A few t-Spanners in the Euclidean plane[1]

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[1] these slides are last updated in 2013; in presenting, blackboard is used

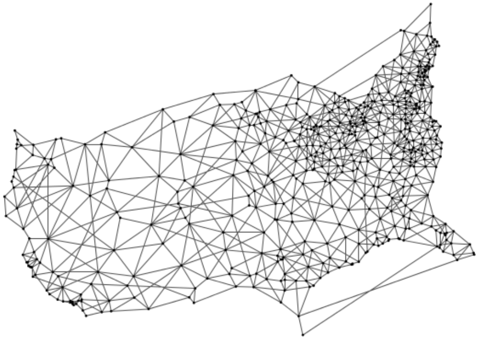
Outline

t-Spanner: definition

Given a set S of points in Euclidean plane, network $G(S, E)$ is a *t-spanner* $(t > 1)$ of S iff for every $u, v \in V$, $dist_G(u, v) \leq t.dist(u, v)$.

Motivation: designing road networks

10-stretch network for US cities



1.2-stretch network for US cities

Making a network by connecting points given in Euclidean plane. [2]

[2] example figs are from [Narasimhan, Smid '07]

Motivation

Asymptotic improvement for algorithms that rely on m.

Problem

Given a set S of points in Euclidean plane, construct a sparse network $G(S,E)$ that obeys one/many of the following factors:

- low stretch factor ($t = max_{p,q \in S} \frac{\delta_G(p,q)}{\delta(p,q)}$)
- $O(|S|)$ number of edges (*sparse*)
- low weight ($\sum_{e \in E} w_e$)
- minimize the maximum degree (*small size*)
- low diameter (*conciseness*)
- high fault tolerance
- small load factor
- small chromatic number

Problem description

Let S be a set of n points in R^2, and let $t > 1$ be a real number.

- Does there exist a t-spanner for S having at most $O(n)$ edges?
- If so, find the lower and upper bounds in constructing the same.
- Is it possible to construct t-spanners in $O(n \lg n)$ time?
- Weight of such spanner as compared with MST?
- Can we minimize the diameter of the spanner?
- Can we minimize the maximum degree of the spanner?

An application: minimum Steiner tree

Given a set S of points and S' of Steiner points,

wight of Steiner minimum tree of S

$\leq$ weight of minimum spanning tree of S

$\leq$ 2.weight of Steiner minimum tree of S [3]

[3] when points are in $\mathbb{R}^2$, factor 2 got improved to $\frac{2}{\sqrt{3}}$ [Du, Hwang '90]

Spanner degree vs diameter

- Any t-spanner whose degree is bounded by a constant must have a spanner diameter $\Omega(\lg n)$

Another application: t-approximate MST

Let G be a t-spanner of S. Then $wt(MST(G)) \leq t.wt(MST(S))$.

> unioning paths in G corresp. to each edge of $MST(S)$ results in a connected spanning graph G' that is a subgraph of G

Lower bounds

In the algebraic computation tree model, the worst-case lower bound in constructing a spanner of n points stands at $\Omega(n \lg n)$.

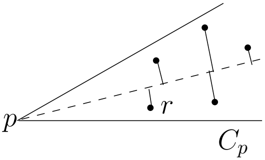
— not proved

Outline

Observation

For each $p \in S$, among all near-parallel edges incident on p in the complete graph, the Θ-graph retains only the shortest one.

Algorithm: preprocessing



input: set S of points in R^2, number of cones κ ($\kappa \geq 9$; so that the cone angle $\theta = \frac{2\pi}{\kappa} \in (0, \frac{\pi}{4})$)

let $\mathcal{C}$ be a set of κ cones partitioning the space around origin

1. introduce each point in S as a vertex in Θ-graph
2. for each point p of S and for each cone C of $\mathcal{C}$, such that the translated cone C_p contains at least one point of $S \backslash \{p\}$, introduce an edge (p, r) into Θ-graph if r is a closest point along the bisector of C to p among all the points in C_p [4]

output: undirected graph $\Theta(S, E)$ with $|E|$ is $O(n\kappa)$

[4] in case of Yao graphs, among all the points in C_p is chosen, closest point to p is chosen

Algorithm: query

input: two query points p and q in S

1. pick a cone C in $\mathcal{C}$ such that $q \in C_p$
2. for $r \in C_p$ and pr being an edge of Θ-graph, output r
3. if $r \neq q$, set $p \leftarrow r$ and go to the stmt (1)

output: a path between p and q

Analysis: stretch factor

for any point q in C_p and closest point r to p along bisector of C_p,

- $|pr| \leq \frac{|pq|}{\cos\theta}$
- $|rq| \leq |pq| - (\cos\theta - \sin\theta)|pr|$

Analysis: stretch factor (cont)

The stretch factor $t = 1/(\cos\theta - \sin\theta)$:

Let $p = p_0, p_1, \ldots, p_m = q$ be the path constructed by the query algorithm.

- $|p_{i+1}q| < |p_i q|$

  implying that each successive point on this path takes us strictly closer to q

- $|p_i p_{i+1}| \leq \frac{1}{\cos\theta - \sin\theta}(|p_i q| - |p_{i+1} q|)$

For each real constant $t > 1$, there exists a sparse t-spanner.

Analysis: complexity

- $O(\kappa n \lg n)$ time (using one plane sweep for each cone)
- using $O(n\kappa)$ space

A sketch of sink spanners

A *q-sink t-spanner* G of a set S of points is a directed graph such that there is a directed t-spanner path from any $p \in S$ to a $q \in S$ in G.

- for each cone C_q, introduce an arc from r to q in C_q, where $r \in C_q$ is the closest point along the bisector of C_q among points in $S - \{q\}$
- recursively define the directed subgraph pivoting at r for points located in C_r by all possible cones in $\mathcal{C}$ centered at r

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The following transformation to any Θ-graph based $\sqrt{t}$-spanner G, leads to achieving a $1/(t-1)^2$ maximum degreed t-spanner, but with slightly worse time complexity to compute the same:

- instead of introducing any edge (p, r) into G while considering any cone C_p, introduce arc (p, r) into G
- for every node q in G, let V_q be the set comprising nodes p in S having arcs (p, q) in G, remove every arc (p, q), and include the edges of q-sink $\sqrt{t}$-spanner of points $V_q \cup q$.

— proofs omitted from this presentation

Outline

Well-separated point sets

Given an n-element point set S in $\mathbb{R}^2$, and a separation factor $s > 0$, two disjoint sets $A \subseteq S$ and $B \subseteq S$ are *s-well separated* if the sets A and B can be enclosed in two Euclidean balls of radius r such that the closest distance between these balls is at least sr.

Well-separated pair decomposition

Given a point set S and a separation factor $s > 0$, a *well-separated pair decomposition (WSPD)* is a collection of pairs of subsets of S, denoted $\{\{A_1, B_1\}, \ldots, \{A_m, B_m\}\}$, such that

1. $\forall_i A_i, B_i \subseteq S$, for $1 \leq i \leq m$,
2. $\forall_i A_i \cap B_i = \phi$, for $1 \leq i \leq m$,
3. $\bigcup_i A_i \bigotimes B_i = S \bigotimes S = \{\{x, y\} | x \in S, y \in S, x \neq y\}$, and
4. $\forall_i A_i, B_i$ are s-well separated, for $1 \leq i \leq m$.

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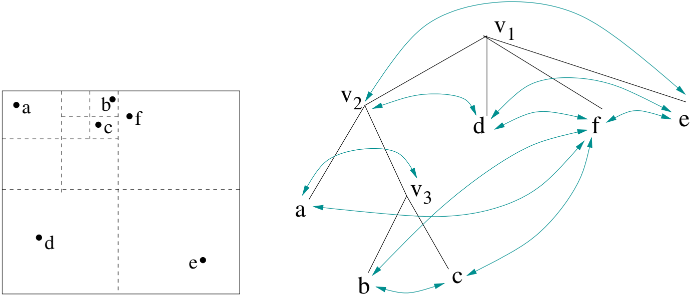
4. $\forall_i A_i, B_i$ are s-well separated, for $1 \leq i \leq m$.

It is immediate there exists a WSPD of size $O(n^2)$ by setting the $\{A_i, B_i\}$ pairs to each of the distinct pair singletons of S; however, the goal is to compute a s-WSPD of size $O(s^2 n)$.

Compressed quadtree for computing a WSPD



(a) point set (b) WSPD with $\epsilon = \frac{1}{2}$

- Each set $\{A_i, B_i\}$ of the pair decomposition is encoded as a pair of nodes $\{u, v\}$ in the quadtree. Implicitly, this pair represents the pairs $S_u \bigotimes S_v$. Here, S_u (resp. S_v) is the set of points stored in the subtree rooted at u (resp. v).

Computing a WSPD (cont)

compute a compressed quadtree T, and augment T: if u is a leaf node that contains a point p (resp. no point), then u's representative, $rep(u) = \{p\}$ (resp. $rep(u) = \phi$); if u is an internal node, then it must have a child v so that the subtree rooted at v has at least one point, set $rep(u) = rep(v)$

Computing a WSPD (cont)

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computeWSPD(u, v)

1. if (u and v are leaves of T and $u = v$) return
   //do not pair a leaf with itself
2. if ($rep(u)$ or $rep(v)$ is empty) return //no pairs to report
3. else if (u and v are s-well separated) return $\{\{u, v\}\}$
   //return the WSP $\{S_u, S_v\}$
4. else
5. if ($level(u) > level(v)$) swap u and v
   //so that u's cell is at least as large as v's
6. return $\bigcup_{u_i \in descendant(u)}$ computeWSPD(u_i, v)

The initial call is computeWSPD(r, r), where r is the root of T.

Proof of correctness

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- For every pair of points p', p'' of S, there exists an i such that $p' \in A_i, p'' \in B_i$, where $\{A_i, B_i\} \in W$.
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- Every unordered pair from S occurs in a *unique* pair $A_i \bigotimes B_i$.

  — homework

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- Every unordered pair from S occurs in a *unique* pair $A_i \bigotimes B_i$.

  — homework
- Including the time to build the compressed quadtree, which is $O(n \lg n)$, the time to compute the WSPD is $O((n \lg n) + s^2 n)$.

  — not presented in this lecture

Algorithm to compute a $(1+\epsilon)$-spanner

- Given a parameter $0 < \epsilon \leq 1$, compute a s-WSPD with $s = \frac{4(2+\epsilon)}{\epsilon}$ and set the representatives for its nodes.
- For every s-WSPD pair $\{S_u, S_v\}$, include an undirected edge between $rep(u), rep(v)$.

Two observations

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  - $\|x' - y'\| \leq \|x' - x\| + \|x - y\| + \|y - y'\|$

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  * $\|x' - y'\| \leq \|x' - x\| + \|x - y\| + \|y - y'\|$
    $\leq \frac{2}{s}\|x - y\| + \|x - y\| + \frac{2}{s}\|x - y\| = (1 + \frac{4}{s})\|x - y\|$.

Stretch of the spanner is $1 + \epsilon$

Proving $\delta_G(x, y) \leq (1 + \epsilon) \cdot \|x - y\|$ by induction on the number of edges of the shortest path between x and y in the spanner:

if there is no edge between x and y in G, then

$\delta_G(x, y) \leq \delta_G(x, x') + \delta_G(x', y') + \delta_G(y', y)$, where x, y respectively lie in S_u, S_v for a WSP $\{S_u, S_v\}$ with $x' = rep(u), y' = rep(v)$

$\leq \delta_G(x, x') + \|x' - y'\| + \delta_G(y', y)$, since there is an edge beween x' and y' in G

$\leq (1 + \epsilon)(\|x - x'\| + \|y - y'\|) + \|x' - y'\|$, indhyp is applied since a shortest path from x to x' (resp. y to y') is a subpath of a shortest path from x to y and hence has lesser number of edges

$\leq (1 + \epsilon)(2\frac{2}{s}\|x - y\|) + (1 + \frac{4}{s})\|x - y\|$, from the above two observations

$= (1 + \frac{4(2+\epsilon)}{s})\|x - y\|$

$= (1 + \epsilon)\|x - y\|$, substituting $s = \frac{4(2+\epsilon)}{\epsilon}$

Further, it is immediate that $\|x - y\| \leq \delta_G(x, y)$.

More analysis

- size is $O(s^2 n)$, that is, $O((\frac{4(2+\epsilon)}{\epsilon})^2 n) = O((\frac{12}{\epsilon})^2 n)$
- time to compute is $O(n \lg n + s^2 n)$, that is, $O(n \lg n + (\frac{12}{\epsilon})^2 n)$.
- degree can be made $O(\frac{1}{\epsilon^3})$ with the help of sink spanners and by choosing the representatives at each node in a specific way

Heavy path decomposition of a binary tree

- By introducing dummy nodes, transform the compressed quadtree T' into a binary tree T.
- Partition the nodes of T into n maximal chains, each containing a unique leaf, wherein for any edge (u, v) belonging to any such path with u being the parent of v, the number of leaves in T_v is greater than or equal to the number of leaves in the tree rooted at the other child of u.

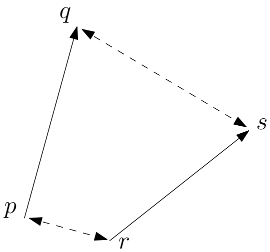
Obtaining a spanner with $(2\lg n) - 1$ diameter

In constructing a WSPD-based spanner, for every node u of T, by choosing the leaf whose chain contains u as the representative of points stored at the leaves of T_u yields a $(1+\epsilon)$-spanner with diameter $2(\lg n) - 1$:

- for any point $p \in S$ with $p \in A_i$ where (A_i, B_i) is a WSP, by induction on $|A_i|$, there is a t-spanner path of length at most $\lg |A_i|$ from p to the $rep(A_i)$
- indeed, due to heavy path decomposition of T, for a WSPD-pair (A_j, B_j) with $p \in A_j$ and $rep(A_i) \in B_j$, $rep(A_i)$ must also be the representative of B_j
- again, due to heavy path decomposition of T, $|A_j| \leq |A_i|/2$
- for any two points $p, q \in S$ with $p \in A_k$ and $q \in B_k$, the length of a $(1+\epsilon)$-spanner path between p and q is at most $(\lg |A_k|) + 1 + (\lg |B_k|)$, which is $2(\lg n) - 1$

Outline

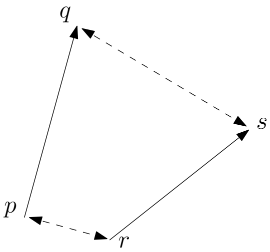
Gap property



Let $w \geq 0$ be a real number, and let E be a set of directed edges in R^d

- E satisfies *w-gap property* whenever for any two distinct edges (p, q) and (r, s) in E, we have $|pr| > w \min(|pq|, |rs|)$
- E satisfies *strong w-gap property* whenever E satisfies w-gap property together with $|qs| > w \min(|pq|, |rs|)$

Gap theorem



Let S be a set of n points in R^d, and let $E \subseteq S \times S$ be a set of directed edges that satisfy the w-gap property.

- if $w \geq 0$, then each point of S is the source of at most one edge of E
- if $w \geq 0$, and E satisfies the strong w-gap property, then each point of S is the sink of at most one edge of E.

Gap theorem (cont)

Let S be a set of n points in R^d, and let $E \subseteq S \times S$ be a set of m directed edges that satisfy the w-gap property. If $w > 0$, then
$wt(E) < (1 + \frac{2}{w}).wt(MST(S)) \lg n$.

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$wt(E) < (1 + \frac{2}{w}).wt(MST(S)) \lg n$.

- claim: E contains a subset E' of size $\frac{m}{2}$, such that $wt(E') < (\frac{2}{w})wt(MST(S))$.

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- claim: E contains a subset E' of size $\frac{m}{2}$, such that $wt(E') < (\frac{2}{w})wt(MST(S))$.
  - * number of edges of E according to the order in which their sources are visited by an optimal TSP of S; consider the portion $T_i = (p_{k_{2i-1}}, p_{k_{2i-1}} + 1, \ldots, p_{k_{2i}})$ of TSP(S) between the sources of two successive edges e_{2i-1} and e_{2i}, where $1 \leq i \leq m/2$

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  - $|p_{k_{2i-1}} p_{k_{2i}}| \leq wt(T_i)$ and
    $|p_{k_{2i-1}} p_{k_{2i}}| > w \min(|e_{2i-1}|, |e_{2i}|) \Rightarrow \min(|e_{2i-1}|, |e_{2i}|) < \frac{1}{w} \cdot wt(T_i)$

Gap theorem (cont)

Let S be a set of n points in R^d, and let $E \subseteq S \times S$ be a set of m directed edges that satisfy the w-gap property. If $w > 0$, then
$wt(E) < (1 + \frac{2}{w}).wt(MST(S)) \lg n$.

- claim: E contains a subset E' of size $\frac{m}{2}$, such that $wt(E') < (\frac{2}{w})wt(MST(S))$.
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    - $|p_{k_{2i-1}} p_{k_{2i}}| \leq wt(T_i)$ and
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    - $\sum_{i=1}^{m/2} \min(|e_{2i-1}, e_{2i}|) \leq \frac{1}{w} \cdot wt(TSP(S))$

Gap theorem (cont)

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$wt(E) < (1 + \frac{2}{w}).wt(MST(S)) \lg n$.

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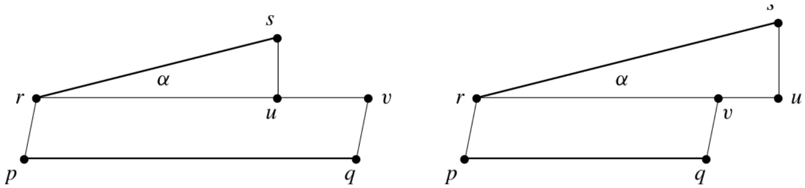
  - number of edges of E according to the order in which their sources are visited by an optimal TSP of S; consider the portion $T_i = (p_{k_{2i-1}}, p_{k_{2i-1}} + 1, \ldots, p_{k_{2i}})$ of TSP(S) between the sources of two successive edges e_{2i-1} and e_{2i}, where $1 \leq i \leq m/2$
  - $|p_{k_{2i-1}} p_{k_{2i}}| \leq wt(T_i)$ and $|p_{k_{2i-1}} p_{k_{2i}}| > w \min(|e_{2i-1}|, |e_{2i}|) \Rightarrow \min(|e_{2i-1}|, |e_{2i}|) < \frac{1}{w} \cdot wt(T_i)$
  - $\sum_{i=1}^{m/2} \min(|e_{2i-1}, e_{2i}) \leq \frac{1}{w} \cdot wt(TSP(S))$
- by induction on m

Observation



Let t, θ, and w be real numbers, such that $0 < \theta < \pi/4$, $0 \leq w < (\cos\theta - \sin\theta)/2$, and $t \geq 1/(\cos\theta - \sin\theta - 2w)$. Let p, q, r, and s be points in R^d, such that

- $p \neq q, r \neq s$,
- $angle(pq, rs) \leq \theta$, (r, s) is almost parallel to (p, q)
- $|rs| \leq |pq|/\cos\theta$, $|rs|$ is not much larger than $|pq|$
- $|pr| \leq w|rs|$. r is close to p

Then $|pr| < |pq|$, $|sq| < |pq|$, and $t|pr| + |rs| + t|sq| \leq t|pq|$. — not proved

Another observation

Let θ, w, and t be real numbers such that $0 < \theta < \pi/4$, $0 \leq w < (\cos\theta - \sin\theta)/2$, and $t \geq 1/(\cos\theta - \sin\theta - 2w)$. Let S be a set of n points in the plane, and let $G(S, E)$ be a directed graph, such that the following holds: for any two distinct points p and q of S, there is an edge $(r, s) \in E$, such that

- $angle(pq, rs) \leq \theta$
- $|rs| \leq |pq| / \cos\theta$
- $|pr| \leq w|rs|$ or $|qs| \leq w|rs|$.

Then, the graph G is a t-spanner for S.

The gap-greedy algorithm[5]

Consider all ordered pairs of distinct points in nondecreasing order of their distances. An edge (p,q) is added iff including (p,q) into the current edge set E does not make the new set to violate the w-strong gap property.

[5] from [Arya, Smid '97]

Analysis

when $0 < \theta < \pi/4$ and $0 \leq w \leq (\cos\theta - \sin\theta)/2$,

- stretch factor $1/(\cos\theta - \sin\theta - 2w)$

Analysis

when $0 < \theta < \pi/4$ and $0 \leq w \leq (\cos\theta - \sin\theta)/2$,

- stretch factor $1/(\cos\theta - \sin\theta - 2w)$
- maximum degree $\leq 2\lceil 2\pi/\theta \rceil$

Analysis

when $0 < \theta < \pi/4$ and $0 \leq w \leq (\cos\theta - \sin\theta)/2$,

- stretch factor $1/(\cos\theta - \sin\theta - 2w)$
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Analysis: optimizing parameters to minimize the weight

when $\theta = (t-1)/2$ and $w = (t-1)/4$,

- stretch factor t
- maximum degree is $O(1/(t-1))$
- weight is $O((1/(t-1)^2) \cdot wt(MST(S)) \lg n)$
- construction time $O(n^3)$ [6]

[6] a modified implementation yields $O(n(\lg n)^2)$ time.

Outline

The path-greedy algorithm[7]

Construct spanner G while onsidering pairs of points in nondecreasing order of distances: add an edge e between the considered pair (u, v) only if $d_G(u, v) > t.d(u, v)$.

[7] from [Das, Heffernan, Narasimhan '93]

Analysis

- t-spanner
- $O(n^2(m + n \lg n))$ time to compute
- node degree is $O(1/(t-1))$ — not proved: analysis is a bit involved
- $O(n)$ size with weight $O((\lg n) w(MST))$ — not proved: analysis is a bit involved

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several optimizations in constructing an approximate path-greedy spanner led to achiving a computation time of $O(n(\lg n)^2/(\lg\lg n))$ while the resulting spanner being sparse with weight $O(\frac{1}{(t-1)^4}w(MST))$ and degree $O(1/(t-1)^3)$

Outline

Comparison of t-spanners

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Comparison of t-spanners

- Parameters based on which spanners presented here are compared: stretch, time to compute, size, weight, diameter, and maximum node degree

- Θ-graph based

- WSPD based

- Greedy algorithms: gap-greedy, path-greedy $\leftarrow$ excellent features but the construction time is a bottleneck

Current research

- specialized: plane, single-source, pairwise
- amid obstacles
- Steiner
- in R^3
- expected analysis
- dynamic spanners
- kinetic spanners
- energy-efficient
- multicriteria

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Thanks!