

11. [13th Nov]

- (i) Given an undirected simple graph, provide pseudocode of a naive polynomial-time algorithm to decompose its edges into blocks.
- (ii) Show the height of any  $k$ -ary tree with  $n$  nodes is at least  $O(\lg_k n)$ .  
Also, show the height of any balanced full  $k$ -ary tree with number of leaves  $\ell$  is equal to  $\lceil \lg_k \ell \rceil$ .

10. [12th Nov]

- (i) Exer 4.2.8, 4.2.9 from [W] pg 173.
- (ii) Verify whether Menger's theorem is correct for any two diametrically opposite nodes  $x, y$  in the graph given on the right side of Exer 1.1.16 from [W] pg 16. In constructing vertex-disjoint  $x, y$ -paths using the constructive inductive proof given in class for Menger's theorem, identify where all the induction hypothesis may require to be used.
- (iii) Prove the size of a minimum cardinality  $x, y'$ -vertex-cut in  $H_1$  is equal to the size of a minimum cardinality  $x, y$ -vertex-cut in  $G$ . For the definitions of  $G$  and  $H_1$ , refer to [W] pg 167.

9. [11th Nov]

- (i) Find  $\kappa(G)$  and  $\lambda(G)$  for the following graphs: hypercube graph  $Q_3$ , biclique  $K_{m,n}$ ,  $K_m \cup K_n$ , and for Petersen graph. Verify whether Whitney's theorem holds good for each of these graphs?
- (ii) Identify the set  $T$  in Whitney's theorem's proof ([W] pg 153) for the following graphs: two graphs drawn on board in class today, and for graph  $H$  in Exer 22 from [R] pg 726. For any of these graphs, determine whether the nodes in  $T$  induce a bipartite graph.
- (iii) Show the graph resulting from contracting the three rightmost nodes in the figure under 4.1.10 of [W] pg 153 yields a multigraph. Precisely define the contraction of a subset of nodes of a graph into a supernode.

8. [6th Nov]

- (i) Write a pseudocode of Heawood's constructive proof of coloring any planar graph with 5-colors. What is its asymptotic worst-case time complexity?
- (ii) Formally write the proof (which was presented in today's lectures) to show that every planar graph is 6-colorable.  
Closely compare it with the proof given to show that every simple graph  $G$  is properly colorable with  $1 + \Delta(G)$  colors.
- (iii) Exer 17, 18 from [R] pg 761.
- (iv) Prove whether the converse of Corollary 3 on [R] pg 758 is valid.

7. [4th Nov]

- (i) Prove that every bipartite graph is a perfect graph.

- (ii) Determine whether there is a family of graphs for which  $\mathcal{X}(G) = \frac{n(G)}{|IS^{max}(G)|}$ .  
What about  $\mathcal{X}(G) > \frac{n(G)}{|IS^{max}(G)|}$ ?
  - (iii) Give a minimum  $k$  for which hypercube graph  $Q_4$  is  $k$ -colorable.
  - (iv) By following the general structure of an incremental algorithm to color vertices (5.1.12 on [W] pg 194), prove the sum of chromatic numbers of any connected undirected simple graph  $G$  and its complement is upper bounded by  $n(G) + 1$ .
6. [1st Nov]
- (i) Prove that a connected simple graph with  $2k$  odd-degreed vertices can be decomposed into  $k$  number of walks such that each of those walks has no repeated edge.
  - (ii) Exer 7.2.12 from [W] pg 295.
  - (iii) Determine whether the hypercube graph  $Q_3$  is Hamiltonian. What about  $Q_4$ ?
5. [30th Oct]
- (i) Provide counterexamples to each of those propositions from the matching theory presented in lectures that are applicable only to bipartite graphs, to prove these are not necessarily correct for general undirected simple graphs.
  - (ii) Among the parameters mentioned to check whether two graphs are isomorphic, identify the one that could be the most efficient one in determining whether the following pairs of graphs are isomorphic: Exer 38-48 from [R] pg 711-712.
  - (iii) Formally prove that every  $EC^{min}$  consists of a collection of star components of the input graph.
  - (iv) Complete the proof given in class to conclude the following: A set  $S$  of vertices is an independent dominating set iff  $S$  is maximal independent set.
4. [29th Oct]
- (i) Exer 3.1.37 from [W] pg 121.
  - (ii) Exer 3.1.1 from [W] pg 118.
  - (iii) For graphs in Exer 7 and 8 from [R] pg 739, identify  $VC^{min}$ ,  $EC^{min}$ ,  $DS^{min}$ ,  $M^{max}$ , and  $IS^{max}$  sets.
3. [28th Oct]
- (i) Find a maximum matching  $M^{max}$  in the Petersen graph, a cycle graph with  $n$  nodes, and a wheel graph with  $n$  nodes.
  - (ii) Given any two distinct maximum matchings  $M$  and  $M'$  of an undirected simple graph  $G(V, E)$ , analogous to Berge's theorem, find the structure of  $M \Delta M'$ .
  - (iii) Determine whether the Petersen graph is a bipartite graph.
2. [22nd Oct]
- (i) Provide proofs for the (in)equalities listed for Fibonacci numbers on the lecture note.
1. [21st Oct]
- (i) Exer 38, 39, 45 from [R] pg 577.