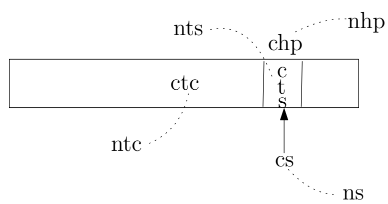
— every partial recursive function is Turing computable —

- DTMs can compute basic functions: successor, zero, projection
- proved that composition of functions can be implemented in DTMs
- if f is defined by primitive recursion from Turing computable functions g and h, then we proved f is Turing computable as well
- unbounded minimalization of a Turing computable total predicate $p(x_1, \ldots, x_n, y)$ is Turing computable: to solve $f(x_1, \ldots, x_n) = \mu z[p(x_1, \ldots, x_n, z)]$, successively substitute $z = 0, 1, \ldots$; computation terminates when the first z for which $p(x_1, \ldots, x_n, z) = 1$, with the value of z written on tape

— every Turing computable function is partial recursive —



[ctc (resp. ntc): Godel number representing nonblank part of the current tape (resp. tape updated); chp (resp. nhp): numerical representation of current (resp. updated) tape head location; cts (resp. nts): numerical representation of symbol in tape cell pointed by chp (resp. nhp); cs (resp. ns): numerical representation of current state (resp. new state)]

- assign a unique natural number to each element of Γ, Q
- encode every configuration into a Godel number[1] : $g = gn$(cs, chp, ctc)

  it is immediate note
  cs = decode(0, g); chp = decode(1, g); cts = decode(decode(1, g), decode(2, g)) [2]

- suppose $\delta(q, \alpha) = (q', \alpha', L)$, $\delta(q, \beta) = (q'', \beta', R)$ be the only transitions from q, then
  $ns = eq(cts, \alpha).q' + eq(cts, \beta).q'' + ne(cts, \alpha).ne(cts, \beta).cs$

  corresponding to $\delta(q, \alpha) = (q', \alpha', L)$,
  $ntc = quo(ctc, primenum(chp)^{cts+1}).primenum(chp)^{nts+1}$, where nts is numerical representation of α'

  $nhp = eq(cs, q).eq(cts, \alpha).(chp-1)+eq(cs, q).eq(cts, \beta).(chp+1)+ne(cs, q).ne(cts, \alpha).ne(cts, \beta).chp$
  (assuming these are the only transitions present)

[1] $gn(x_0, \ldots, x_n) = \Pi_{i=0}^{n} primenum(i)^{x_i+1}$
[2] $decode(i, x) = \mu^x z[complsgn(divides(x, primenum(i)^{z+1}))] - 1$

- initial configuration with tape having string w: $config(0) = gn(0, 0, \Pi_{i=1}^{|w|} primenum(i)^{w[i]+1})$

  subsequent configurations: $config(y + 1) = gn(ns(config(y)), nhp(config(y)), ntc(config(y)))$

  computation terminates after it undergoes $\mu z[eq(config(z), config(z + 1))]$ number of transitions