CE 501: Surface Water Hydrology

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Example 3:

The annual runoff of a stream is assumed to follow normal distribution with mean of 2390 ha-m and standard deviation of 567 ha-m. Find the probability that the annual runoff in any year is greater than 3000 ha-m. What is the probability that the annual runoff in any year will be between 1800 and 2800 ha-m?

$$z = \frac{X-2390}{567}$$

(i) $P[X \geq 3000] = P\left[z \geq \frac{3000-2390}{567}\right] = P[z \geq 1.076] = 1 - F(1.076) = 1 - 0.859 = 0.141$

(ii) $P[1800 \leq X \leq 3000] = P\left[\frac{1800-2390}{567} \leq z \leq \frac{2800-2390}{567}\right] = P[-1.0406 \leq z \leq 0.7231]$

$= F(0.7231) - F(-1.0406) = F(0.7231) - [1 - F(1.0406)] = 0.76493 - (1 - 0.85074) = 0.616$

Binomial distribution

At any time a event may occur with a probability P

At any time a event may not occur with a probability $Q = 1 - P$

The probability that one event in two years is $PQ + QP = 2PQ = 2P(1-P) = C_1^2 P(1-P)$

The probability that one event in three years is $PQQ + QPQ + QQP = 3PQ^2 = 3P(1-P)^2 = C_1^3 P(1-P)^2$

The probability that there will be exactly k events in n years $C_k^n P^k (1-P)^{n-k}$

In hydrology it is not important to know the probability that even will occur k times in n year. But we would like to know the probability that an event will occur one or more in n years.

$$f_x\{1\ or\ more\ event\ in\ n\ years\} = 1 - f_x\{zero\ event\ in\ n\ years\}$$

$$f_x\{1\ or\ more\ event\ in\ n\ years\} = 1 - C_0^n P^0 (1-P)^{n-0} = 1 - (1-P)^n$$

Question: Analysis of data on maximum one day rainfall depth indicated that a depth of 280 mm had a return period of 50 years. Determine the probability of a one day rainfall depth equal to or greater than 280 mm occurring (a) once in 20 successive years, (b) twice in 15 successive years, and (c) at least once in 20 successive years.

Ans: Here $P = \frac{1}{T} = \frac{1}{50} = 0.02$

a) $f_x\{once\ in\ 20\ successive\ years\} = C_1^{20}P^1(1-P)^{20-1} = C_1^{20}(0.02)^1(1-0.05)^{20-1} = 0.272$

b) $f_x\{twice\ in\ 15\ successive\ years\} = C_2^{15}P^2(1-P)^{15-2} = C_2^{15}(0.02)^2(1-0.05)^{15-2} = 0.323$

c) $f_x\{at\ lest\ once\ in\ 20successiveyears\} = 1-(1-P)^n = 1-(1-0.02)^{20} = 0.332$

PLOTING POSITIONS

Plotting position refers to the probability value assigned to each piece of data to be plotted

$$P(X \geq x_m) = \frac{m}{n}$$ California's formula

$$P(X \geq x_m) = \frac{m-1}{n}$$ Modified formula

$$P(X \geq x_m) = \frac{m-0.5}{n}$$ Hazen (1930) formula

$$P(X \geq x_m) = \frac{m-0.3}{n+0.4}$$ Chegodayev's formula

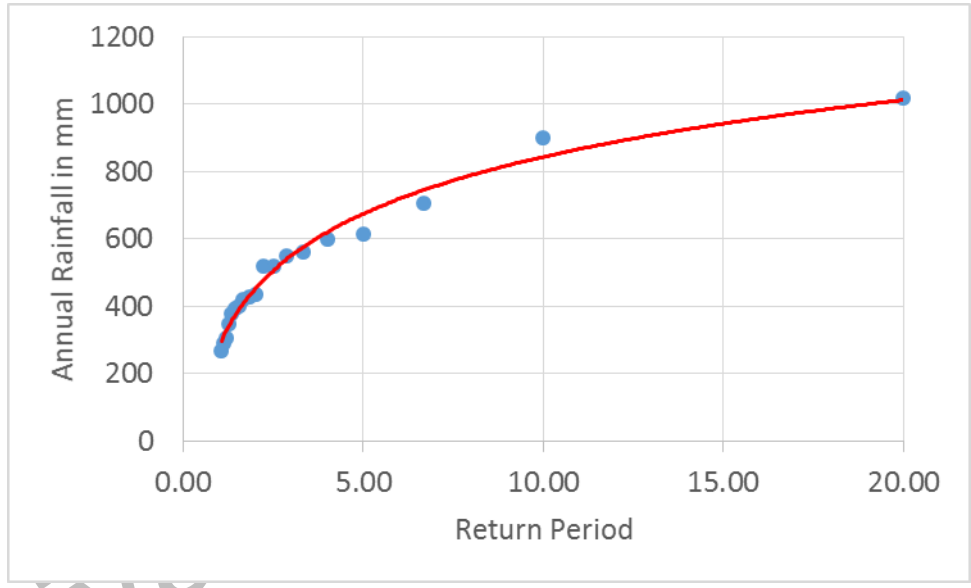
$$P(X \geq x_m) = \frac{m}{n+1}$$ Weibull formula

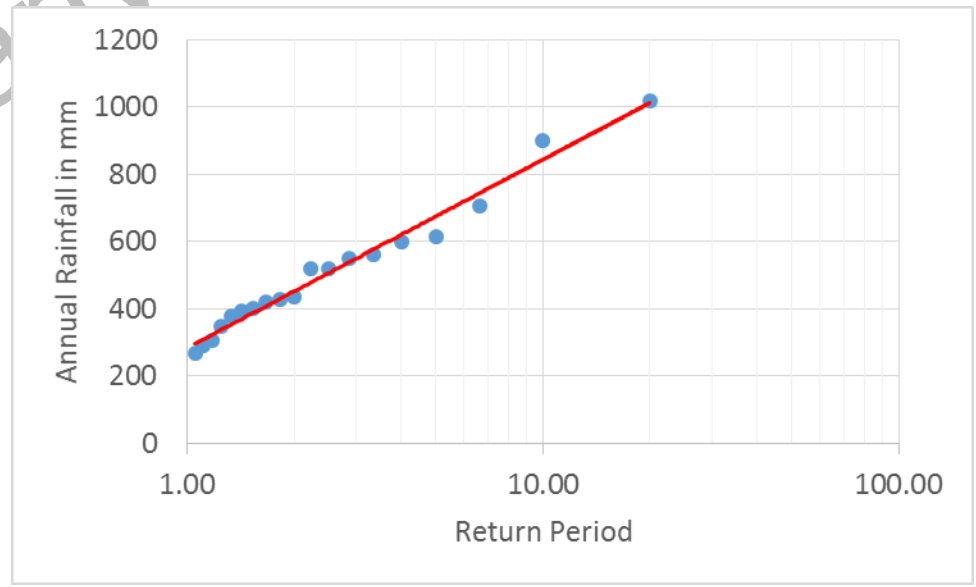
$$P(X \geq x_m) = \frac{m-b}{n+1-2b}$$

$b = 0.5$	For Hazen (1930) formula
$b = 0.3$	For Chegodayev's formula
$b = 0$	For Weibull formula
$b = 3/8$	For Blom's formula
$b = 1/3$	For Tukey's formula
$b = 0.44$	For Gringorten's formula

PLOTING POSITIONS

Year	Annual Rainfall	Year	Annual Rainfall	Rank	Exceedance Probability	Return Period
1970	520	1987	1020	1	0.05	20.00
1971	615	1988	900	2	0.10	10.00
1972	420	1976	705	3	0.15	6.67
1973	270	1971	615	4	0.20	5.00
1974	305	1977	600	5	0.25	4.00
1975	380	1980	560	6	0.30	3.33
1976	705	1979	550	7	0.35	2.86
1977	600	1970	520	8	0.40	2.50
1978	350	1982	520	9	0.45	2.22
1979	550	1983	435	10	0.50	2.00
1980	560	1986	430	11	0.55	1.82
1981	400	1972	420	12	0.60	1.67
1982	520	1981	400	13	0.65	1.54
1983	435	1984	395	14	0.70	1.43
1984	395	1975	380	15	0.75	1.33
1985	290	1978	350	16	0.80	1.25
1986	430	1974	305	17	0.85	1.18
1987	1020	1985	290	18	0.90	1.11
1988	900	1973	270	19	0.95	1.05





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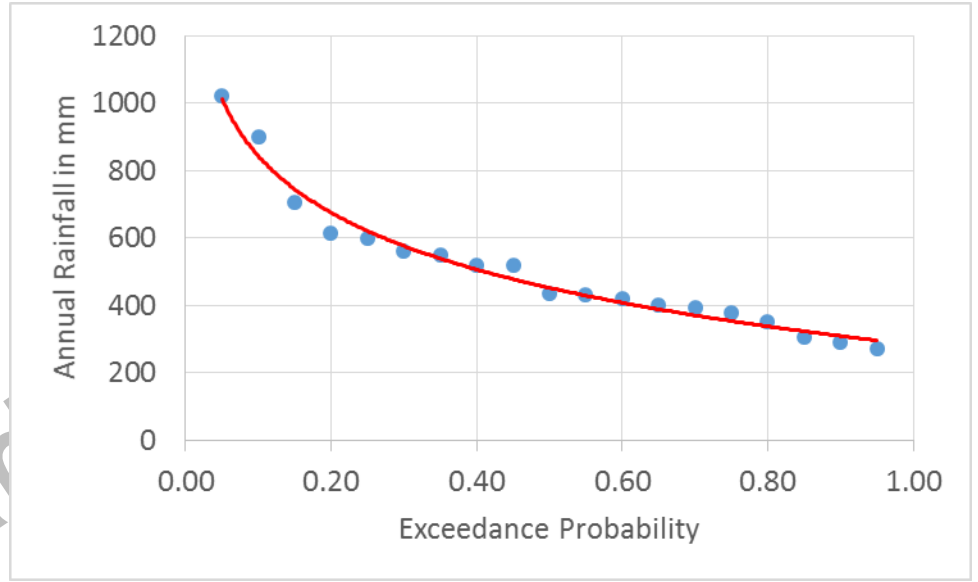
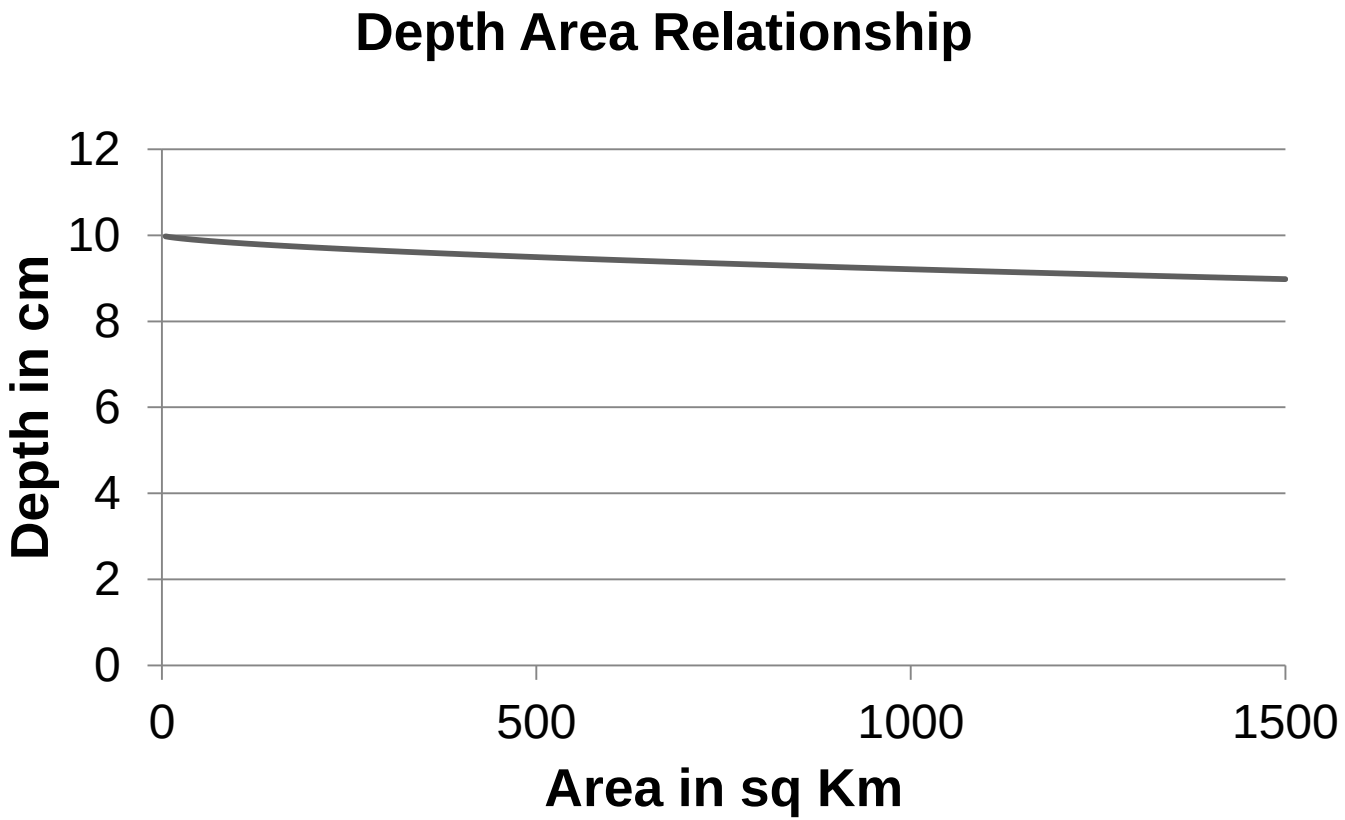


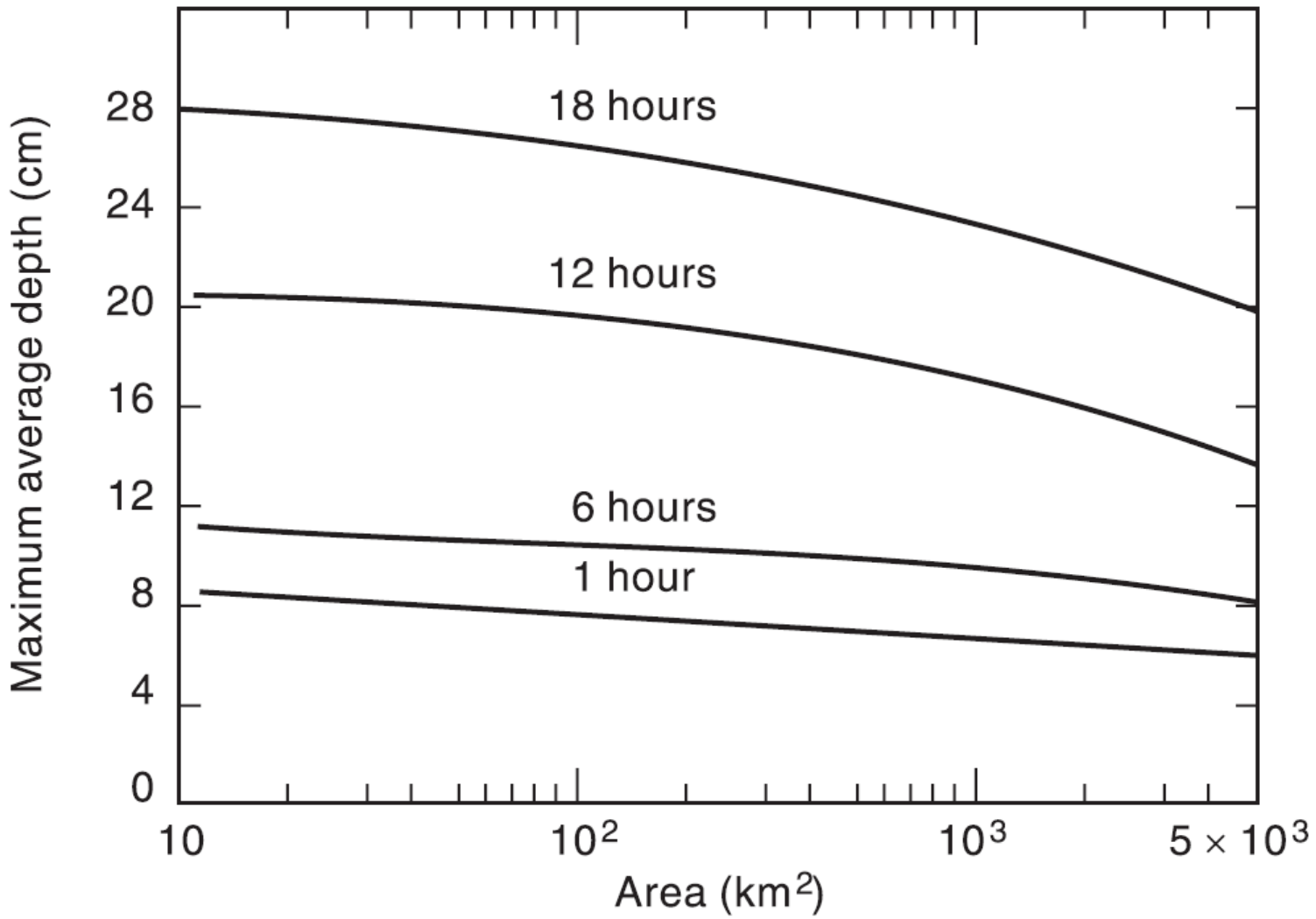
Depth area duration relationship

$$\bar{P} = P_o exp(-KA^n)$$

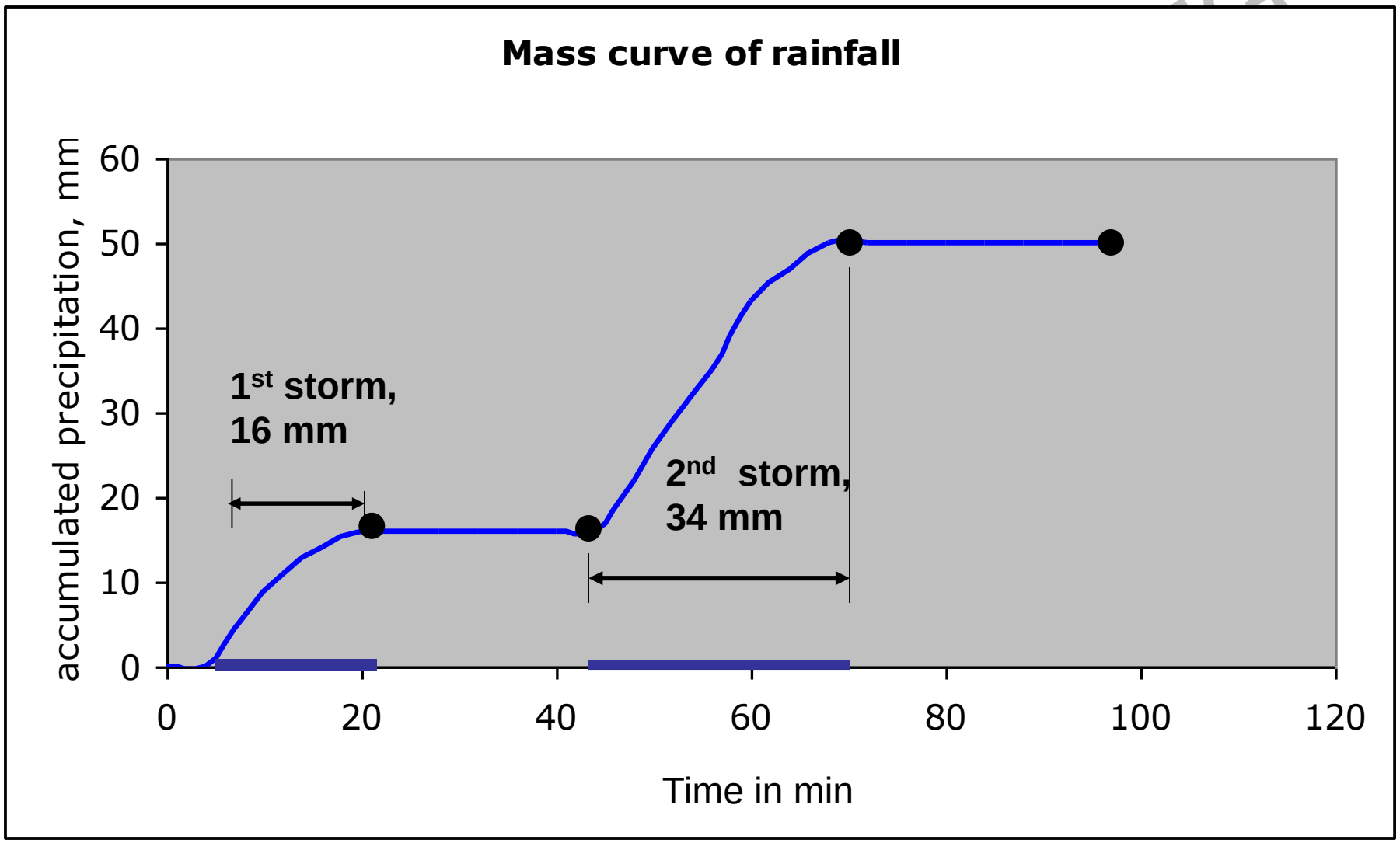
$\bar{P}$ is the average depth in cm
P_o is the highest amount of rainfall in cm
K and n are constant for a given region



Depth Area Duration Curve

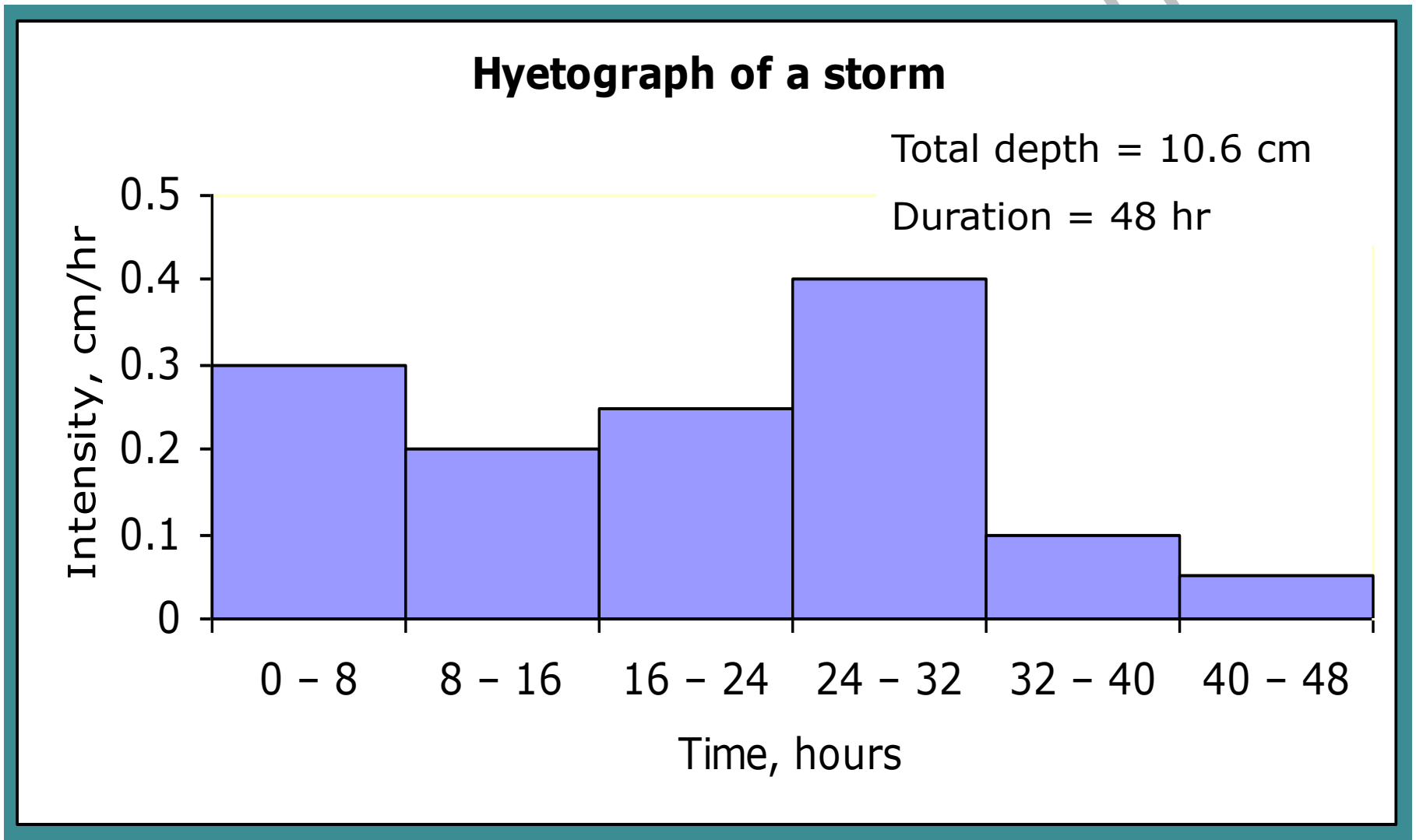


Intensity – Duration – Frequency (IDF) Relationship



Intensity – Duration – Frequency (IDF) Relationship

Hyetograph is a plot of the accumulated precipitation against time, plotted in chronological order



Intensity – Duration – Frequency (IDF) Relationship

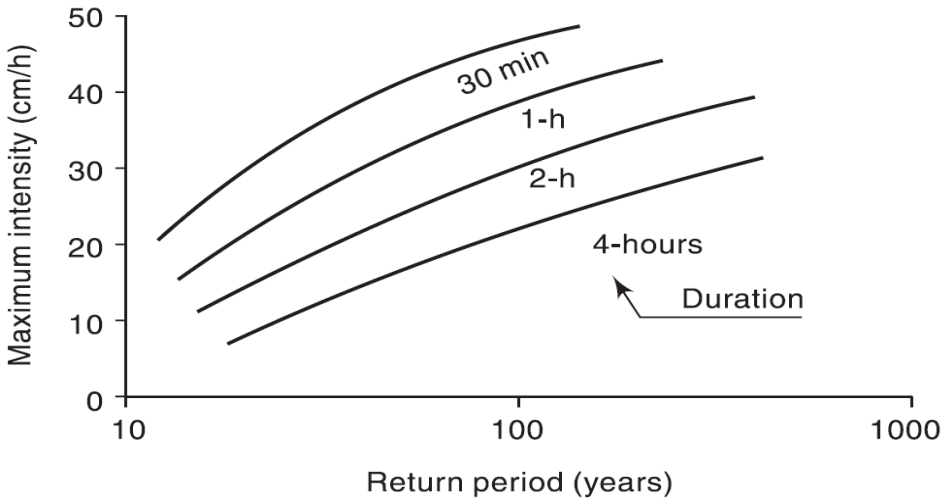


Fig. 2.18 Maximum Intensity-Return Period-Duration Curves

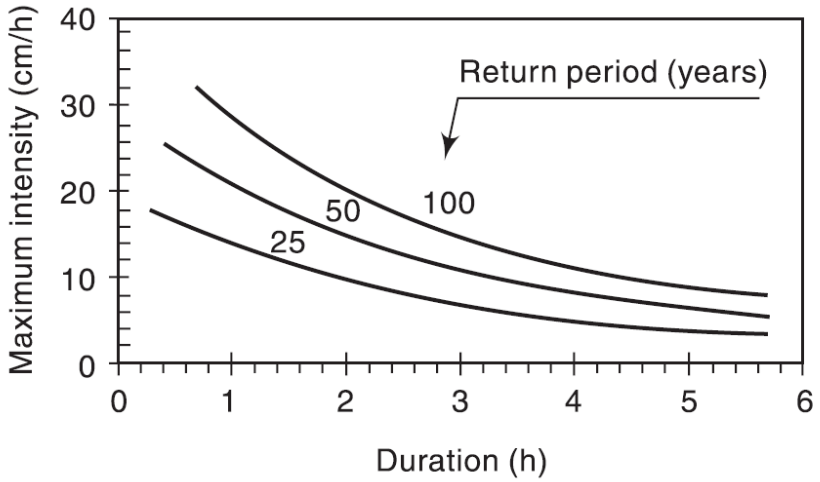


Fig. 2.19 Maximum Intensity-Duration-Frequency Curves

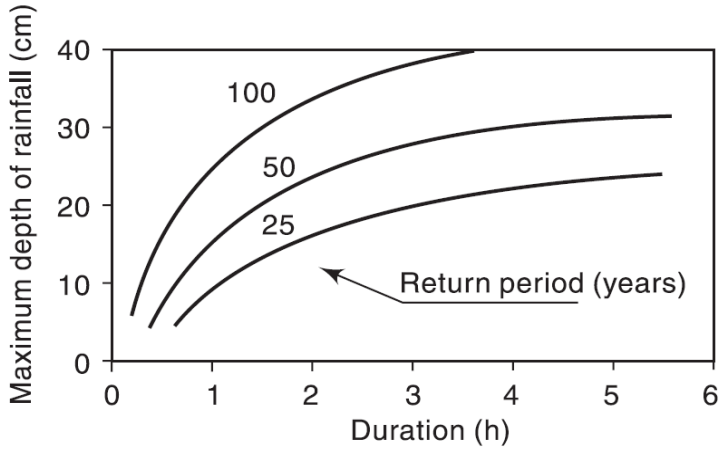


Fig. 2.20 Maximum Depth-Duration-Frequency Curves

Intensity – Duration – Frequency (IDF) Relationship

- ✓ In many design problems related to watershed such as runoff disposal, erosion control, highway construction, culvert design, it is necessary to know the rainfall intensities of different durations and different return periods.
- ✓ The curve that shows the inter-dependency between i (cm/hr), D (hour) and T (year) is called IDF curve.
- ✓ The relation can be expressed in general form as:

$$i = \frac{k\,T^{x}}{(D+a)^{n}}$$

i – Intensity (cm/hr)

D – Duration (hours)

k, x, a, n – are constants for a given catchment

Maximum Intensity-Duration Relation

Incremental depth of rainfall (mm) in various durations

Time (min.)	Cumulative Rainfall (mm)	Durations(min)								
		30	60	90	120	150	180	210	240	270
0	0									
30	6	6								
60	18	12	18							
90	21	3	15	21						
120	36	**15**	18	**30**	36					
150	43	7	**22**	25	**37**	43				
180	49	6	13	28	31	**43**	**49**			
210	52	3	9	16	31	34	46	**52**		
240	53	1	4	10	17	32	35	47	**53**	
270	54	1	2	5	11	18	33	36	48	**54**

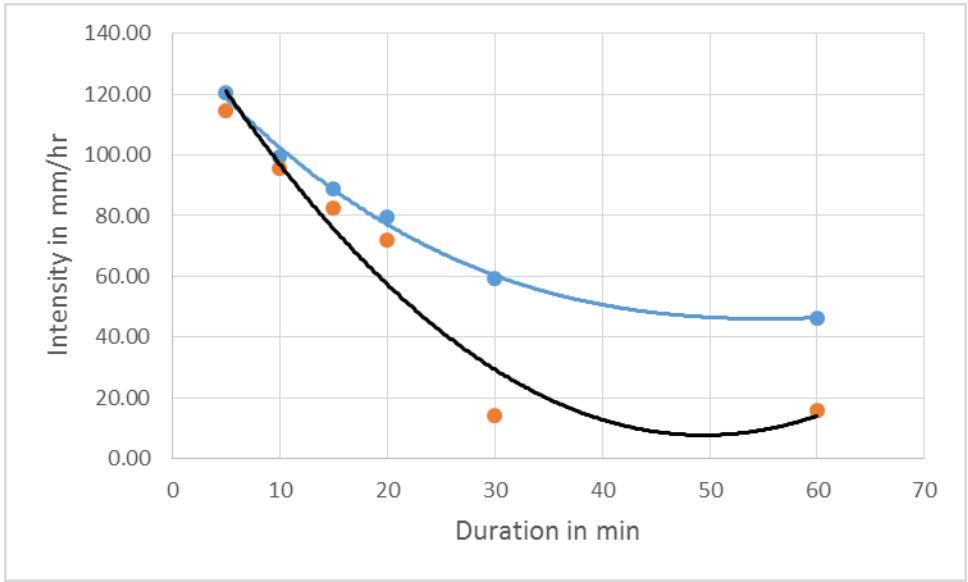
Maximum Intensity-Maximum Depth-Duration Relation

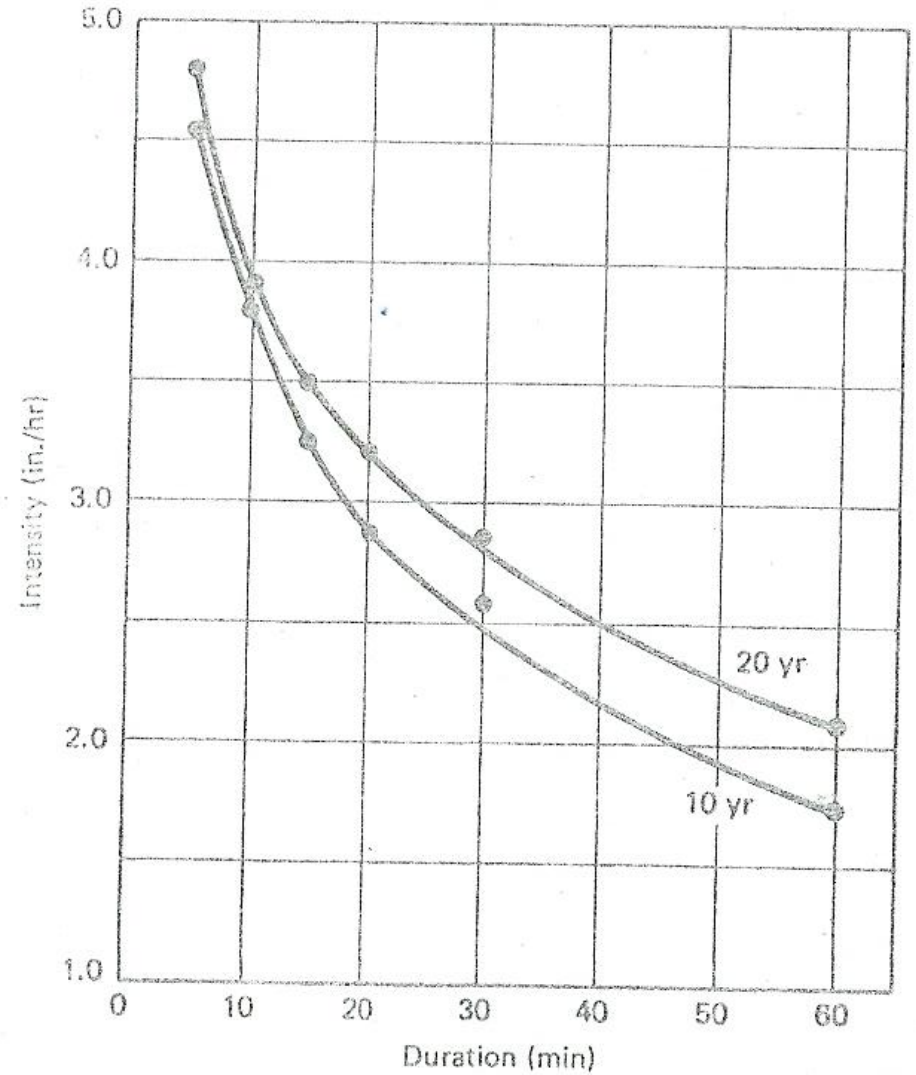
Maximum Intensity (mm/h)	30.0	22.0	20.0	18.5	17.2	16.3	14.9	13.3	12.0
Duration in min.	30	60	90	120	150	180	210	240	270
Maximum Depth (mm)	15.0	22.0	30.0	37.0	43.0	49.0	52.0	53.0	54.0

Rank	Precipitation						
	Duration						
	5	10	15	20	30	60	T
1	10.16	16.76	22.61	27.18	37.59	54.61	23.00
2	9.65	16.00	21.08	24.64	7.37	23.37	11.50
3	9.40	15.75	20.07	23.11	6.60	4.57	7.67
4	9.14	15.24	19.30	21.84	23.11	26.92	5.75
5	8.89	15.24	18.54	20.32	21.08	24.38	4.60
6	8.38	14.73	18.29	19.56	20.83	23.88	3.83
7	8.38	12.70	18.29	19.56	19.30	22.86	3.29
8	7.87	12.70	16.00	17.78	19.05	22.10	2.88
9	7.62	12.45	14.48	16.51	17.02	19.56	2.56
10	7.11	11.18	14.22	15.75	16.76	19.05	2.30
22	0.13	0.23	0.32	0.4	0.4	0.43	1.05

T							
	5	10	15	20	30	60	
20	120.33	99.39	88.83	79.55	59.41	46.46	
10	114.63	95.42	82.74	72.12	14.14	16.01	

Rank	Precipitation						
	Duration						
	5	10	15	20	30	60	T
1	10.16	16.76	22.61	27.18	37.59	54.61	23.00
	10.03	16.57	22.21	26.52	29.71	46.46	20.00
2	9.65	16.00	21.08	24.64	7.37	23.37	11.50
	9.55	15.90	20.68	24.04	7.07	16.01	10.00
3	9.40	15.75	20.07	23.11	6.60	4.57	7.67
4	9.14	15.24	19.30	21.84	23.11	26.92	5.75
5	8.89	15.24	18.54	20.32	21.08	24.38	4.60
6	8.38	14.73	18.29	19.56	20.83	23.88	3.83
7	8.38	12.70	18.29	19.56	19.30	22.86	3.29
8	7.87	12.70	16.00	17.78	19.05	22.10	2.88
9	7.62	12.45	14.48	16.51	17.02	19.56	2.56
10	7.11	11.18	14.22	15.75	16.76	19.05	2.30
22	0.13	0.23	0.32	0.4	0.4	0.43	1.05





EXAMPLE 2.6

For the precipitation data arranged for different durations in Table 2.6, prepare intensity-duration-frequency curves for 20-year and 10-year frequencies.

TABLE 2.6 FREQUENCY ANALYSIS OF DIFFERENT DURATION OF PRECIPITATION DEPTHS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rank	Precipitation (in.) of Duration						Return Period
m	5 min.	10 min.	15 min.	20 min.	30 min.	60 min.	$T = n + 1/m$
1	0.40	0.66	0.89	1.07	1.48	2.15	23
2	0.38	0.63	0.83	0.97	1.29	1.92	11.5 ← 20 yr
3	0.37	0.62	0.79	0.91	1.26	1.48	7.7 ← 10 yr
4	0.36	0.60	0.76	0.86	0.91	1.06	5.8
5	0.35	0.60	0.73	0.80	0.83	0.96	4.6
6	0.33	0.58	0.72	0.77	0.82	0.94	3.8
7	0.33	0.50	0.72	0.77	0.78	0.90	3.3
8	0.31	0.50	0.63	0.70	0.75	0.87	2.9
9	0.30	0.49	0.57	0.65	0.67	0.77	2.6
10	0.28	0.44	0.56	0.62	0.66	0.75	2.3
⋮							
22	0.13	0.23	0.32	0.40	0.40	0.43	1.05

SOLUTION

1. For each duration, the precipitation depths are arranged in descending order. The highest value has been assigned a rank of 1 and the lowest a rank of 22. The return periods are obtained in column 8 of Table 2.6.
2. The depths of different duration corresponding to 20-year frequency are interpolated from the values for 23- and 11.5-year frequencies and converted to intensities in Table 2.7. Similar calculations are done for 10-year frequency. These are plotted in Figure 2.6.

TABLE 2.7 VALUES FOR 20-YEAR AND 10-YEAR PRECIPITATION INTENSITIES OF DIFFERENT DURATION

Return Period (years)	Intensity (in./hr) of Duration					
	5 min.	10 min.	15 min.	20 min.	30 min.	60 min.
20	4.74	3.9	3.50	3.13	2.86	2.09
10	4.51	3.78	3.24	2.85	2.58	1.75

Probable maximum precipitation (PMP)

$$PMP = \bar{P} + K\sigma$$

$\bar{P}$ is the mean annual maximum precipitation

σ is the standard deviation of the series

K is the frequency factor