R. Inkulu

3. [21st Aug]

   (i) Prove that if S and T are infinite sets then $S \cup T$ is an infinite set.
   Also, prove if S and T are finite sets then $S \cap T$ is a finite set.

   (ii) Find the width and height of posets in Figs. 4 and 5 of [R] pg 656.
   Give a chain decomposition of these posets such that the number of chains in the decomposition is equal to their respective widths.
   Also, give an antichain decomposition of these posets such that the number of antichains in the decomposition is equal to their respective heights.

   (iii) In proving Dilworth's decomposition theorem, identify all the places in induction step where the induction hypothesis is used. Also, state the significance of separating out a maximal element (instead of an arbitrary element) of S in the induction hypothesis.

2. [20th Aug]

   (i) Determine whether using a maximal element in place of minimal element in the pseudocode given on [R] pg 660 affects its correctness.

   (ii) Find two compatible linear orderings corresponding to Hasse diagram given on [R] pg 665 Exer 66.

   (iii) Argue both the maximum element of any poset and the supremum of any subset of any poset must be unique, if they exist at all.

   (iv) Give examples for a chain, a maximal chain, and a maximum antichain of the poset $(\{1, 2, 3, 4, 6, 8, 12\}, |)$.

   (v) Exer 34 from [R] pg 663.

1. [19th Aug]

   (i) Argue that $(Z^+ \times Z^+, \leq)$ is well-ordered.

   (ii) Exer 6, 8, and 22 from [R] pg 662-663.

   (iii) Prove or disprove the following: A relation R on set S cannot be both symmetric and antisymmetric if it contains $(a, b) \in R$ in which $a \neq b$ for any $a, b \in S$.