CE 501: Surface Water Hydrology

Prof. (Dr.) Rajib Kumar Bhattacharjya
Indian Institute of Technology Guwahati
Guwahati, Assam
Email: rkbc@iitg.ernet.in Web: www.iitg.ernet.in/rkbc

Adequacy of Raingauge stations

The optimal number is calculated $$N = \left(\frac{C_v}{\varepsilon}\right)^2$$

N is the optimal number of raingauge stations

ε is the allowable degree of error in the estimate of the mean rainfall

C_v is the coefficient of variation of the rainfall at the

If there are m stations in the catchment each recording rainfall values $P_1, P_2, \ldots P_m$ in a known time, the coefficient C_v is calculated as

$$C_v = \frac{100\sigma_{m-1}}{\bar{P}}$$

$$\sigma_{m-1} = \sqrt{\left[\frac{\sum_1^m (P_i - \bar{P})^2}{m-1}\right]}$$

$$\bar{P} = \frac{1}{m}\left(\sum_1^m P_i\right)$$

Usually ε of 10% is considered

Probabilistic Approach:

A random variable X is a variable described by a probability distribution.

- The distribution specifies the chance that an observation x of the variable will fall in a specific range of X.
- A set of observations n_1, n_2, n_3,…………., n_n of the random variable is called a sample.
- It is assumed that samples are drawn from a hypothetical infinite population possessing constant statistical properties.
- Properties of sample may vary from one sample to others.

The probability P (A) of an event is the chance that it will occur when an observation of the random variable is made.

$$P(A) = \lim_{n\to\infty}\left(\frac{n_A}{n}\right)$$

n_A --- number in range of event A.

n----- Total observations

1. Total Probability

$$P(A_1) + P(A_2) + P(A_3) + \ldots \ldots \ldots \ldots \ldots \ldots + P(A_n) = P(\Omega) = 1$$

2. Complementarity $P(\overline{A}) = 1 - P(A)$

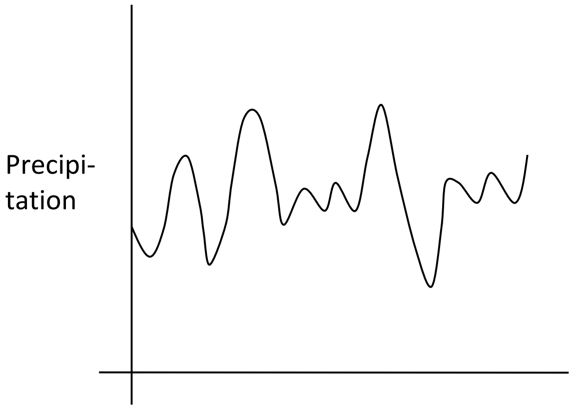
3. Joint Probability $P(A \cap B) = P(A)P(B)$

Example:

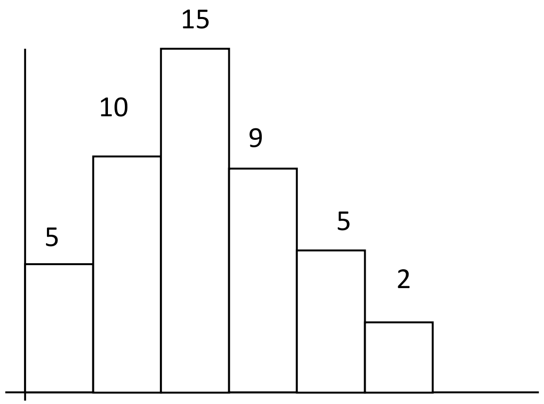
The probability that annual precipitation will be less than 120 mm is 0.333. What is the probability that there will be two successive year of precipitation less than 120 mm.

$P(R<35) = 0.333$

$P(C) = 0.333^2 = 0.111$



Year



Frequency Histogram

Relative Frequency:

$$f_s(x_i) = \frac{n_i}{n}$$

Which is equivalent to $P(x_i - \Delta x \leq X \leq x_i)$, the probability that the random variable X will lie in the interval $[x_i - \Delta x, x_i]$

Cumulative Frequency Function:

$$F_s(x_i) = \sum_{j=1}^{i} f_s(x_j)$$

This is estimated as $P(X \leq x_i)$, the cumulative probability of x_i.
This is estimated for sample data, corresponding function for population will be

Probability density function:

$$f(x) = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} \frac{f_s(x)}{\Delta x}$$

Probability distribution function:

$$F(x) = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} F_s(x)$$

Whose derivative is the probability density function

$$f(\gamma) = \frac{dF(\gamma)}{dx}$$

The cumulative probability as $P(X \leq x)$, can be expressed as

$$P(X \leq x) = F(x) = \int_{-\infty}^{x} f(u)du$$

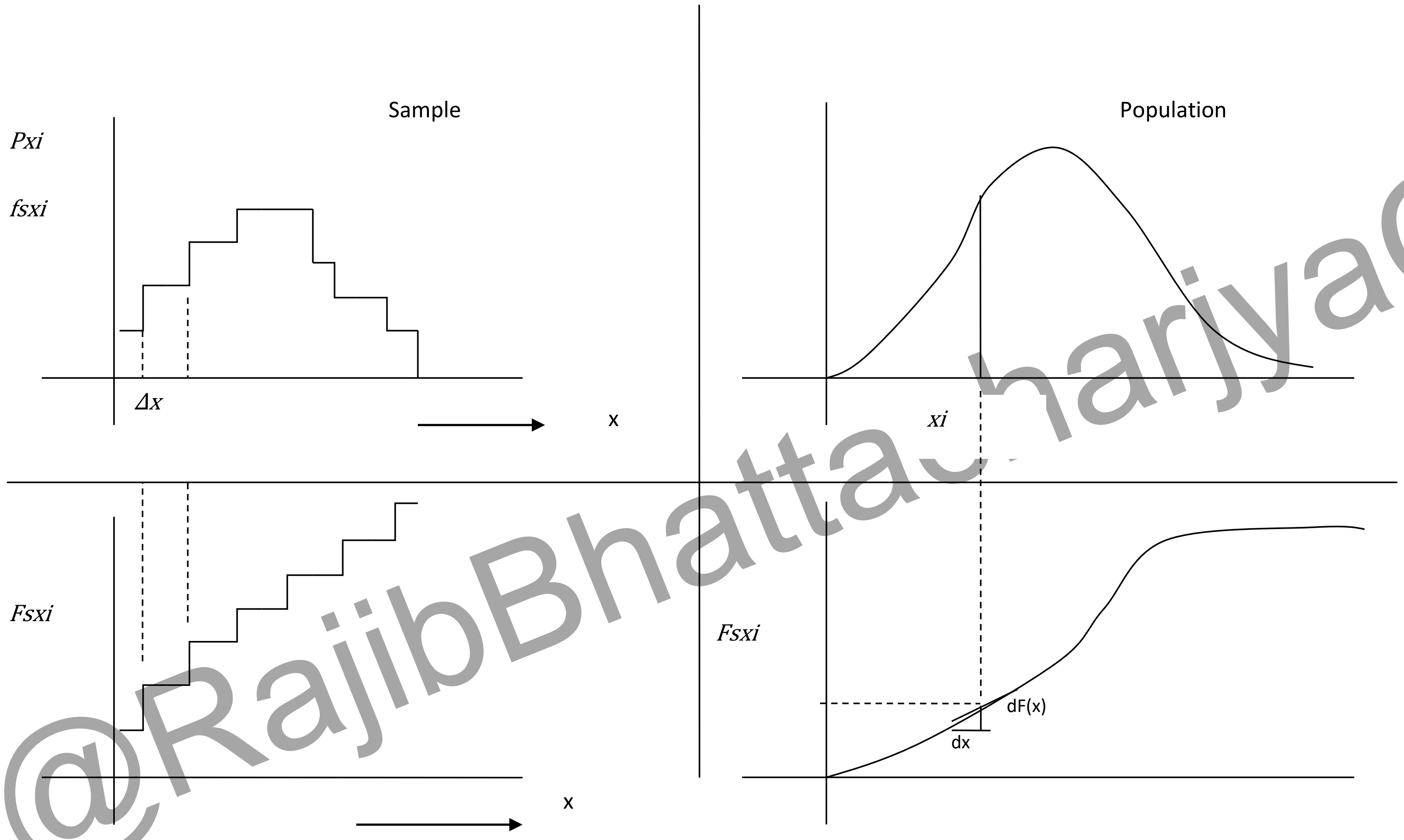
$$P(x_i) = P(x_i - \Delta x \leq X \leq x_i)$$

$$= \int_{x_i - \Delta x}^{x_i} f(x)dx$$

$$= \int_{-\infty}^{x_i} f(x)dx - \int_{-\infty}^{x_i - \Delta x} f(x)dx$$

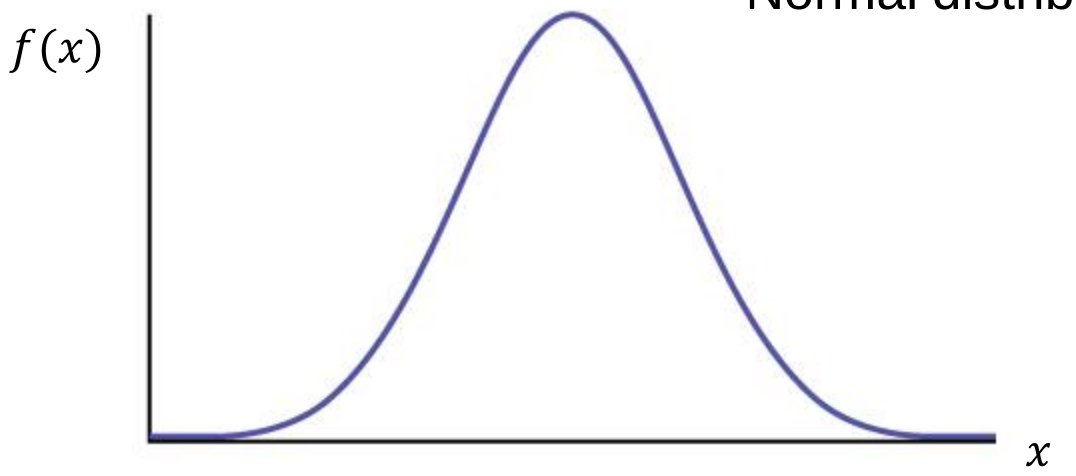
$$= F(x_i) - F(x_i - \Delta x)$$

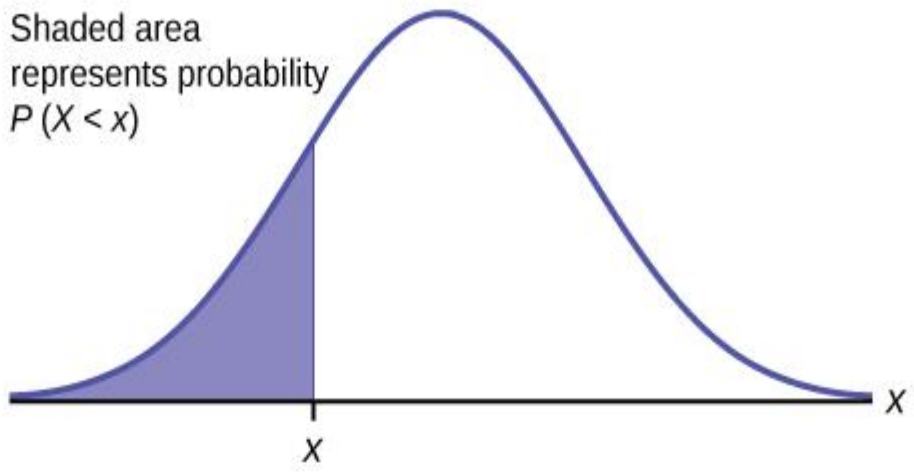
$$= F(x_i) - F(x_{i-1})$$

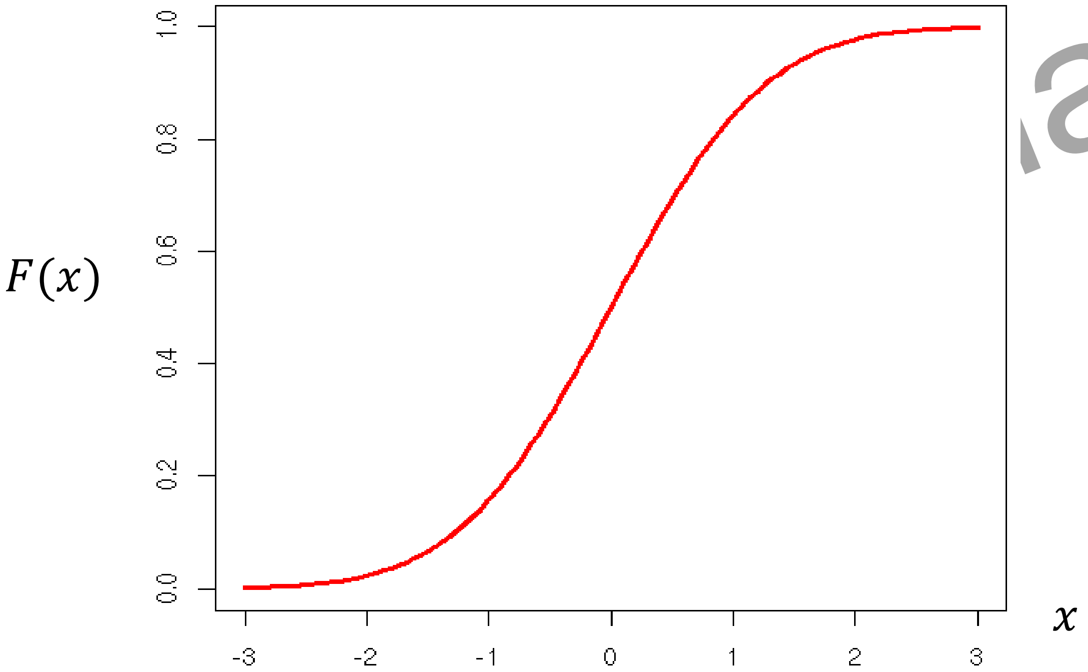

Sample
Population
Pxi
fsxi
Δx
x
xi
Fsxi
Fsxi
dF(x)
dx
x

Probability density function

Normal distribution







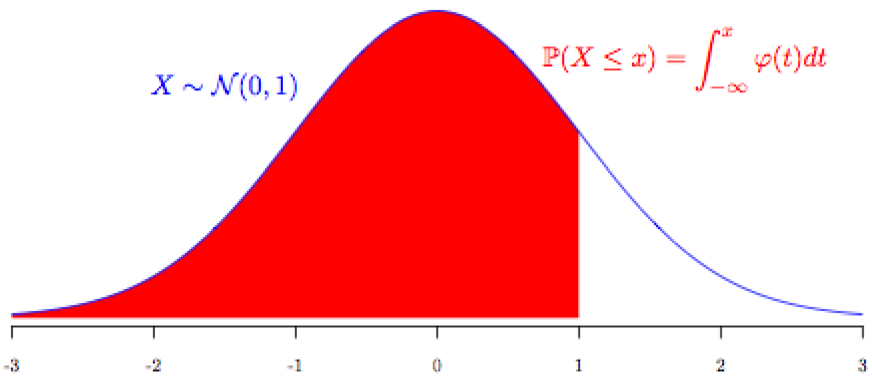
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$f(z) = \frac{1}{\sqrt{2\pi}} exp\left[-\frac{z^2}{2}\right] \qquad z = \frac{x-\mu}{\sigma} \qquad -\infty \leq z \leq \infty$$

$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

Cumulative probability of standard normal distribution

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990



$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

$$F(z) = B \qquad \text{for } z < 0$$

$$F(z) = 1 - B \qquad \text{for } z \geq 0$$

$$B = \frac{1}{2}[1 + 0.196854|z| + 0.115194|z|^2 + 0.000344|z|^3 + 0.019527|z|^4]^{-4}$$

Ex. What is the probability that the standard normal random variable z will be less than -2? Less than 1? What is $P(-2 < Z < 1)$?

Solution

$$P(Z < -2) = F(-2) = 1 - F(2) = 1 - 0.9772 = 0.228$$

$$P(Z < 1) = F(1) = 0.8413$$

$$P(-2 < Z < 1) = F(1) - F(2) = 0.841 - 0.023 = 0.818$$

Example 2:

The annual runoff of a stream is modeled by a normal distribution with mean and standard deviation of 5000 and 1000 ha-m respectively.

i. Find the probability that the annual runoff in any year is more than 6500 ha-m.

ii. Find the probability that it would be between 3800 and 5800 ha-m.

Solution:

Let X is the random variable denoting the annual runoff. Then z is given by

$z = \frac{(X-5000)}{1000}$ N (0, 1)

(i) P (X≥6500) = P (z ≥$\frac{(6500-5000)}{1000}$)

= P (z≥1.5)

= 1-P (z≤1.5)

= 1- F (1.5) F (1.5) = 0.9332

P (X≥6500) =1-0.9332

P (X≥6500) =0.0688

(ii) P (3800≤X≤5800)

$= P\left[\frac{(3800-5000)}{1000} \leq z \leq P\ \frac{(5800-5000)}{1000}\right]$

= P [-1.2≤z≤0.8]

= F (0.8)-F (-1.2)

= F (0.8)-[1-F (1.2)]

=0.7881-(1-0.8849)

=0.673