

- $coNP = \{L : \bar{L} \in NP\}$ .  
 $L \in NP$ : every yes instance of  $L$  has a polynomial length proof of  $x \in L$ .  
 $L \in coNP$ : every no instance of  $L$  has a polynomial length refutation of  $x \in L$ .
- For example,  $\overline{SAT}$ ,  $\overline{HAMPATH}$ , and  $TAUTOLOGY$  are in class  $coNP$ .
- $P \subseteq NP \cap coNP$ .
- $FACTORING = \{ \langle n, a, b \rangle \mid n, a, b \text{ are positive integers in binary and there exists a prime number } p \in [a, b] \text{ that divides } n \} \in NP \cap coNP$ .
- $PRIMES = \{n \mid \text{positive integer } n \text{ is a prime number}\} \in NP \cap coNP$ , and  $PRIMES$  was proven to be in class  $P$ .
- If  $P = NP$  then  $NP = coNP$ .
- $coNP \subseteq PSPACE$ .
- $L \in NP\text{-hard} \Rightarrow \bar{L} \in coNP\text{-hard}$ .
- $TAUTOLOGY = \{ \phi : \phi \text{ is a boolean formula that is satisfied by every truth assignment} \}$  is coNP-complete.
- If an NP-complete language is in coNP, then  $NP = coNP$ .