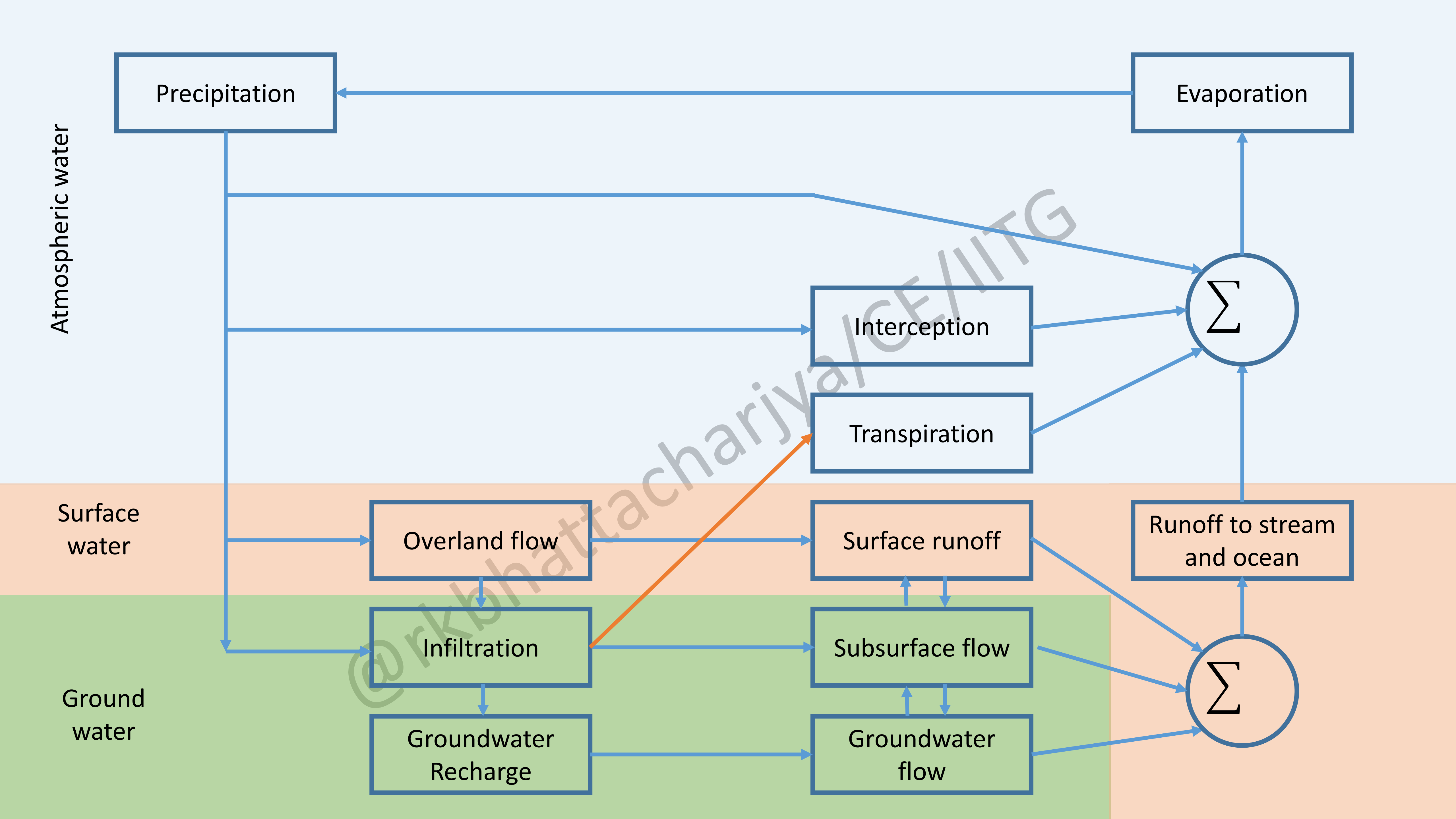


This is the one dimensional form of Richard’s Equation, the governing equation for unsteady unsaturated flow in porous medium


Infiltration curve
Infiltration rate (cm/h)
0
1
2
3
4
5
6
0
1
2
3
4
5
6
Infiltration


Infiltration curve
Infiltration rate (cm/h)
Depression storage
Start of overland flow
Overland flow
Infiltration
0
1
2
3
4
5
6
0
1
2
3
4
5
6
Time (h)


Start of overland flow
Depression storage
Infiltration curve
Infiltration rate (cm/h)
Overland flow
Infiltration
0
1
2
3
4
5
6
0
1
2
3
4
5
6
Time (h)



Atmospheric water
Precipitation
Evaporation
$\sum$
Interception
Transpiration
Surface water
Overland flow
Surface runoff
Runoff to stream and ocean
Ground water
Infiltration
Subsurface flow
$\sum$
Groundwater Recharge
Groundwater flow

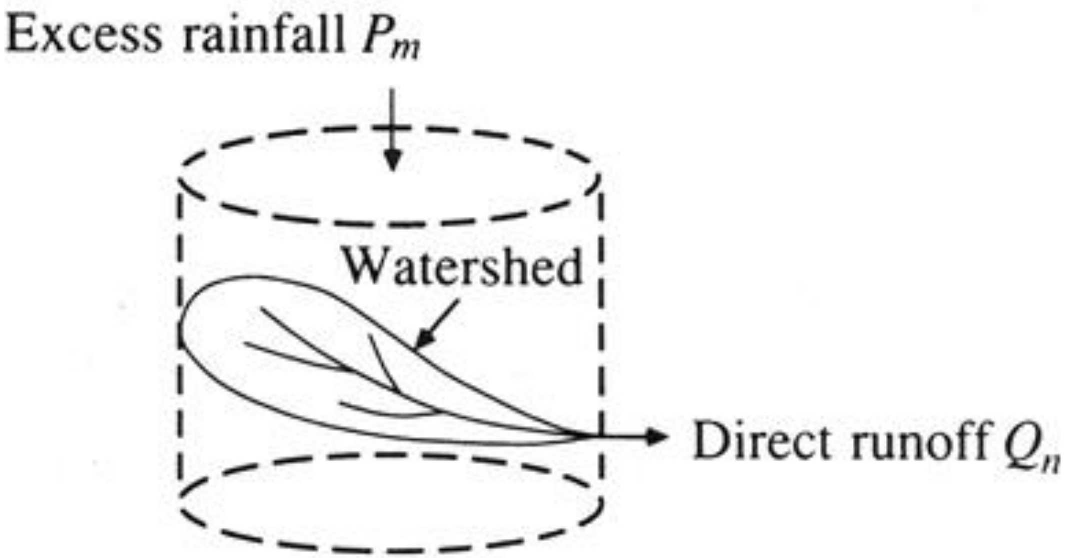
Hydrologic system model

$Q(t) = \Omega I(t)$ Where $Q(t)$ is the input to the system and $I(t)$ is output from the system

$Q(t) = CI(t)$ C is a constant

The transfer function is $\Omega = \dfrac{Q(t)}{I(t)} = C$

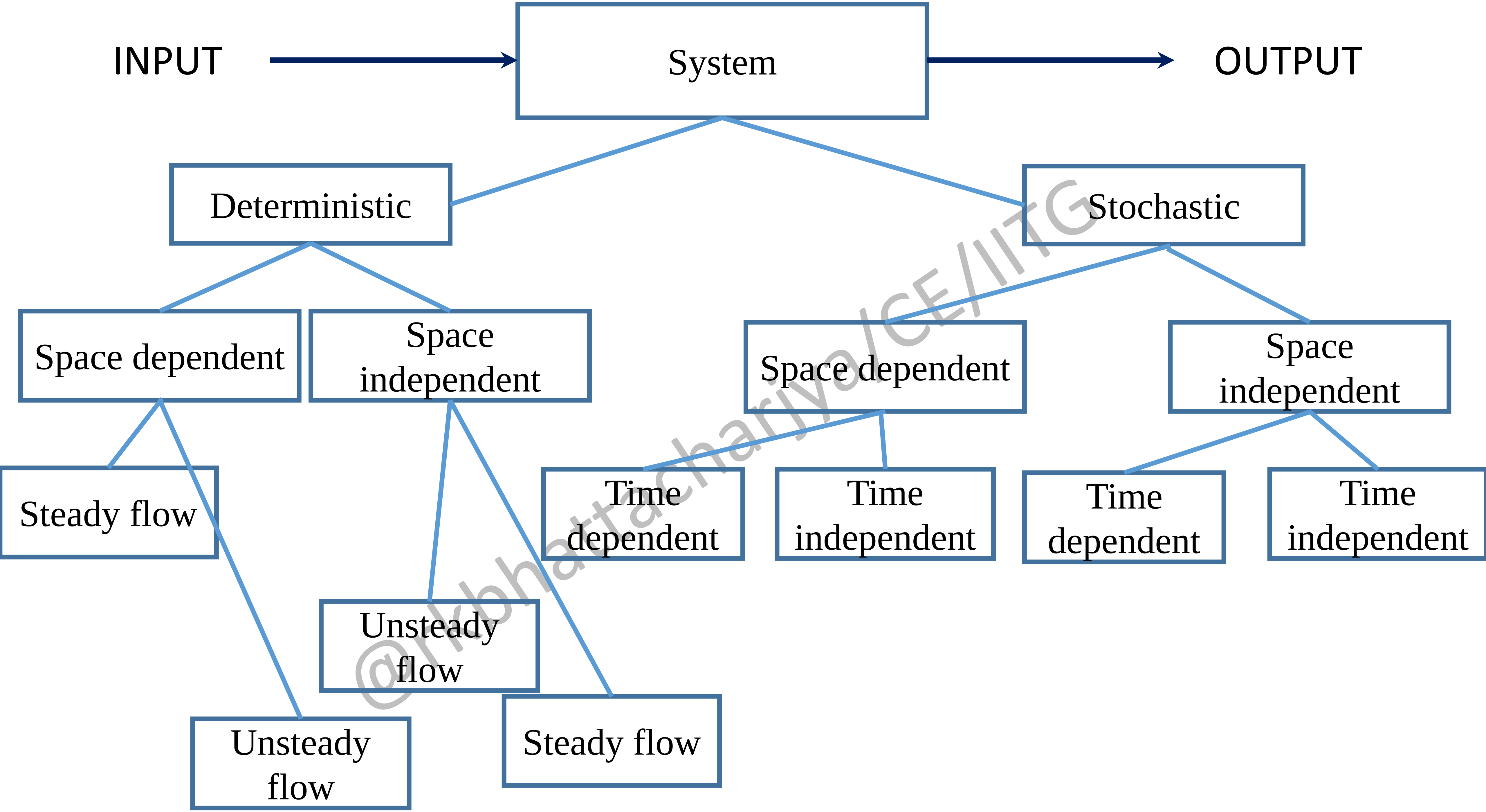
Now consider a linear reservoir system $S = kQ$



$$\frac{dS}{dt} = I - Q$$

$$K\frac{dQ}{dt} + Q = I$$

$$\boxed{\Omega = \frac{Q(t)}{I(t)} = \frac{1}{1 + kD}}$$



INPUT
System
OUTPUT
Deterministic
Stochastic
Space dependent
Space independent
Space dependent
Space independent
Steady flow
Time dependent
Time independent
Time dependent
Time independent
Unsteady flow
Unsteady flow
Steady flow
@rkbhattachariya/CE/IITG

Excess rainfall

Excess rainfall or effective rainfall is that rainfall which is neither retained on the land surface nor infiltrated into the soil

ϕ −index is the constant rate of abstraction (cm/hr) that will yield on excess rainfall hyetograph (ERH) with a total depth equal to the depth of direct runoff (r_d) over the watershed.

$$r_d = \sum_{m=1}^{M} (R_m - \phi \Delta t)$$

R_m is the observed rainfall
r_d is the direct runoff
M is the total pulses of rainfall

Runoff coefficient

Abstraction may be accounted by means of runoff coefficient. A runoff coefficient is defined by the ratio of runoff to rainfall over a time period.

$$C = \frac{r_d}{\sum_{m=1}^{M} R_m}$$

SCS method of abstractions

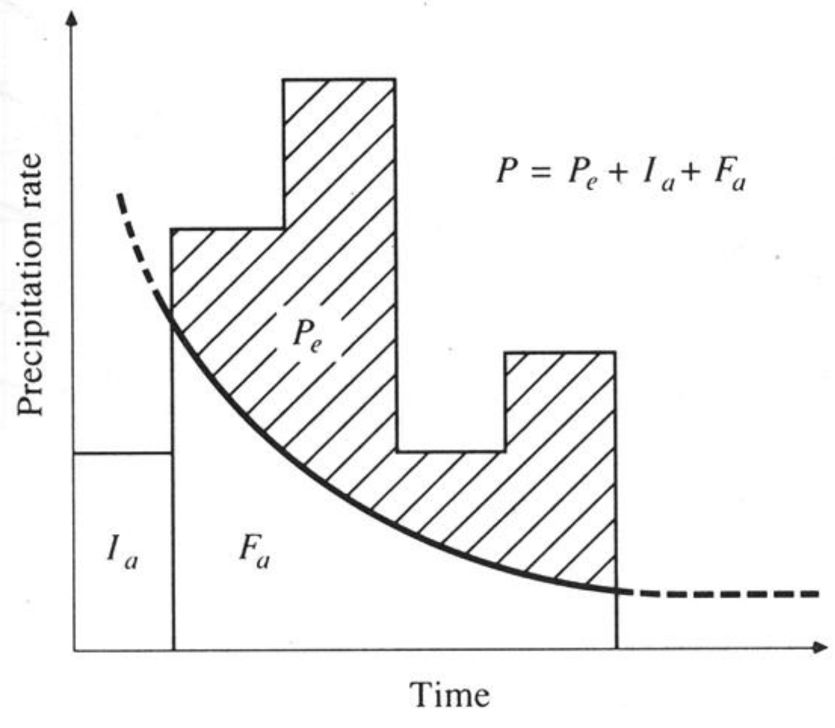
P_e is the depth of excess precipitation or direct runoff

P is the precipitation

F_a is the additional depth of water retained in the watershed

S is potential maximum retention

I_a is the initial abstraction



The depth of excess precipitation or direct runoff P_e is always less than or equal to the depth of precipitation P.

The additional depth of water retained in the watershed F_a is less than or equal to some potential maximum retention S

There are some amount of rainfall I_a for which no runoff will occur. So potential runoff is $P - I_a$.

The hypothesis of the SCS method is that the ratio of two actual to the two potential quantities are equal

$$\frac{F_a}{S} = \frac{P_e}{P - I_a}$$

$$P = P_e + I_a + F_a$$

$$P_e = \frac{(P - I_a)^2}{P - I_a + S}$$

P_e is the depth of excess precipitation or direct runoff

P is the precipitation

F_a is the additional depth of water retained in the watershed

S is potential maximum retention

I_a is the initial abstraction

This is the equation for computing depth of excess rainfall or direct runoff from a storm by SCS method

Experimental studies conducted showed that $I_a = 0.2S$

$$P_e = \frac{(P - 0.2S\,)^2}{P - 0.2S + S} \qquad \boxed{P_e = \frac{(P - 0.2S\,)^2}{P + 0.8S}}$$

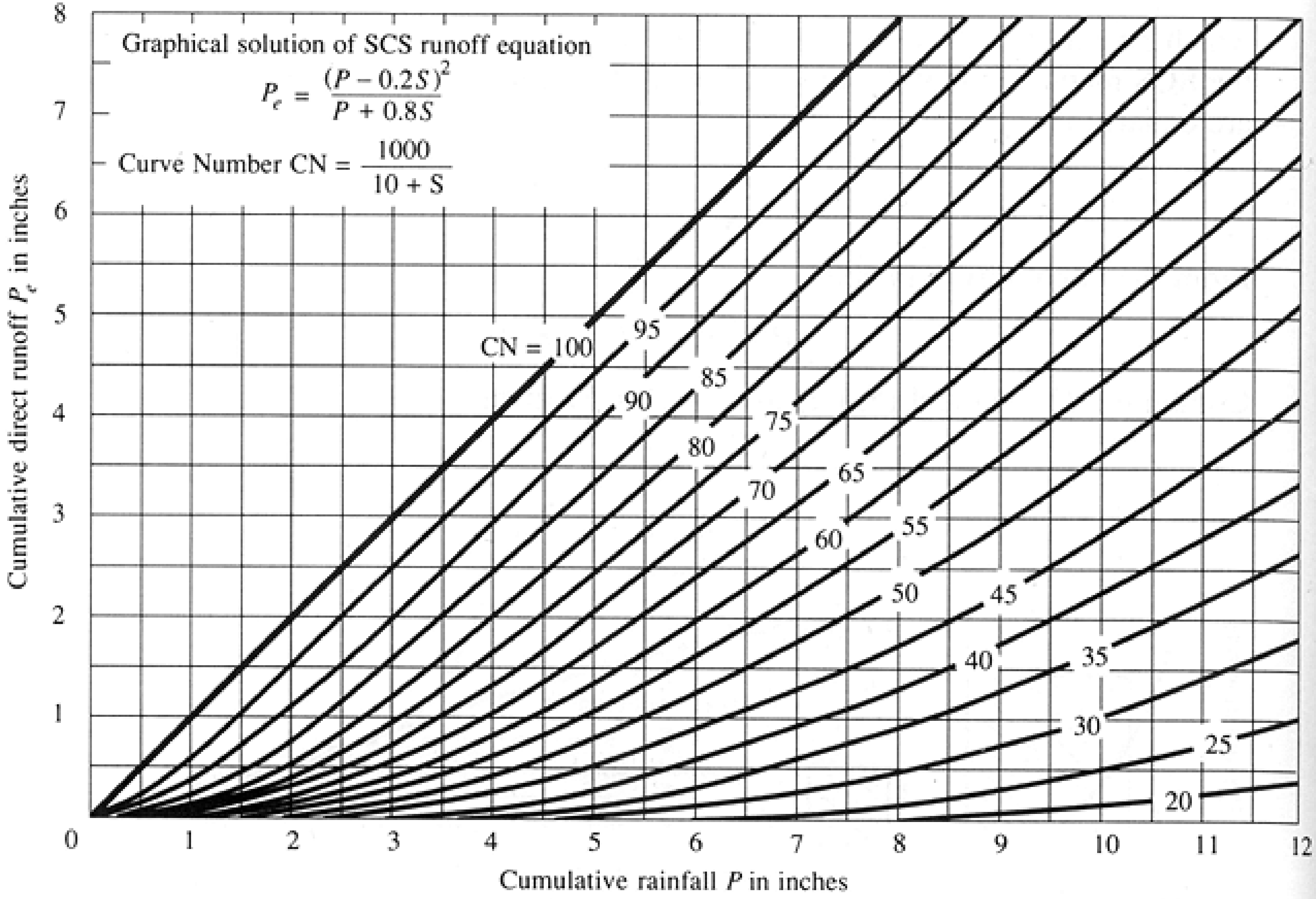


FIGURE 5.5.2
Solution of the SCS runoff equations. (*Source:* Soil Conservation Service, 1972, Fig. 10.1, p. 10.21)

The curve number and S are related by

$$S = \frac{1000}{CN} - 10$$

Where S is in inches. The curve number shown in the Fig. apply for normal antecedent moisture content condition (AMC II). For the Dry (AMC I) or wet condition (AMC III), the equivalent curve number can be computed by

$$CN(I) = \frac{4.2CN(II)}{10 - 0.058CN(II)}$$

$$CN(III) = \frac{23CN(II)}{10 + 0.13CN(II)}$$

Group A: Deep sand, deep loess, aggregated silts

Group B: Shallow loess, sandy loam

Group C: Clay loams, shallow sandy loam, soil in low organic content, and soils usually high in clay

Group D: Soils that swell significantly when wet, heavy plastic clays and contain saline soils

For impervious and water surface CN = 100

For natural surface CN <100

TABLE 5.5.2
Runoff curve numbers for selected agricultural, suburban, and urban land uses (antecedent moisture condition II, $I_a = 0.2S$)

Land Use Description		Hydrologic Soil Group			
		A	B	C	D
Cultivated land[1]: without conservation treatment		72	81	88	91
with conservation treatment		62	71	78	81
Pasture or range land: poor condition		68	79	86	89
good condition		39	61	74	80
Meadow: good condition		30	58	71	78
Wood or forest land: thin stand, poor cover, no mulch		45	66	77	83
good cover[2]		25	55	70	77
Open Spaces, lawns, parks, golf courses, cemeteries, etc.					
good condition: grass cover on 75% or more of the area		39	61	74	80
fair condition: grass cover on 50% to 75% of the area		49	69	79	84
Commercial and business areas (85% impervious)		89	92	94	95
Industrial districts (72% impervious)		81	88	91	93
Residential[3]:					
Average lot size	Average % impervious[4]				
1/8 acre or less	65	77	85	90	92
1/4 acre	38	61	75	83	87
1/3 acre	30	57	72	81	86
1/2 acre	25	54	70	80	85
1 acre	20	51	68	79	84
Paved parking lots, roofs, driveways, etc.[5]		98	98	98	98
Streets and roads:					
paved with curbs and storm sewers[5]		98	98	98	98
gravel		76	85	89	91
dirt		72	82	87	89

[1]For a more detailed description of agricultural land use curve numbers, refer to Soil Conservation Service, 1972, Chap. 9

[2]Good cover is protected from grazing and litter and brush cover soil.

[3]Curve numbers are computed assuming the runoff from the house and driveway is directed towards the street with a minimum of roof water directed to lawns where additional infiltration could occur.

[4]The remaining pervious areas (lawn) are considered to be in good pasture condition for these curve numbers.

[5]In some warmer climates of the country a curve number of 95 may be used.

Q. Calculate the runoff from 5 inches of rainfall on a 1000 acre watershed. The soil is 60% in Group B and 40 % in Group C. Assume antecedent moisture condition II. The land use is open with fair grass cover.

Ans: The curve numbers for open land with fair grass cover are CN = 69 for Group B and 79 for Group C.

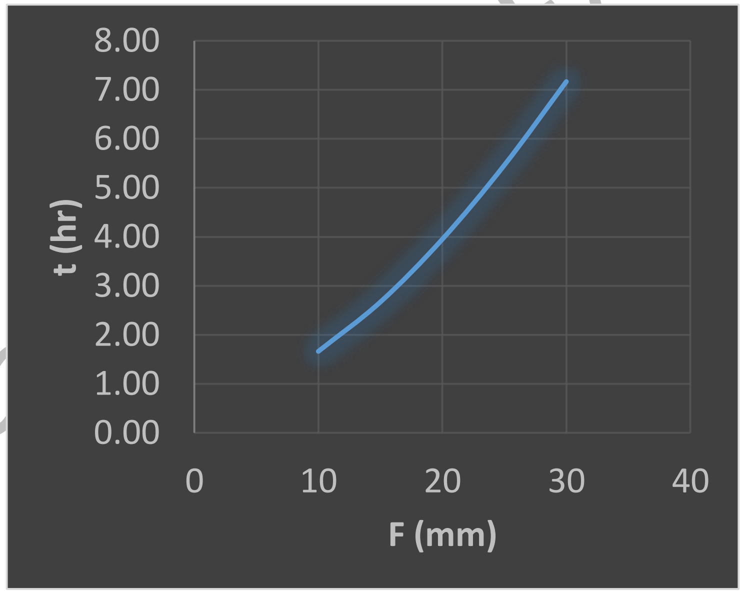
The average CN =0.6x69+0.4x79= 73

$$S = \frac{1000}{73} - 10 = 3.69$$

$$P_e = \frac{(P-0.2S)^2}{P+0.8S} = \frac{(5-0.2\times3.69)^2}{5+0.8\times3.69} = 2.28 \text{ in}$$

Ex. 01 Rainfall at a constant intensity of 6mm/hr falls on a homogeneous soil which has an initial uniform moisture content of 0.23. The soil property data obtained are $K = 1.24$mm/hr, η =0.48, ψ =150 mm. Determine rainfall excess. Assume no depression storage.

F (mm)	t (h)
10	1.66
15	2.67
20	3.95
25	5.46
30	7.17



t (hr)	F (mm)	Δt (hr)	ΔF (mm)	$i\Delta t$ (mm)	$ER = i\Delta - \Delta F$ (mm)
1.63	9.80				
2.00	11.85	0.37	2.05	2.22	0.17
3.00	16.39	1.00	4.54	6.00	1.46
4.00	20.18	1.00	3.79	6.00	2.21
5.00	23.55	1.00	3.37	6.00	2.63

Ex. 02 Rainfall occurs at a constant rate of 10.5 mm/hr on a sandy loam having an initial moisture content of 0.20. At saturation, the moisture content is 0.52 and the hydraulic conductivity is 2×10^{-4} mm/sec. The average capillary suction is 250 mm. Determine the rainfall excess for every hour up to 4 hours using Green-Ampt Model. Assume no depression storage.

Ex. 03 Following are the rates of rainfall in cm/hr for a successive 30 min period from a storm of 180 min: 0.5, 0, 0, 4.5, 5.0, 3.0. Determine the rainfall excess in each successive period using Green-Ampt model. The soil is sandy with a porosity of 0.59 and a saturated hydraulic conductivity of 3.3×10^{-4}cm/sec. Initial moisture content is 0.15. and average capillary suction is 6.5 cm. Assume no depression storage.

Time (min)	i(cm/hr)	F (cm)	f(cm/hr)	ΔF (cm)	$i\Delta t$ (cm)	ER = $i\Delta t - \Delta F$ (cm)
0						
30	0.5	0.25	14.93	0.25	0.25	0.00
60	0	0.25	14.93	0.00	0.00	0.00
90	0	0.25	14.93	0.00	0.00	0.00
100.5	4.5	1.04	4.50	0.79	0.79	0.00
120	4.5	2.14	2.80	1.10	1.46	0.36
150	5	3.36	2.22	1.22	2.50	1.28
180	3	4.41	1.98	1.05	1.50	0.45